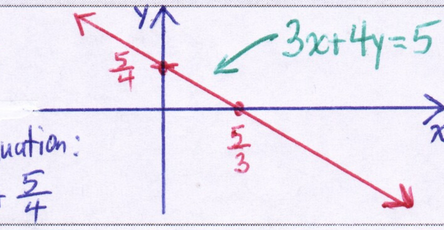
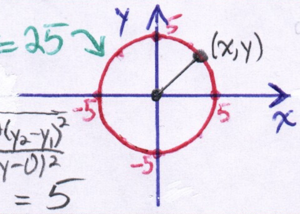
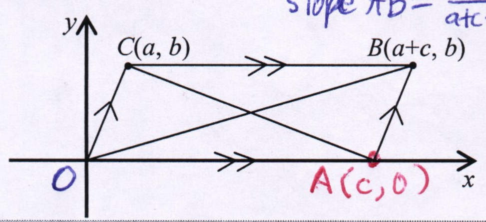
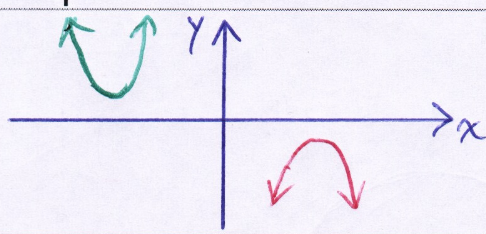
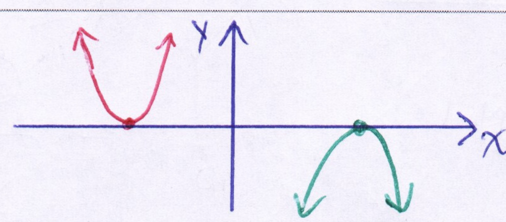
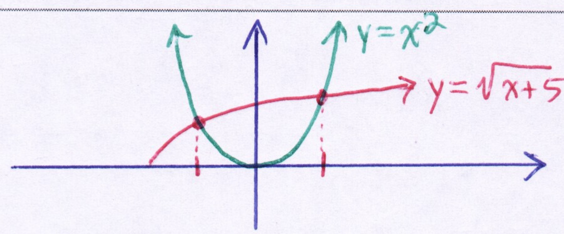
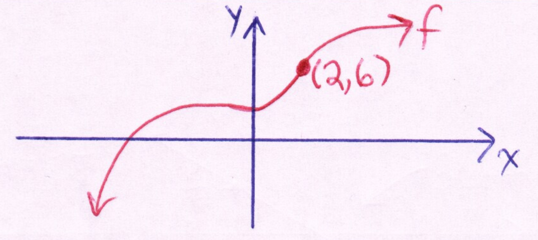
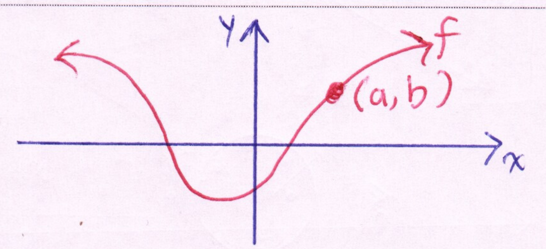
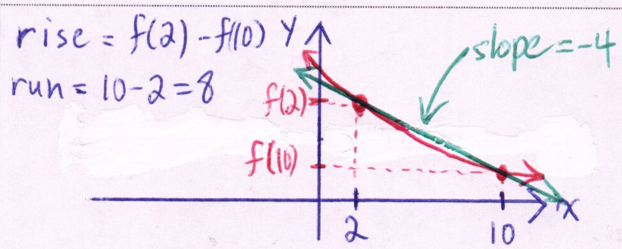
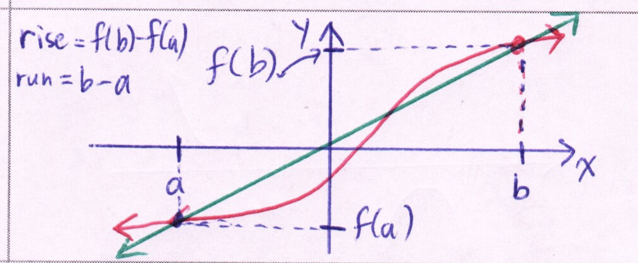
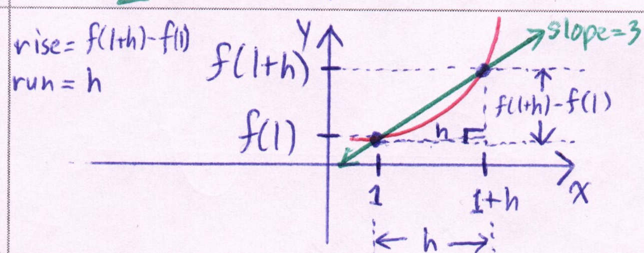
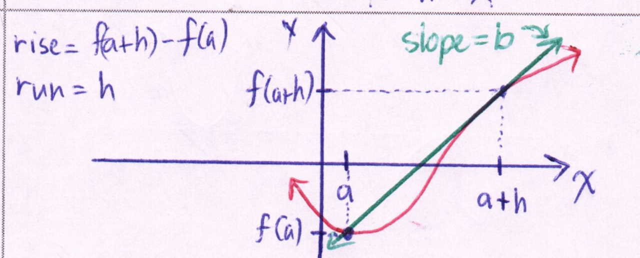
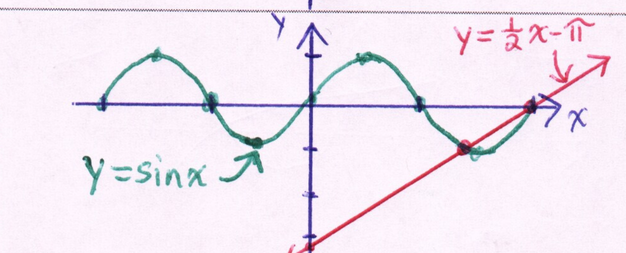
MHF4U0 – Review # 4 – The Language of Mathematics

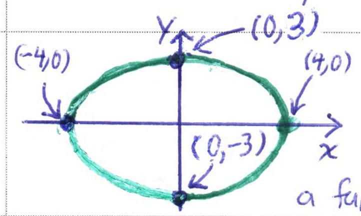
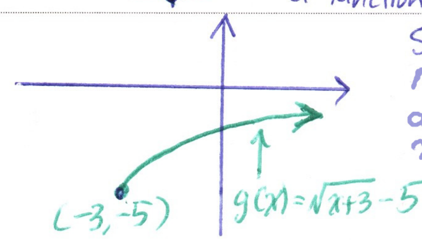
1. Complete the tables given below. This kind of exercise will help you to develop the ability to associate mathematical symbols, expressions and equations with concrete ideas.

Expression, Equation or Inequality	*Diagram*	*Conclusion, Interpretation or Explanation*
$3x+4y=5$	slope $= -\frac{3}{4}$ $y=mx+b$ form of equation: $y=-\frac{3}{4}x+\frac{5}{4}$  	• x and y are linearly related • the sum of triple x and quadruple y is 5
$x^2+y^2=25$	 $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(x-0)^2+(y-0)^2}$ $=\sqrt{x^2+y^2}=5$	The distance squared from $(0,0)$ to (x,y) is 25. Therefore, the distance from $(0,0)$ to (x,y) is 5. The equation describes a circle of radius 5 with centre at the origin.
There are many possibilities here e.g. An equation of the line through AB is $y=\frac{b}{a}x-\frac{bc}{a}$	slope $AB=\frac{b-0}{a+c-c}=\frac{b}{a}$  	There are many possibilities e.g. OABC is a parallelogram etc.
$b^2-4ac<0$	 	The quadratic equation $ax^2+bx+c=0$ has no real roots. Therefore, the graph of $f(x)=ax^2+bx+c$ does not cross the x-axis.
$b^2-4ac>0$	 	The quadratic equation $ax^2+bx+c=0$ has two distinct real roots. Therefore, the graph of $f(x)=ax^2+bx+c$ crosses the x-axis twice.
$b^2-4ac=0$	 	The quadratic equation $ax^2+bx+c=0$ has one real root. Therefore, the graph of $f(x)=ax^2+bx+c$ intersects the x-axis at exactly one point.
$\sqrt{x+5}=x^2$	 	The graph of $y=x^2$ intersects the graph of $y=\sqrt{x+5}$ at exactly two points. Therefore, the equation $\sqrt{x+5}=x^2$ has exactly two solutions.

<table>
<tr><th>Expression, Equation or Inequality</th><th>Diagram</th><th>Conclusion, Interpretation or Explanation</th></tr>
<tr><td>$f(2)=6$</td><td>
</td><td>The point (2,6) lies on the graph of f.</td></tr>
<tr><td>$f(a)=b$</td><td>
</td><td>The point (a,b) lies on the graph of f.</td></tr>
<tr><td>$\frac{f(10)-f(2)}{10-2}=-4$</td><td>
</td><td>The slope of the secant line passing through (2, f(2)) and (10, f(10)) is -4.</td></tr>
<tr><td>$\frac{f(b)-f(a)}{b-a}=c$</td><td>
</td><td>The slope of the secant line passing through (a, f(a)) and (b, f(b)) is c.</td></tr>
<tr><td>$\frac{f(1+h)-f(1)}{h}=3$</td><td>
</td><td>The slope of the secant line passing through (1, f(1)) and (1+h, f(1+h)) is 3.</td></tr>
<tr><td>$\frac{f(a+h)-f(a)}{h}=b$</td><td>
</td><td>The slope of the secant line passing through (a, f(a)) and (a+h, f(a+h)) is b.</td></tr>
<tr><td>$\sin x=\frac{1}{2}x-\pi$</td><td>
</td><td>The graph of y = sin x intersects the graph of y = ½x - π at exactly two points. Therefore, the equation sin x = ½x - π has exactly two solutions.</td></tr>
</table>

2. State whether each of the following is true or false. Provide an explanation to support each response. Keep the following points in mind:

- If a mathematical statement is said to be *true*, it must be true in *all possible cases*.
- A *general proof* is required to demonstrate that a statement is *true*. The proof must demonstrate the truth of the statement in all possible cases! Clearly, any number of examples cannot accomplish this goal.
- To demonstrate that a statement is *false*, it is only necessary to produce a *single example* that contradicts the statement. Such an example is called a *counterexample*.

Statement	*True or False?*	*Proof, Counterexample or Explanation*
$(a+b)^2 = a^2 + b^2$	F	Suppose that $a=b=1$. Then, $\left.\begin{array}{l} L.S. = (1+1)^2 = 2^2 = 4 \\ R.S. = 1^2+1^2 = 1+1 = 2 \end{array}\right\} \therefore L.S. \neq R.S.$ $(a+b)^2 = a^2+2ab+b^2$ is the correct equation
$\sqrt{a+b} = \sqrt{a}+\sqrt{b}$	F	Suppose that $a=16$, $b=9$. Then, $\left.\begin{array}{l} L.S. = \sqrt{16+9} = \sqrt{25} = 5 \\ R.S. = \sqrt{16}+\sqrt{9} = 4+3 = 7 \end{array}\right\} \therefore L.S. \neq R.S.$
For all functions f and all real numbers u and c, $f(u+c) = f(u) + f(c)$	F	Let $f(x) = x^2$, $u=c=1$. Then, $\left.\begin{array}{l} L.S. = f(1+1) = f(2) = 2^2 = 4 \\ R.S. = f(1)+f(1) = 1^2+1^2 = 2 \end{array}\right\} \therefore L.S. \neq R.S.$
The slope of the line $3x+4y-6=0$ is 3.	F	When expressed in $y=mx+b$ form, the equation becomes $y = -\frac{3}{4}x + \frac{3}{2}$. Therefore, the slope is $-\frac{3}{4}$, not 3.
The equation $\frac{x^2}{16}+\frac{y^2}{9}=1$ describes a function.	F	 The given equation describes an ELLIPSE, which is clearly not a function (fails vertical line test).
For the function $g(x) = \sqrt{x+3} - 5$, $D = \{x \in \mathbb{R} : x \geq -3\}$ and $R = \{y \in \mathbb{R} : y \geq -5\}$. (Here D and R represent domain and range respectively.)	T	 Since square roots of negative numbers are not real #'s, $x+3 \geq 0 \rightarrow x \geq -3$. Since $\sqrt{x+3} \geq 0$, $g(x) \geq -5$.
Suppose that $g(x) = -3f(2x-8)+6$. To obtain the graph of g, the following transformations must be performed to f: • Vertical stretch by a factor of -3 followed by a shift up by 6 units • Horizontal compression by a factor of $1/2$ followed by a shift 8 units right.	F	The given answer would be correct if the expression within the parentheses were given in factored form (i.e. $2(x-8)$). Since $2x-8$ is NOT in factored form, the correct way to reverse the operations is to add 8, then divide by 2. The transformations should be 1. Shift 8 right 2. Compress horizontally by factor of $\frac{1}{2}$