REVIEW OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

1. Label the following.

$$\underset{\text{Power}}{y} = \underset{\text{Base}}{a}^{\overset{\text{Exponent}}{x}}$$

$$\underset{\text{Exponent}}{y} = \overset{\text{Name of Function}}{\log}_{\underset{\text{Base}}{a}} \overset{\text{Power}}{x}$$

2. Complete the following table.

Exponential Form	*Logarithmic Form*
$10^6 = 1000000$	$\log 1000000 = 6$
$3^{-4} = \frac{1}{81}$	$\log_3 \frac{1}{81} = -4$
$y = 6^x$	$\log_6 y = x$
$4^y = x$	$y = \log_4 x$
$a = b^c$	$\log_b a = c$
$n^m = p$	$m = \log_n p$

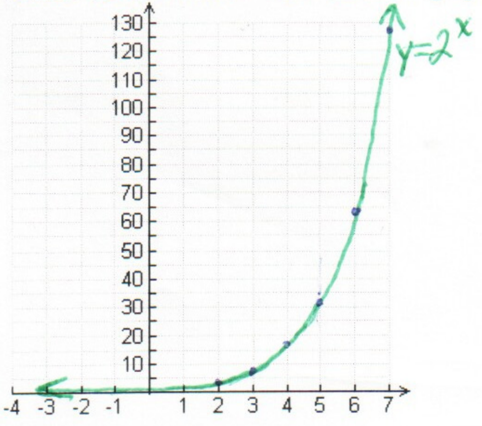
3. In the equation $y = \log_a x$, what is the *meaning* of $\log_a$? Does it represent a number? If not, what does it represent?

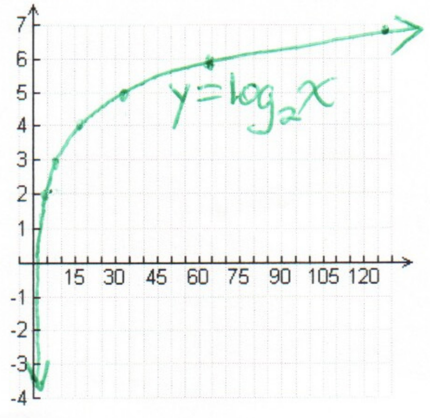
By itself, "$\log_a$" is simply the NAME of the the logarithmic function to the base a.

4. Explain the *meaning* of the expression $\log_n p$.

As a whole, "$\log_n p$" means "the exponent to which n must be raised to obtain p."

5. Use the provided grids to sketch the graphs of the *functions* $f(x) = 2^x$ and $g(x) = \log_2 x$. How are the two functions related to each other? How are their graphs related





The relationship between *f* and *g* is that f and g are inverses of each other.

The relationship between the graphs of *f* and *g* is that they are reflections of each other in the line $y = x$

6. After consuming 16 energy drinks and 22 hamburgers, Andrew decided that he had enough energy to tackle his math homework. The excessive food and drink made Andrew so hyper that he hurried through his work without giving it much thought. The following are samples of his work. Has Andrew applied valid mathematical reasoning? Explain.

$$\cancel{\frac{\log_{10}x}{\log_7 x}=\frac{\log_{10}\cancel{x}}{\log_7\cancel{x}}=\frac{\log_{10}}{\log_7}} \qquad \cancel{\frac{\log_{10}x}{\log_5 x}=\frac{\cancel{\log_{10}}\cancel{x}}{\cancel{\log_5}\cancel{x}}=\frac{10}{5}=2}$$

Obviously, Andrew's reasoning is invalid. All that is allowed is either to multiply or divide BOTH numerator and denominator by the same value! (Notice that he treats $\log_{10}$ as if it were a number.)

7. Suppose that $f(x)=2^x$ and $g(x)=\log_2 x$. Use the provided grids to sketch the graphs of $p(x)=-1.5g(-2(x-3))+1$ and $q(x)=\frac{1}{2}f\left(\frac{1}{3}x+\frac{4}{3}\right)+5$

(a) Write equations of p and q without using the symbols f and g.

$p(x)=-1.5\log_2(-2(x-3))+1$ $\qquad$ $q(x)=\frac{1}{2}\left(2^{\boxed{\frac{1}{3}x+\frac{4}{3}}}\right)+5$

→ 1. Shift left $\frac{4}{3}$ units
2. Stretch by factor of 3

→ OR, this can be rewritten as $\frac{1}{3}(x+4)$ → 1. Stretch by 3
2. Shift left 4 units

(b) State the transformations required to obtain q from f and p from g.

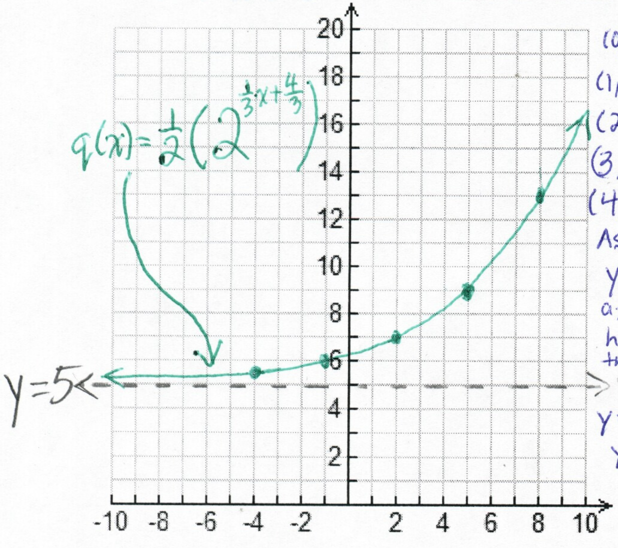
$g\to p$		$f\to q$	
Horizontal	*Vertical*	*Horizontal*	*Vertical*
1. Compress by a factor of $\frac{1}{2}$, reflect in y-axis 2. Translate 3 units to the right	1. Stretch by a factor of 1.5, reflect in x-axis 2. Translate 1 unit up.	1. Shift left $\frac{4}{3}$ units 2. Stretch by a factor of 3 OR 1. Stretch by a factor of 3 2. Shift 4 units left	1. Compress by a factor of $\frac{1}{2}$ 2. Translate 5 units up

$3(x-\frac{4}{3})$ alternative answer

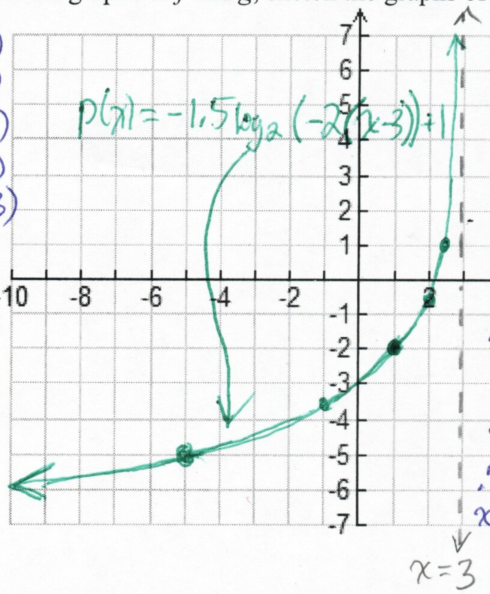
(c) Now express both transformations in *mapping notation*.

$g\to p$ $\qquad (x,y)\to\left(-\frac{1}{2}x+3,\ -1.5y+1\right)$ $\qquad f\to q$ $\qquad (x,y)\to\left(3x-4,\ \frac{1}{2}y+5\right)$

(d) Finally, by applying the transformations to a few key points on the graphs of f and g, sketch the graphs of q and p.



$(0,1)\to(-4,\frac{11}{2})$
$(1,2)\to(-1,6)$
$(2,4)\to(2,7)$
$(3,8)\to(5,9)$
$(4,16)\to(8,13)$
Asymptote
y=0 → not affected by horizontal transformations
y=0 →
$y=\frac{1}{2}(0)+5$
∴ y=5



$(1,0)\to(\frac{5}{2},1)$
$(2,1)\to(2,-\frac{1}{2})$
$(4,2)\to(1,-2)$
$(8,3)\to(-1,-3.5)$
$(16,4)\to(-5,-5)$
$(128,7)$
$\to(-61,-9.5)$
Asymptote
x=0 not affected by vertical transf.
∴ x=0 →
$x=-\frac{1}{2}(0)+3$
∴ x=3

8. Complete the following table. Remember that a counterexample is sufficient to demonstrate that a statement is false. However, a general proof is required to demonstrate that a statement is true. Also recall that when the base of a logarithm is omitted, it is usually assumed to mean "log to the base 10."

Statement	*True or False?*	*Proof, Counterexample or Explanation*
$\log 5b^2 = 2\log 5b$	F	$\log 5b^2 = \log 5 + \log b^2$ $= \log 5 + 2\log b$
$\log 3x^2 = \log 3x + \log x$	T	$\log 3x + \log x$ $= \log[3x(x)]$ (law 1) $= \log 3x^2$ (simplification)
graph each of these using calculator or program to see difference The graphs of $y = \log 3x^2$ and $y = \log 3x + \log x$ are identical.	F	Since $3x^2 \geq 0$ for all $x \in \mathbb{R}$, the domain of $y=3x^2$ is $\{x\in\mathbb{R} \mid x\neq 0\}$ while the domain of $y=\log 3x+\log x$ is $\{x\in\mathbb{R} \mid x>0\}$
To obtain the graph of $y=\log_a\sqrt{x}$, compress the graph of $y=\log_a x$ vertically by a factor of $\frac{1}{2}$.	T	$y=\log_a\sqrt{x} = \log_a x^{\frac{1}{2}} = \frac{1}{2}\log_a x$ $\therefore$ $y = \frac{1}{2}\log_a x$, which is a vertical compression by a factor of $\frac{1}{2}$
$\log_a a^{n+1} = n+1$	T	$\log_a a^{n+1}$ means "To what exponent must a be raised to obtain a^{n+1}." Obviously, the exponent must be n+1.
$a^{3\log_a(5b)} = 125b^3$	T	$a^{3\log_a(5b)} = a^{\log_a(5b)^3}$ ← The exponent to which a must be raised to obtain $(5b)^3$ $= (5b)^3$ $= 125b^3$
$\log\frac{x}{10} = \frac{\log x}{\log 10} = \frac{\log x}{1} = \log x$	F	$\log\frac{x}{10} = \log x - \log 10$ $= \log x - 1$
To obtain the graph of $y=\log\frac{x}{10}$, translate the graph of $y=\log x$ down 1 unit.	T	$y=\log\frac{x}{10} = \log x - \log 10 = \log x - 1$ $\therefore$ $y = \log x - 1$ $\therefore$ the graph of $y=\log\frac{x}{10}$ can be obtained by shifting the graph of $y=\log x$ DOWN 1 unit.

9. Why is it not possible to evaluate the logarithm of zero or a negative number? Give examples to illustrate your answer.

Suppose that $y = \log_a 0$

Then $a^y = 0$. The only way this can happen is if $a = 0$. However, the base $a = 0$ is not allowed for exponential functions.

$\therefore a^y \neq 0$ for any $y \in \mathbb{R}$

Let $x < 0$ (i.e. x is a negative number) and suppose that $y = \log_a x$.

Then $a^y = x < 0$. But for all $y \in \mathbb{R}$, $a^y > 0$ since $a > 0$. Therefore, $\log_a x$ is undefined for $x < 0$.

e.g. $\log_2(-32) = y$ means $2^y = -32$, which is impossible

10. Does it make sense to write expressions such as $\log_{-2}(-32)$? Explain.

Recall that for all exponential functions $f(x) = a^x$, $a > 0$ and $a \neq 1$. Since logarithmic functions are the inverses of exponential functions, the same restriction must apply to them. If a were allowed to be negative, the resulting functions would be very badly behaved, just like functions of the form $f(x) = a^x$ where $a < 0$

11. You are given a solution of hydrochloric acid with a pH of 1.7 and are asked to increase its pH by 1.4.

(a) Determine the factor by which you would need to dilute the solution.

$pH = -\log[H^+]$

If $pH = 1.7$, then

$1.7 = -\log[H^+]$

$\therefore \log[H^+] = -1.7$

$\therefore [H^+] = 10^{-1.7}$ mol/L

If $pH = 1.7 + 1.4 = 3.1$, then

$\log[H^+] = -3.1$

$\therefore [H^+] = 10^{-3.1}$ mol/L

$\therefore$ dilution factor $= \dfrac{10^{-1.7}}{10^{-3.1}} = 10^{-1.7-(-3.1)} = 10^{1.4} \doteq 25.1$

(b) If the solution originally had a pH of 2.2 and you were asked to increase its pH by 1.4, would you dilute by the same factor that you calculated in (a)? Explain.

$2.2 + 1.4 = 3.6$

$\dfrac{10^{-2.2}}{10^{-3.6}} = 10^{1.4}$

The dilution factor is the same.

If the original pH were x and it had to be raised by y, then

dilution factor $= \dfrac{10^{-x}}{10^{-x-y}} = 10^{-x-(-x-y)} = 10^y$

If the pH is to be raised by y, the dilution factor is 10^y.