Review # 1 - Second log Graph - How to find Equation

By observing the given graph carefully, we see that it has a vertical asymptote at $x=-3$. Therefore, the given function must be a transformation of $f(x)=\log_{\frac{1}{4}}(x+3)$.

Let g represent the given function. Then

$$g(x) = A\log_{\frac{1}{4}}(x+3)+b$$

(Recall from the solution given for the first log graph that K can have any value >0. Thus, $K=1$ is the simplest choice.)

Since $(2,-1)$ and $(7,-2)$ lie on g, they must satisfy the equation of g

$$\therefore\ g(2) = A\log_{\frac{1}{4}}(2+3)+b = -1$$

$$\text{and}\ g(7) = A\log_{\frac{1}{4}}(7+3)+b = -2$$

$$\therefore\ A\log_{\frac{1}{4}}5 + b = -1 \quad ①$$

$$\text{and}\ A\log_{\frac{1}{4}}10 + b = -2 \quad ②$$

② - ①, $A\log_{\frac{1}{4}}10 - A\log_{\frac{1}{4}}5 = -1$

$$\therefore\ A(\log_{\frac{1}{4}}10 - \log_{\frac{1}{4}}5) = -1$$

$$\therefore\ A\log_{\frac{1}{4}}\frac{10}{5} = -1$$

$$\therefore\ A\log_{\frac{1}{4}}2 = -1$$

$$\rightarrow A\left(-\frac{1}{2}\right) = -1$$

$$\therefore\ A = 2$$

Substituting in ① we obtain

$$b = -1 - 2\log_{\frac{1}{4}}5$$

$$= -1\log_{\frac{1}{4}}\frac{1}{4} + (-2)\log_{\frac{1}{4}}5$$

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$$\therefore\ b = \log_{\frac{1}{4}}\left(\tfrac{1}{4}\right)^{-1} + \log_{\frac{1}{4}} 5^{-2}$$

$$= \log_{\frac{1}{4}} 4 + \log_{\frac{1}{4}}\left(\tfrac{1}{25}\right)$$

$$= \log_{\frac{1}{4}}\left[4\left(\tfrac{1}{25}\right)\right)$$

$$= \log_{\frac{1}{4}} \frac{4}{25}$$

$\therefore,\quad g(x) = 2\log_{\frac{1}{4}}(x+3) + \log_{\frac{1}{4}}\frac{4}{25}$ is

an equation of the given function

Note:

① The laws of logarithms could be used to write the equation of g in different forms

② If k were chosen to have a value of 0.1, then the equation would turn out to be

$$g(x) = 2\log_{\frac{1}{4}}(0.1(x+3)) - 2,$$

which is equivalent to the equation derived above.