MHF4U0 – Review # 2 – Going beyond the Mechanical

1. Several equations are given below.

(a) $x^2-3x-22=4(x-1)$ (b) $f(x)=ax^2+bx+c$ (c) $t_n=4\left(2^{n-1}\right)$ (d) $x^2+y^2=1$

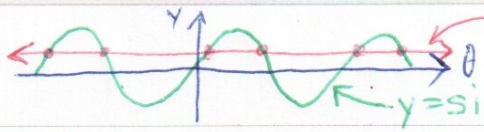
(e) $\sin x=\frac{1}{2}$ (f) $y=2x-1$ (g) $f(n)=a+(n-1)d$ (h) $\cos^2 x+\sin^2 x=1$

(i) $(x+y)^2=x^2+2xy+y^2$ (j) $c^2=a^2+b^2$ (k) $1+\tan^2 x=\sec^2 x$ (l) $x^2+x=1$

Circle the letters in the table that correspond to the equations that match the type given in the heading of each column.

Equations that are Solved to find the Value(s) of the Unknown				*Equations that Describe a Relationship between two or more Quantities*				*Identities*			
(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)	(e)	(f)	(g)	(h)	(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)	(i)	(j)	(k)	(l)	(i)	(j)	(k)	(l)

2. The following table contains a series of mathematical statements, some of which contain terminological and/or notational errors. Give a geometric interpretation of each statement that does not contain any errors. Suggest corrections for the statements that do contain errors.

Statement(s)	*Give a geometric interpretation of the correct statements. Suggest possible corrections of the statements that contain errors.*
$\sin\theta=\frac{1}{2}$ ✓	 $y=\frac{1}{2}$ The solutions to the equation are found at the points of intersection.
The *factored* form of x^2-5x+6 is $\left(x-\frac{5}{2}\right)^2-\frac{1}{4}$ X	The factored form of x^2-5x+6 is $(x-2)(x-3)$ The given form is the vertex form
$2^x=-1$ $\therefore x$ is *impossible* X	Since $2^x>0$ for all $x\in\mathbb{R}$, there are NO SOLUTIONS ✓ to the equation $2^x=-1$
$\frac{6x^2+17x+12}{2x^2+11x+12}=\frac{3+17+1}{1+11+1}=\frac{21}{13}$	$\frac{6x^2+17x+12}{2x^2+11x+12}=\frac{(3x+4)(2x+3)}{(x+4)(2x+3)}=\frac{3x+4}{x+4}$ When both the top and bottom are in factored form, the top and bottom can be divided by a common factor
The graph of $f(x)=\sec x$ is called a *parabola* because it consists of "u-shaped" sections.	ONLY the graphs of QUADRATIC functions are called PARABOLAS.
When the *equation* X $x^2y^3-5x^2y^3+3x^3-5a^2-7x^3$ is X *evaluated*, the result is $-4x^2y^3-4x^3-5a^2$ However, $5a^2-7b^2$ cannot be *evaluated*. X	equation → EXPRESSION ✓ evaluated → SIMPLIFIED ✓

3. State whether each of the following is true or false. Provide an explanation to support each response.

Statement	*True or False?*	*Explanation*
If $f(x)=2^x$ then $f(x+y)=f(x)+f(y)$	False	Let $x=y=2$. Then, L.S. $= f(x+y)=f(2+2)=f(4)=2^4=16$ R.S. $=f(x)+f(y)=f(2)+f(2)=2^2+2^2=8$ $\therefore$ L.S. $\neq$ R.S.
$t_n=t_{n-1}+t_{n-2}$ $\therefore nt=(n-1)t+(n-2)t$	False	$t_n=t_{n-1}+t_{n-2}$ means "term n of a sequence is found by adding terms n-1 and n-2" Also note that $(n-1)t+(n-2)t=(n-1+n-2)t=(2n-3)t \neq nt$
$125^{\frac{1}{3}}=(1/3)125=\frac{125}{3}$	False	$125^{\frac{1}{3}}=\sqrt[3]{125}=5 \neq \frac{125}{3}$
For all functions f, $f^{-1}(x)=\frac{1}{f(x)}$	False	$f^{-1}(x)$ is the inverse of the function $f(x)$ $\frac{1}{f(x)}$ is the reciprocal of the function $f(x)$
The quadratic expression $253x^2-72x-493$ *can be* factored over the integers. (DO NOT TRY TO FACTOR! JUST EXPLAIN WHETHER IT IS POSSIBLE TO FACTOR!)	True	A quadratic expression ax^2+bx+c factors over the integers if and only if the discriminant b^2-4ac is a perfect square. $b^2-4ac=(-72)^2-4(253)(-493)=504100=710^2$ Since b^2-4ac is a perfect square, the given quadratic factors over the integers.

4. Although we often think of radioactive substances as dangerous and harmful, they also have many useful applications. Consider the following examples.

- The radioactive substance carbon-14 can be used to estimate the age of fossils that are up to about 40000 years old.
- Sodium-24, another radioactive substance, is used as a tracer in medical diagnostics. For example, it can be used to measure the rate of flow of blood in an artery or a vein.

Since radioactive substances are unstable, they decay (break down) over time. For instance, if a 1 kg sample of sodium-24 were left undisturbed, after 14.9 hours only about 500 g of it would be left. This process of decreasing mass over time due to the emission of radiation is called *radioactive decay*.

The table at the right lists measurements made to determine the rate of decay of sodium-24.

Time (hours)	*Mass (kg)*
0	0.1
2	0.0911
4	0.0830
6	0.0756
8	0.0689
10	0.0628
12	0.0572
14	0.0521
16	0.0475
18	0.0433
20	0.03944
22	0.03594
24	0.03274

(a) Use regression to find a function that fits the data well. (**Hint:** We have only used quadratic, sinusoidal and exponential regressions. Think of which of these three best fits this situation. If you are unsure, experiment with each type of regression until you find the most logical fit.)

Only an exponential function makes sense. TI-Interactive gives the equation $m(t)=0.099968(0.954557^t)$

(b) Now use your equation to predict how much of the sample would be left after 14 *days*.

14 days = 14(24) hours = 336 hours

$m(336)=0.099968(0.954557^{336})=0.000000016$ kg

Only 0.000000016 kg of the sodium-24 would remain.