| MHF 4U0<br>Grade 12 Advanced Functions (University Prepara<br>Unit 3 – Major Test on Polynomial Functions  | tion)          | Seme          | ster 2, 20   | 008 - 2009 |
|--|----------------|---------------|--|------------|
| Mr. N. Nolfi   | KU             | APP           | TIPS   | COM        |
| Victim:  | /7             | /25           | /18  | /10        |
| <b>1.</b> Determine an equation of the polynomial function $y = f(x)$ whose graph is shown<br>right. Express the polynomial expression in your equation in <i>factored form</i> . (4 K | n at the<br>U) | y = f(<br><-4 | x)<br>24-<br>16-<br>(x)<br>8-<br>-2<br>0<br>-2<br>-2<br>-2<br>-2<br>-2<br>-2<br>-2<br>-2<br>-2<br>-2 | 2 4        |

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2. Consider the quintic function  $p(x) = 4x^5 + 32x^2$ .

| (a) Fully factor the polynomial.<br>(3 KU) | (b) Use the factored form of the polynomial to sketch the graph of $y = p(x)$ . (3 APP) | <ul> <li>(c) Lisa took the graph of y = p(x) and applied the following transformations to produce the function g.</li> <li>1. Vertical stretch by a factor of 2.</li> <li>2. Vertical translation one unit up. How many zeros does g have? (2 TIPS)</li> </ul> |
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**3.** The following transformations are applied to the function  $y = x^4$ .

- (i) Vertical stretch by a factor of 3, reflection in the *x*-axis, vertical translation 2 units down.
- (ii) Horizontal compression by a factor of 1/4, reflection in the y-axis, horizontal translation 5 units left.

| (a) Write an equation of the transformed function. (4 APP) | (b) State the image of the point $(1,1)$ under the |
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|  | transformation. (3 APP)                            |
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4. Solve the equation  $3x^3 - 3x^2 - 7x + 5 = x^3 - 2x^2 - 1$ . (6 APP)

5. Solve the following inequalities. In both cases, express the solution both as a set and with the use of a number line.

| (a) $-4 \le -3x - 3 \le 5$ (4 APP) | <b>(b)</b> $(2x-4)^2(x+3) \le 0$ <b>(5 APP)</b> |
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6. Write an inequality that corresponds to the diagram given at the right. In addition, state the solution set. (Do not solve the inequality. You should be able to see the solution just by looking at the graphs.) (3 TIPS)



7. Explain why the polynomial equation  $2x^{10} + 13x^8 + 5x^6 + x^4 + 19x^2 + 1 = 0$  has *no real roots*. In addition, sketch a possible graph of the polynomial function  $f(x) = 2x^{10} + 13x^8 + 5x^6 + x^4 + 19x^2 + 1$ . (4 TIPS)

8. The function  $f(x) = kx^4 + 8x^2$  has three turning points, an absolute (global) maximum value of 8 and a zero at x = 2. Determine the value of k as well as the other zero(s) of f. Then sketch the graph of y = f(x). (4 TIPS)

9. A box that holds an expensive pen has square ends and its length is 13 cm greater than its width. If the volume of the box is 60 cm<sup>3</sup>, determine its dimensions. (5 TIPS)

## **MHF 4U0**

Semester 2, 2008 - 2009

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Grade 12 Advanced Functions (University Preparation) Unit 3 – Major Test on Rational Functions

| Mr. N. Nolfi | KU  | APP | TIPS | COM |
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| Victim:      | /14 | /22 | /17  | /10 |
|              | -   |     |      |     |

1. Match each function with its graph. (4 KU)

a) 
$$h(x) = \frac{x+4}{2x+5}$$
  
b)  $m(x) = \frac{2x-4}{x-2}$   
c)  $f(x) = \frac{3}{x-1}$   
d)  $g(x) = \frac{2x-3}{x+2}$   
a) \_\_\_\_\_ b) \_\_\_\_  
c) \_\_\_\_ d) \_\_\_\_



2. Complete the following table. (10 KU, 6 APP)

| Equation of<br>Function                | Characteris   | tics of Function                        | Graph  |
|--|---------------|---|--|
|  | Zeros:        | As $x \to \infty$ , $f(x) \to$          | 101  |
|  | y-intercept:  | As $x \to -\infty$ , $f(x) \to \_\_\_$  | 8  |
| f(x) = x+2                             | Domain:       | As $x \to 3^-$ , $f(x) \to \_\_\_$      |  |
| $\int (x)^{-} \frac{1}{x^{2} - x - 6}$ | Range:        | As $x \to 3^+$ , $f(x) \to \_$          | -10 -8 -6 -4 -2 - 2 4 6 8 10<br>-2 -         |
|  | Asymptote(s): | As $x \to -1^-$ , $f(x) \to $           |  |
|  |               | As $x \to -1^+$ , $f(x) \to $           | -10L   |
| $f(x) = \frac{-12x - 3}{6x - 6}$       | Zeros:        | As $x \to \infty$ , $f(x) \to$          |  |
|  | y-intercept:  | As $x \to -\infty$ , $f(x) \to -\infty$ | 6  |
|  | Domain:       |   |  |
| Ku —                                   | Range:        | As $x \to 1^-$ , $f(x) \to \_$          | -10 -8 -6 -4 -2 - 2 4 6 8 10<br>-2 -<br>-4 - |
| App –<br>Tips –<br>Com –               | Asymptote(s): | As $x \to 1^+$ , $f(x) \to $            | -6   |

| 3. | Sol | lve |
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4. The graph of the rational function *f* is shown at the right. If the equation of *f* has the form  $f(x) = \frac{m}{x^2 + kx + n}$ , determine the values of *k*, *m* and *n*. (5 TIPS)



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**5.** As the date of the MHF4U0 exam drew nearer, Manavjot and Niroj became increasingly nervous. To help relieve their stress, they decided to walk along a straight line on which Mr. Nolfi had marked an origin.

At the time t seconds,

Niroj's distance from the origin is given by the function  $s_N(t) = \frac{7t}{t^2 + 1}$  and Manavjot's distance from the origin is given by the function  $s_M(t) = t + \frac{5}{t+2}$ .

(a) At what time(s) do Niroj and Manavjot collide? (4 TIPS)

(b) When is Niroj closer to the origin than Manavjot? (4 TIPS)

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- 6. Three employees work at a shipping warehouse. Paco can fill an order two minutes faster than Tom but Carl fills an order one minute slower than Tom. When Tom and Paco work together they can fill an order in one minute and twenty seconds. When Paco and Carl work together, they take one minute and thirty seconds to fill an order.
  - (a) How long does each person take to fill an order? (5 APP)

(b) How long would it take all three of them working together to fill an order? (4 TIPS)

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**MHF 4U0** Semester 2, 2008 - 2009 Grade 12 Advanced Functions (University Preparation) Unit 3 - Major Test on Polynomial Functions COM leton work is thorough APP TIPS KU Mr. N. Nolfi 18/18 5/25 10 /10 Victim: 1. Determine an equation of the polynomial function y = f(x) whose graph is shown at the (3,28) right. Express the polynomial expression in your equation in factored form. (4 KU) Assume that the given polynomial function has degree 3 16 copposite and behaviours -> degree must be add) y = f(x)8 : the equation of f has the form  $f(x) = \alpha(x+4)(x+2)(x-2)$  ("zeros are -4, -2,2) -8-Since the point (3,28) lies on the curve, f(3) = 28 -16 :, a(3+4)(3+2)(3-2) = 28 24 Can also use the (0, -16) $q = \frac{28}{35} = \frac{4}{5}$ point (0, -16).  $f(\chi) = \underbrace{\#(\chi+4)(\chi+2)(\chi-2)}_{\text{Consider the quintic function } p(x) = 4x^5 + 32x^2.$ which case the equation is tound to be In f(x) = (x+4)(x+2)(x-3)(a) Fully factor the polynomial. (b) Use the factored form of the (c) Lisa took the graph of y = p(x) and polynomial to sketch the graph of (3 KU) applied the following transformations to y = p(x). (3 APP) produce the function g. 4x 5+ 32x2 50 1. Vertical stretch by a factor of 2.  $=4\chi^2(\chi^3+8)$ 2. Vertical translation one unit up. 40 How many zeros does g have? (2 TIPS) 30 The function g should have  $(\chi + 2)(\chi^2 - 2\chi + 4)$ 20 only one zero. The vertical 10 stretch has no effect on the does not factor -3 zeros because any point on the x-axis has a y-co-ordinate of zero. The vertical shift, however, -10 -20 causes the point (0,0) to 3. The following transformations are applied to the function  $y = x^4$ move up, which means that the (ii) Horizontal compression by a factor of  $\frac{2}{3}$ , reflection in the y-axis, horizontal translation 5 units left. (intersect 5 the x-axis) (a) Write an equation of the transformed function. (4 APP) (b) State the image of the point (1,1) under the transformation. (3 APP)  $y=-3(-4(x+5))^{4}-2$  $(x,y) \rightarrow (-4x-5, -3y-2)$  $((1,1) \rightarrow (-4(1) - 5, -3(1) - 2)$ :.  $Y = -768(x+5)^4 - 2$  $(1,1) \rightarrow (-\frac{21}{4},-5)$ 

4. Solve the equation  $3x^3 - 3x^2 - 7x + 5 = x^3 - 2x^2 - 1$ . (6 APP)

$$i 2x^{3} - x^{2} - 7x + 6 = 0$$

$$i (x - 1)(2x^{2} + x - 6) = 0$$

$$i (x - 1)(2x - 3)(x + 2) = 0$$

$$i x - 1 = 0 \text{ or } 2x - 3 = 0 \text{ or } x + 2 = 0$$

$$i x = 1 \text{ or } x = \frac{3}{2} \text{ or } x = -2$$

Let 
$$f(x) = 2\pi^{3} - \pi^{2} - 7\pi + 6$$
  
"."  $f(1) = 0$ ,  $\pi - 1$  must be  
a factor of  $f(\pi)$  (factor theorem)  
 $\frac{2\pi^{2} + \pi - 6}{2\pi^{3} - \pi^{2} - 7\pi + 6}$   
 $\frac{2\pi^{3} - 2\pi^{2}}{\pi^{2} - 7\pi}$   
 $\frac{\pi^{2} - 7\pi}{-6\pi + 6}$   
 $-6\pi + 6$   
 $-6\pi + 6$ 

5. Solve the following inequalities. In both cases, express the solution both as a set and with the use of a number line.

(a) 
$$-4 \le -3x - 3 \le 5$$
 (4 APP)  
(a)  $-4 \le -3x - 3 \le 5$  (4 APP)  
(b)  $(2x-4)^2(x+3) \le 0$  (5 APP)  
(c)  $-4+3 \le -3x - 3 + 3 \le 5 + 3$   
(c)  $-4+3 \le -3x - 3 + 3 \le 5 + 3$   
(c)  $-4+3 \le -3x - 3 + 3 \le 5 + 3$   
(c)  $2x-4)^2 \ge 0$  and  $x+3 \le 0$  or  $(2x+4)^2 \le 0$  and  $x+3 \ge 0$   
(c)  $2x-4)^2 \ge 0$  for all  $x \in \mathbb{R}$ ,  
(c)  $2x-4)^2 \ge 0$  for all  $x \in \mathbb{R}$ ,  
(c)  $2x-4)^2 \ge 0$  for all  $x \in \mathbb{R}$ ,  
(c)  $2x-4)^2 \ge 0$  for all  $x \in \mathbb{R}$ ,  
(c)  $2x-4)^2 \ge 0$  for all  $x \in \mathbb{R}$ ,  
(c)  $2x-4)^2 \ge 0$  and  $x+3 \ge 0$   
(c)  $2x-4)^2 \ge 0$  and  $x+3 \ge 0$   
(c)  $2x-4)^2 \ge 0$  and  $x \ge -3$   
(c)  $2x-4)^2 \ge 0$ 

6. Write an inequality that corresponds to the diagram given at the right. In v = 4648 addition, state the solution set. (Do not solve the inequality. You should be 44 able to see the solution just by looking at the graphs.) (3 TIPS) 40 May also use 36 v = 3032  $30 \leq 3(2x+4) - 2(x+1) \leq 46$ 28 24  $y = 3(2x+4) - 2(x+1)^{20}$ Solution Set  $\left\{x \in \mathbb{R} \mid 5 \le x \le 9\right\}$  or  $\left[5, 9\right]$  in interval notation 7. Explain why the polynomial equation  $2x^{10} + 13x^8 + 5x^6 + x^4 + 19x^2 + 1 = 0$  has no real roots. In addition, sketch a possible graph of the polynomial function  $f(x) = 2x^{10} + 13x^8 + 5x^6 + x^4 + 19x^2 + 1$ . (4 TIPS) Since all the coefficients are positive and all the powers are even powers of x, (0,1 2x10+13x8+5x6+x4+19x2 >0 For all xFR A possible graph of y = f(x) $2x^{10} + 13x^8 + 5x^6 + x^4 + 19x^2 + 1 \ge 1$  for all xER The graph must lie entirely  $f(x) \ge 1$  for all  $x \in \mathbb{R}$ above the x-axis, have same end behaviours and up to 9 turning points. That is, no matter what value of x is chosen, the value of f(x) must be at least 1. Therefore, the graph of cannot cross the x-axis, which means that the equation cannot have any real roots 8. The function  $f(x) = kx^4 + 8x^2$  has three turning points, an absolute (global) f(x)=-2x+8x maximum value of 8 and a zero at x = 2. Determine the value of k as well as the other zero(s) of f. Then sketch the graph of y = f(x). (4 TIPS) > f(2)=0 Since f has a zero at x=2, f(2)=0  $k(a^{4}) + 8(a^{2}) = 0$ 3 -3 0 0 Note that 16F + 32 = 0Fisan 8 EVEN function : 16K = -32  $f(-x) = -2(-x)^{4} + 8(-x)$  $=-2x^{4}+8x^{2}=f(x)$ : K=-2 is symmetric Since the order in the y-axis (also called multiplicity)  $f(x) = -2x^4 + 8x^2$ of this zero is 2  $= -2x^{2}(x^{2}-4)$ the graph looks like a = -272(x-2)(x+2) parabola in the vicinity of x=0 and is tangent . the other zeros are 0 and -2. to the x-axis.

9. A box that holds an expensive pen has square ends and its length is 13 cm greater than its width. If the volume of the box is 60 cm<sup>3</sup>, determine its dimensions. (5 TIPS)

Let V(x) represent the volume of the box  $\chi_{13} = \chi(\chi)(\chi + 13)$   $\chi_{13} = \chi(\chi)(\chi + 13) \times \square$ Since the volume of the box is 60 cm3 V(x) = 60Let f(x) = x3+13x2-60  $x^{2}(x+13) = 60$ "f(2)=0  $x^{3} + 13x^{2} = 60$ : x-2 must be a factor of f(x) (factor theorem)  $x^{3} + 13x^{2} - 60 = 0$ x2+15x+30 x-2)x3+13x2+0x-60  $(x-2)(x^2+15x+30)=0$  $\frac{\chi^3 - 2\chi^2}{15\chi^2 + 0\chi}$ 15x-30x : x-2=0 or x2+15x+30=0 +30x-60  $\chi = 2$  or  $\chi = \frac{-15 \pm \sqrt{15^2 - 4(1)(30)}}{2}$ =  $\frac{-15 \pm \sqrt{105}}{2} < 0$ 1. 30x -60 \* inadmissab Since a length cannot be negative The dimensions of the box are 2 cm x 2 cm × 15 cm . //



3. Solve.

$$\begin{array}{c} (a) \frac{-b}{b-1} = \frac{-3}{b+7} (5 \text{ APP}) \quad (M_{u} \text{ Hiply B.S.} \\ by (b-1)(b-7)) \\ (a) \frac{-b}{b-1} = \frac{-3}{b+7} (5 \text{ APP}) \quad (M_{u} \text{ Hiply B.S.} \\ by (b-1)(b-7)) \\ (b) \frac{2}{c+5} > \frac{3c}{c+10} (6 \text{ APP}) \quad (b-1)(b-7)) \\ (c+5)(c+10) \text{ because} \\ (c+5)(c+10) \text{ because}$$

4. The graph of the rational function f is shown at the right. If the equation of f has the form  

$$f(x) = \frac{m}{x^2 + kx + n}, \text{ determine the values of } k, m \text{ and } n. \text{ (5 TIPS)}$$
Since f has vertical asymptotes  $x = -1$  and  $x = 1$   

$$\therefore x^2 + kx + n = (x+1)(x-1)$$

$$\therefore x^2 + kx + n = x^2 - 1$$

$$\therefore x = 0 \text{ and } n = -1$$

$$\therefore f(x) = \frac{m}{x^2 - 1}$$
Since  $(0, -3)$  lies on the graph of f,  $f(0) = -3$   

$$\therefore \frac{m}{-1} = -3$$

$$\therefore m = 3$$
Therefore,  $k = 0, n = -1$  and  $m = 3$ 

5. As the date of the MHF4U0 exam drew nearer, Manavjot and Niroj became increasingly nervous. To help relieve their stress, they decided to walk along a straight line on which Mr. Nolfi had marked an origin.

At the time t seconds,

Niroj's distance from the origin is given by the function  $s_N(t) = \frac{7t}{t^2 + 1}$  and Manavjot's distance from the origin is given by the function  $s_M(t) = t + \frac{5}{t+2}$ .

(a) At what time(s) do Niroj and Manavjot collide? (4 TIPS)

Nirej and Maravjet collide when their distances from the origin are the same at a particular time. That is, they collide whenever
$$S_{N}(t) = S_{M}(t)$$

$$\therefore \frac{7t}{t^{2}+1} = t + \frac{5}{t+2}$$

$$\therefore 7t(t+2) = t(t+2)(t^{2}+1) + 5(t^{2}+1)$$

$$\therefore 7t^{2}+14t = t(t^{3}+2t^{2}+t+2) + 5t^{2}+5$$

$$\therefore 7t^{2}+14t = t^{4}+2t^{3}+t^{2}+2t + 5t^{2}+5$$

$$\therefore 7t^{2}+14t = t^{2}-12t + 5 = 0$$
By using a graphing calculator or graphing software,  
we find that  $t = 0.4168$  or  $t = 1.705$ . Therefore,  
Manavjet and Nirej collide at about 0.425 and 1.75. //  
(b) When is Wirej closer to the origin than Manavjot? (4 TIPS)  
Nirej is closer to the origin than Manavjot whenever  

$$\frac{7t}{t^{2}+1} < t + \frac{5}{t+2}$$
Since  $t^{2}+1 > 0$  for all  $t \in \mathbb{R}$  and  $t + 2 > 0$  for all  $t > 0$ ,  
we can multiply both sides by  $(t+2)(t^{2}+1)$  without reversing  
the inequality.  $(t+2>0)$  since negative times are maxingless in this context).

Following the same steps as in 5(a), we obtain

 $t^{4}+2t^{3}-t^{2}-12t+5<0$ .

From the graph in Sa, we can conclude that t must lie between about 0.4168s and 1.705s. Therefore, Niroj is closer to the origin than Manavjot roughly between 0.4s and 1.7s.

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6. Three employees work at a shipping warehouse. Paco can fill an order two minutes faster than Tom but Carl fills an order one minute slower than Tom. When Tom and Paco work together they can fill an order in one minute and twenty seconds. When Paco and Carl work together, they take one minute and thirty seconds to fill an order. 1 ROS

(a) How long does each person take to fill an order? (5 APP)

t -> time that it takes Tom to fill an order (in seconds)  $t-120 \rightarrow 11$  11 11 11 Paco to 11 11 11 (11 11)  $t+100 \rightarrow 11$  11 11 11 Paco to 11 11 11 (11 11)  $t+100 \rightarrow 11$  11 11 (Carl to 11 11 11 (11 11)) fraction of order completed hims alone by tous in one second when working alone fraction of order completed in one order completed in one second (by Tom and Paco together) working alone :: 80(t-120)+ 30t = t(t-120) (in one second) :: 1, 160t-9600= t2-120t -inadmissable  $t^2 - 230t + 9600 = 0$ because this would mean that it Paco (t - 240)(t - 40) = 0could fill an order : t=240 or t=40E in 40-120=-80 s Therefore, it takes Tom 4 minutes, Paco 2 minutes and Carl 5 minutes to fill an order. (b) How long would it take all three of them working together to fill an order? (4 TIPS) In one minute, Tom completes to of an order

Paco i 1 of an order Carl 11 to of an order Working together, the three of them can complete  $\frac{1}{4} + \frac{1}{2} + \frac{1}{5} = \frac{5}{20} + \frac{10}{20} + \frac{4}{20} = \frac{19}{20}$  of an order in one minute

Therefore, it takes them 20 = 1.05 minutes to complete an order when working together. //

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