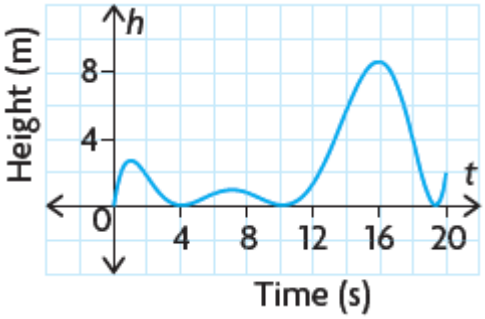
MHF4U0 Final Exam Review #3 – Rates of Change and Modelling

1. Shown at the right is a graph of a cyclist's elevation relative to her elevation above sea level at the start of a race. The graph covers the first 20 s of the race.



(a) List the intervals for which the instantaneous rate of change of height with respect to time is ***positive***. What does a positive instantaneous rate of change imply about the elevation?

(b) List the intervals for which the instantaneous rate of change of height with respect to time is ***negative***. What does a negative instantaneous rate of change imply about the elevation?

(c) Find the average rate of change of height with respect to time on the interval $[0,20]$.

(d) Estimate the instantaneous rate of change of height with respect to time at 15.5 s.

(e) Over which intervals is the rate of change of height ***speeding up***? Over which intervals is it ***slowing down***?

2. Consider the function $f(x)=x^3-4x^2+4x$.

(a) Sketch the graph of f by using its zeros and your knowledge of the end behaviour of a polynomial of odd degree.

(b) Calculate the average rate of change of $f(x)$ with respect to x over the interval $[-1,4]$. Sketch the secant line whose slope equals the rate of change that you just calculated.

(c) Estimate the instantaneous rate of change of $f(x)$ with respect to x at $x=1$. Sketch the tangent line whose slope equals the rate of change that you just calculated.

(d) Use your graph to estimate the intervals over which the instantaneous rate of change of $f(x)$ with respect to x is positive, negative and zero.

3. The following table shows the monthly average number hours of sunshine for Toronto.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Average Monthly Sunshine (h)	95.5	112.6	150.5	187.7	229.7	254.9	278.0	244.0	184.7	145.7	82.3	72.6

Source: Environment Canada

(a) Create a scatter plot of the number of hours of sunshine versus time. (Let $t=1$ represent January, $t=2$ represent February, and so on.

(b) Sketch the curve of best fit. (Do this part using graph paper.)

(c) Find a function that models the data. (Use a graphing calculator or a computer to perform an appropriate regression. If necessary, consider a few alternatives.)

(d) According to the function, when will the number of hours of sunshine reach a maximum? When will it reach a minimum?

(e) How well does your model fit the data?

4.

The population of Canada is measured on a regular basis by taking a census. The table shows the population of Canada at the end of each period. From 1851 to 1951, each period is a 10-year interval. From 1951 to 2006, each period is a five-year interval.

Period	Census Population at the End of a Period (in thousands)	Period	Census Population at the End of a Period (in thousands)
1851–1861	3 230	1951–1956	16 081
1861–1871	3 689	1956–1961	18 238
1871–1881	4 325	1961–1966	20 015
1881–1891	4 833	1966–1971	21 568
1891–1901	5 371	1971–1976	23 450
1901–1911	7 207	1976–1981	24 820
1911–1921	8 788	1981–1986	26 101
1921–1931	10 377	1986–1991	28 031
1931–1941	11 507	1991–1996	29 672
1941–1951	13 648	1996–2001	30 755
		2001–2006	31 613

Source: Statistics Canada, Demography Division

a) Use technology to investigate polynomial and exponential models for the relationship of the population and years since 1861. Describe how well each model fits the data.

b) Use each model to estimate Canada's population in 2016.

c) Which model gives the most realistic answer? Explain.

d) Use the model you chose in part c) to estimate the rate at which Canada's population was increasing in 2000.