MHF4U0 Review #1 Solutions

1. Simplify

(a) $3(x^2-2xy+2y^2)-5(2x^2-2xy-y^2)$
$$= 3x^2-6xy+6y^2-10x^2+10xy+5y^2$$
$$= -7x^2+4xy+11y^2$$

(b) $6(x-y)-2(2x+7y)-(3x-2y)$
$$= 6x-6y-4x-14y-3x+2y$$
$$= -x-18y$$

(c) $\dfrac{\sin^2 x}{1-\cos x}$
$$= \frac{1-\cos^2 x}{1-\cos x}$$
$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}}$$
$$= 1+\cos x$$

(d) $\cot x \csc x (\sec x - 1)$
$$= \frac{\cos x}{\sin x}\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}-1\right)$$
$$= \frac{\cancel{\cos x}}{\sin^2 x}\left(\frac{1-\cos x}{\cancel{\cos x}}\right)$$
$$= \frac{1-\cos x}{\sin^2 x}$$
$$= \frac{1-\cos x}{1-\cos^2 x} = \frac{\cancel{1-\cos x}}{\cancel{(1-\cos x)}(1+\cos x)} = \frac{1}{1+\cos x}$$

(e) $\dfrac{2x}{4x^2+6x} - \dfrac{3}{2x+3}$
$$= \frac{\cancel{2x}}{\cancel{2x}(2x+3)} - \frac{3}{2x+3}$$
$$= \frac{1}{2x+3} - \frac{3}{2x+3}$$
$$= -\frac{2}{2x+3}$$

(f) $\dfrac{a^{-3}b^2}{a^4 b^{-6}} = (a^{-3-4})(b^{2-(-6)})$
$$= a^{-7}b^8 = \frac{b^8}{a^7}$$

(g) $\dfrac{(m^{-3}n^{-2})^{-1}}{(m^{-4}n^{-6})^3} = \dfrac{m^3n^2}{m^{-12}n^{-18}} = m^{15}n^{20}$

(h) $10a^2b+5ab-15a$ cannot be simplified because there are no like terms

2. Factor

(a) $10a^2b+5ab-15a$
$$= 5a(2ab+b-3)$$

(b) $d^2+3d-10$
$$= (d+5)(d-2)$$

(c) $y^2-10y+25$
$$= (y-5)^2$$

(d) $2x^2+7x+3$
$$= (2x+1)(x+3)$$

(e) $4x^2+12x+9$
$$= 4x^2+6x+6x+9$$
$$= 2x(2x+3)+3(2x+3)$$
$$= (2x+3)(2x+3) = (2x+3)^2$$

$4(9)=36$
$6\times6=36,\ 6+6=12$
Rough work

(f) $6q^3s^7-7q^3s^6-3q^3s^5$
$$= q^3s^5(6s^2-7s-3)$$
$$= q^3s^5(6s^2-9s+2s-3)$$
$$= q^3s^5[3s(2s-3)+1(2s-3)]$$
$$= q^3s^5(2s-3)(3s+1)$$

$6(-3)=-18$
$-9(2)=-18$
$-9+2=-7$

(g) $9a^2-16$
$$= (3a)^2-4^2$$
$$= (3a-4)(3a+4)$$

(h) $9a^2b-6ab+b$
$$= b(9a^2-6a+1)$$
$$= b(3a-1)^2$$

(i) $\sin^2 x - \cos^2 x$
$$= (\sin x - \cos x)(\sin x + \cos x)$$

(j) $\cot^2 x - 4\cot x + 4$
$$= (\cot x - 2)^2$$

3. Solve

(a) $\dfrac{x}{3}+\dfrac{1}{2}=0$
$$\therefore 6\left(\frac{x}{3}+\frac{1}{2}\right)=6(0)$$
$$\therefore \frac{6}{1}\left(\frac{x}{3}\right)+\frac{6}{1}\left(\frac{1}{2}\right)=0$$
$$\therefore 2x+3=0$$
$$\therefore 2x=-3$$
$$\therefore x=-\frac{3}{2}$$

(b) $\dfrac{m+2}{2}=\dfrac{m-1}{3}$
$$\therefore 6\left(\frac{m+2}{2}\right)=6\left(\frac{m-1}{3}\right)$$
$$\therefore 3(m+2)=2(m-1)$$
$$\therefore 3m+6=2m-2$$
$$\therefore m=-8$$

(c) $1.2(10x-5)-(4x+7)=8$
$$\therefore 12x-6-4x-7=8$$
$$\therefore 8x-13=8$$
$$\therefore 8x=21$$
$$\therefore x=\frac{21}{8}$$

3(d) $2y^2+7y+3=0$

$\therefore (2y+1)(y+3)=0$

$\therefore 2y+1=0$ or $y+3=0$

$\therefore y=-\frac{1}{2}$ or $y=-3$

(e) $(q+4)(q-4) = -9(q+1)(q-1)$

$\therefore q^2-16 = -9(q^2-1)$

$\therefore q^2-16 = -9q^2+9$

$\therefore 10q^2 = 25$

$\therefore q^2 = \frac{5}{2}$

$\therefore q = \pm\sqrt{\frac{5}{2}}$

(f) $(z+5)(2z-3)=(z+3)(z+4)$

$\therefore 2z^2+7z-15 = z^2+7z+12$

$\therefore z^2 = 27$

$\therefore z = \pm\sqrt{27} = \pm 3\sqrt{3}$

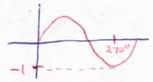
(g) $\sin^2 x + 2\sin x + 1 = 0$

$\therefore (\sin x+1)^2 = 0$

$\therefore \sin x + 1 = 0$

$\therefore \sin x = -1$

$\therefore x = 270^\circ$ (if θ is restricted to $0 \le \theta \le 360^\circ$)



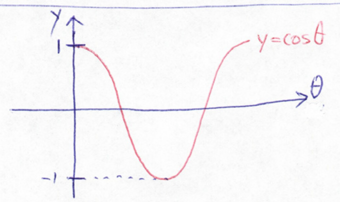
(h) $\cos^2\theta - 4\cos\theta = -4$

$\therefore \cos^2\theta - 4\cos\theta + 4 = 0$

$\therefore (\cos\theta - 2)^2 = 0$

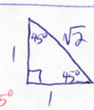
$\therefore \cos\theta - 2 = 0$

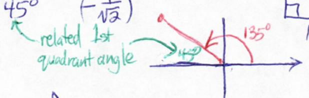
$\therefore \cos\theta = 2$, which is impossible because $-1 \le \cos\theta \le 1$ for all θ



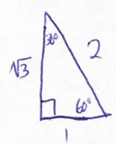
4. Evaluate

(a) $\sec 135^\circ = \frac{1}{\cos 135^\circ} = \frac{1}{-\cos 45^\circ} = \frac{1}{\left(-\frac{1}{\sqrt{2}}\right)} = -\sqrt{2}$





(b) $\tan 120^\circ = -\tan 60^\circ = -\sqrt{3}$



(c) $\sin 180^\circ = 0$ (see graph)

(d) $\tan 90^\circ$ is undefined

4(e) $64^{\frac{2}{3}}$

$=(\sqrt[3]{64})^2$

$=4^2$

$=16$

(f) $32^{\frac{11}{5}}$

$=(\sqrt[5]{32})^{11}$

$=2^{11}$

$=2048$

(g) $h(-5) = 4^{-(-5)}$

$=4^5$

$=1024$

(h) $f(135^\circ) = 5\cos^2 135^\circ - 2\sin 135^\circ + \frac{1}{\sin 135^\circ}$

$= 5\left(-\frac{1}{\sqrt{2}}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\left(\frac{1}{\sqrt{2}}\right)}$

$= \frac{5}{1}\left(\frac{1}{2}\right) - \frac{2}{\sqrt{2}} + \sqrt{2}$

$= \frac{5}{2} - \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{1}$

$= \frac{5\sqrt{2}}{2\sqrt{2}} - \frac{4}{2\sqrt{2}} + \frac{4}{2\sqrt{2}} = \frac{5\sqrt{2}}{2\sqrt{2}} = \frac{5}{2}$

(i) $g(0) = -1.3(2^{5(0)})+27$

$= -1.3(2^0)+27$

$= -1.3(1)+27$

$= -1.3+27$

$= 25.7$

(j) $t_5 = -6(2^5)$

$= -6(32)$

$= -192$

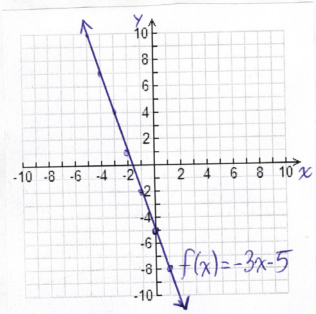
5. (a) $f(x) = -3x - 5$ (Linear Function)

slope $= m = -3$, y-intercept $= b = -5$

$b = -5 \rightarrow (0, -5)$ lies on the graph

To find another point on the graph, use the slope:

$m = \frac{\Delta y}{\Delta x} = \frac{-3}{1}$ ← 3 down, ← 1 to the right or $\frac{\Delta y}{\Delta x} = \frac{3}{-1}$ ← 3 up, ← 1 left



(b) $g(x) = -3x^2 - 5x + 1$

$\therefore g(x) = -3(x^2 + \frac{5}{3}x) + 1$

$\therefore g(x) = -3(x^2 + \frac{5}{3}x + (\frac{5}{6})^2 - (\frac{5}{6})^2) + 1$

$\therefore g(x) = -3(x^2 + \frac{5}{3}x + (\frac{5}{6})^2) + 3(\frac{5}{6})^2 + 1$

$\therefore g(x) = -3(x + \frac{5}{6})^2 + \frac{3}{1}(\frac{25}{36}) + 1$

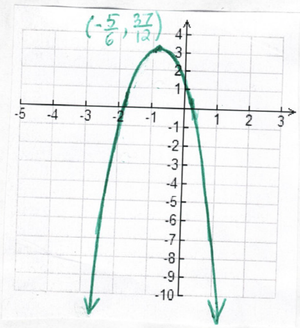
$\therefore g(x) = -3(x + \frac{5}{6})^2 + \frac{25}{12} + \frac{12}{12}$

$\therefore g(x) = -3(x + \frac{5}{6})^2 + \frac{37}{12}$

opens downward

vertical stretch by a factor of 3

Vertex: $(-\frac{5}{6}, \frac{37}{12})$



5(c) $h(\theta) = 1.5\sin(2(\theta - 45°)) + 1$

A = Amplitude = 1.5 (stretch vertically by factor of 1.5)

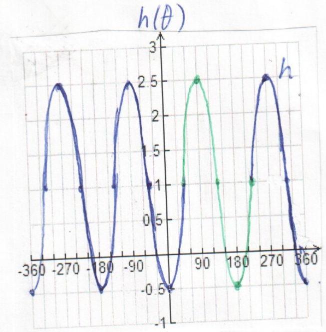
d = vertical displacement = +1 (shift up by 1 unit)

T = period = $360°(\frac{1}{2}) = 180°$ (360° x horizontal compression factor)

p = phase shift = +45° = (45° shift to the right)

Parent / Base / Mother function: $f(\theta) = \sin\theta$

Pre-image	Image
(x, y)	$\rightarrow (\frac{1}{2}x + 45°, 1.5y + 1)$
$(0, 0)$	$\rightarrow (45°, 1)$
$(90°, 1)$	$\rightarrow (90°, 2.5)$
$(180°, 0)$	$\rightarrow (135°, 1)$
$(270°, -1)$	$\rightarrow (180°, -0.5)$
$(360°, 0)$	$\rightarrow (225°, 1)$



(d) $p(z) = -3(2^{5(z-1)}) + 7$

Base Function: $p(z) = 2^z$

Horizontal Transformations	Vertical Transformations
1. Compress by a factor of $\frac{1}{5}$	1. Stretch by a factor of -3 (stretch by 3, reflect in x-axis)
2. Translate 1 unit right	2. Translate 7 units up

$(x, y) \rightarrow (\frac{1}{5}x + 1, -3y + 7)$

$(0, 1) \rightarrow (1, 4)$

$(1, 2) \rightarrow (\frac{6}{5}, 1)$

$(2, 4) \rightarrow (\frac{7}{5}, -5)$

$(3, 8) \rightarrow (\frac{8}{5}, -17)$

$(4, 16) \rightarrow (\frac{9}{5}, -41)$

$(-1, \frac{1}{2}) \rightarrow (\frac{4}{5}, \frac{11}{2})$

$(-2, \frac{1}{4}) \rightarrow (\frac{3}{5}, \frac{25}{4})$

$(5, 32) \rightarrow (2, -89)$