

7. Solve for θ to the nearest hundredth, where $0 \leq \theta \leq 2\pi$.

a) $2 \cos^2 \theta + \cos \theta - 1 = 0$

b) $2 \sin^2 \theta = 1 - \sin \theta$

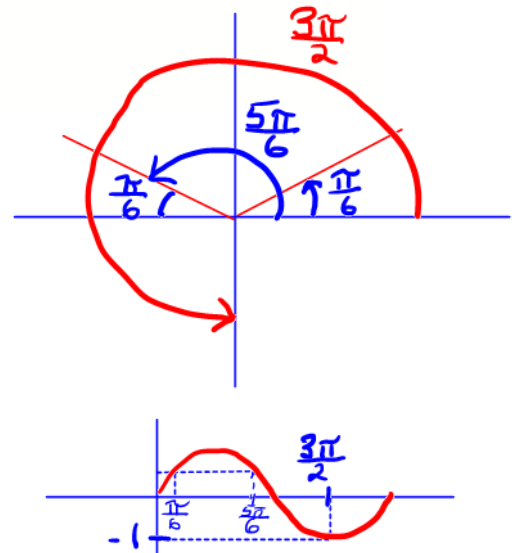
$\therefore 2 \sin^2 \theta + \sin \theta - 1 = 0$

$\therefore (2 \sin \theta - 1)(\sin \theta + 1) = 0$

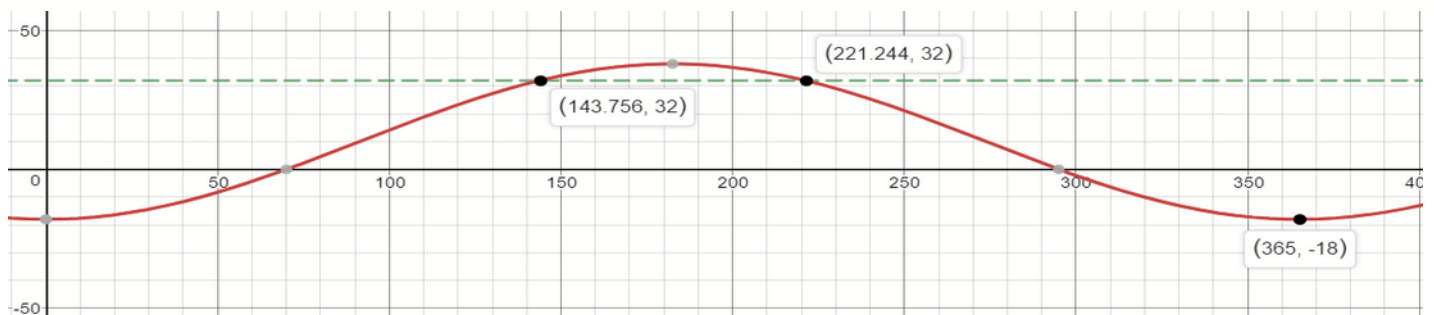
$\therefore 2 \sin \theta - 1 = 0$ or $\sin \theta + 1 = 0$

$\therefore \sin \theta = \frac{1}{2}$ or $\sin \theta = -1$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$



11. A city's daily high temperature, in degrees Celsius, can be modelled by the function $t(d) = -28 \cos \frac{2\pi}{365}d + 10$, where d is the day of the year and $1 = \text{January } 1$. On days when the temperature is approximately 32°C or above, the air conditioners at city hall are turned on. During what days of the year are the air conditioners running at city hall?



$-28 \cos\left(\frac{2\pi}{365}d\right) + 10 \geq 32$

$\therefore -28 \cos\left(\frac{2\pi}{365}d\right) \geq 22$

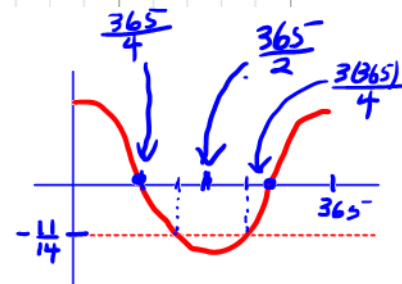
$\therefore \cos\left(\frac{2\pi}{365}d\right) \leq \frac{22}{-28} = -\frac{11}{14}$

$\therefore \cos^{-1}\left(-\frac{11}{14}\right) \leq \frac{2\pi}{365}d \leq 2\pi - \cos^{-1}\left(-\frac{11}{14}\right)$

$\therefore 2.4746 \leq \frac{2\pi}{365}d \leq 2\pi - 2.4746$

$\therefore \frac{365(2.4746)}{2\pi} \leq d \leq \frac{365(2\pi - 2.4746)}{2\pi}$

$143.8 \leq d \leq 221.2$



Extending

16. Prove $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$.

$$\text{Let } x = \frac{C+D}{2} \quad y = \frac{C-D}{2}$$

$$\text{Then } x+y = \frac{C}{2} + \frac{D}{2} + \frac{C}{2} - \frac{D}{2} = C$$

$$x-y = \frac{C}{2} + \frac{D}{2} - \left(\frac{C}{2} - \frac{D}{2} \right) = D$$

$$\begin{aligned} \therefore \text{L.S.} &= \sin C + \sin D \\ &= \sin(x+y) + \sin(x-y) \\ &= \sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y \\ &= 2 \sin x \cos y \\ &= 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right) \\ &= \text{R.S.} \end{aligned}$$