MHF 4U0 **Semester 1, 2008 - 2009**

Grade 12 Advanced Functions (University Preparation)
Rates of Change of Trigonometric Functions – Performance Task

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Victim: Mr. Solutions Great work Mr. S. !!

KU	APP	TIPS	COM
9 /9	9 /9	12 /12	10 /10

Consider the data given in the following table for the number of hours of daylight in 2009 for Central Peel Secondary School.
Source: United States Naval Observatory (USNO) http://aa.usno.navy.mil/data/docs/RS_OneYear.php

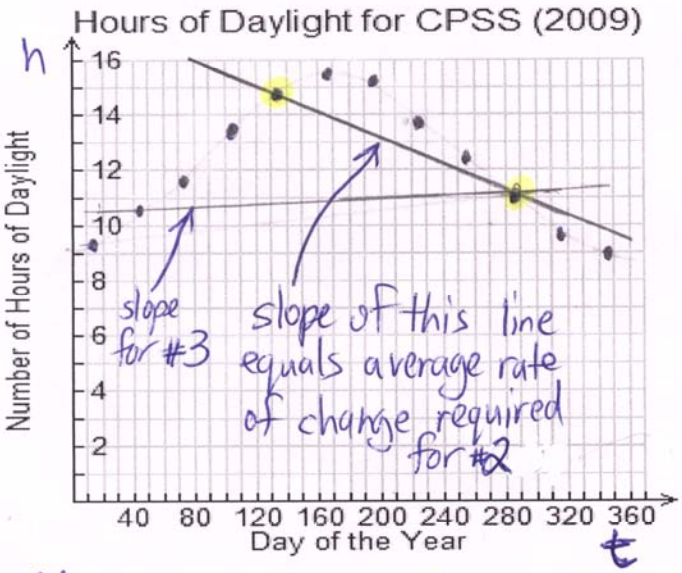
Date	*Day of the Year*	*Sunrise (hh:mm)*	*Sunset (hh:mm)*	*Daylight (hh:mm)*	*Number of daylight hours to the nearest 100th of an hour*
January 16	15	7:47	17:11	9:24	9.40
February 14	44	7:18	17:49	10:31	10.52 (+1.45)
March 16	74	6:29	18:27	11:58	11.97 (+1.45)
April 15	104	5:37	19:02	13:25	13.42 (+1.3)
May 16	135	4:54	19:37	14:43	14.72
June 15	165	4:39	20:00	15:21	15.35
July 16	196	4:54	19:56	15:02	15.03 (-1.13)
August 16	227	5:26	19:20	13:54	13.90 (-1.4)
September 15	257	5:59	18:29	12:30	12.50 (-1.5)
October 16	288	6:34	17:34	11:00	11.00
November 15	318	7:12	16:55	9:35	9.58
December 16	349	7:45	16:45	9:00	9.00

30 days
30 days

IMPORTANT **(Highlight or underline the following two sentences for one bonus mark. Otherwise, lose one mark.)**

- **TO ANSWER QUESTIONS 1 THROUGH 4, ONLY USE THE DATA PROVIDED IN THE TABLE!**
- **CORRECT UNITS MUST ACCOMPANY ALL NUMERIC ANSWERS!**

1. Use the provided grid to create a *scatter plot* for the data given in the above table. (Note that the time $t = 0$ corresponds to January 1, 2009 exactly at 12:00 AM.) **(3 KU)**



Let h represent # of hours of daylight
Let t represent time in days.

2. Now calculate the *average rate of change* of number of hours of daylight with respect to time from day 135 to 288. In addition, use the grid in question 1 to sketch a straight line whose slope equals the required average rate of change. On average, does the number of daylight hours between day 135 and day 288 increase or decrease? **(3 APP)**

average rate of change

$$= \frac{\Delta h}{\Delta t}$$

$$= \frac{11.00 - 14.72}{288 - 135}$$

$$\doteq -0.243 \text{ h/day}$$

Between day 135 and day 288, the number of hours of daylight decreased at an average rate of 0.243 h/day. //

3. Would you expect the average rate of change of number of daylight hours from day 44 to day 288 to be higher or lower than the value that you obtained in question 2? Explain. (Do not perform any calculations!) **(3 TIPS)**

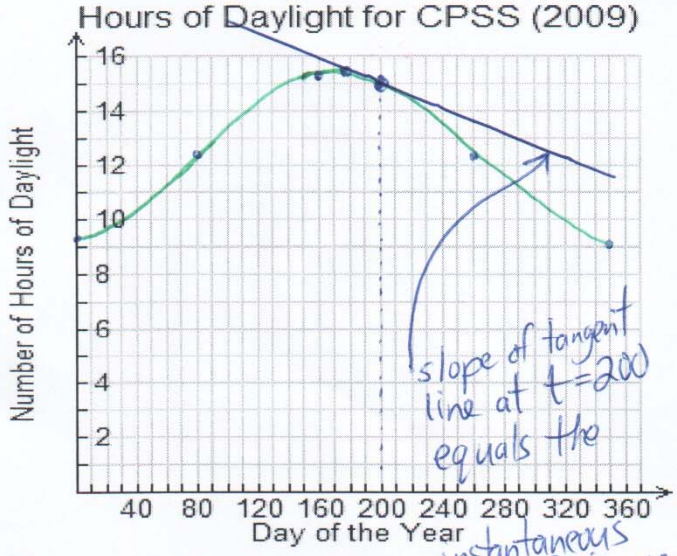
By sketching a secant line through the points (44, 10.52) to (288, 11.00), we can see that the line has a small positive slope. Doing the same for question 2, we obtain a line with a negative slope. Since average rate of change is equal to the slope of a corresponding secant line, the average rate from day 44 to day 288 must be higher.

4. Use only the data in the table to *estimate* the times at which the number of hours of daylight are increasing and decreasing at the fastest rate. Explain your reasoning. **(3 TIPS)**

By observing the values in the table carefully, we can see that the largest changes occur from Feb 14 to Apr 15 and from Sep 15 to Oct 16. Therefore, the greatest rate of increase should occur at some point between Feb 14 and Apr 15 and the greatest rate of decrease should occur at some point between Sep 15 and Oct. 16

5. Use TI-Interactive or a graphing calculator to create a *mathematical model* for the given data by performing a *sinusoidal regression*. Write the equation that you obtain in the provided space. In addition, include a rough sketch of the graph of the sinusoidal function that you obtain. **(3 KU)**

Equation: $h(t) = 3.16052 \sin(0.01754 - 1.28913) + 12.1217$



6. Use the equation that you obtained in question 5 and a very small interval of time to estimate the instantaneous rate of change of number of hours of daylight with respect to time for day 200. **(3 APP)**

instantaneous rate of change of hours of daylight with respect to time on day 200

$$\doteq \frac{h(200.1) - h(199.9)}{200.1 - 199.9}$$

$$\doteq \frac{14.9066 - 14.9116}{0.2}$$

(obtained using the "Calculate" tool in TI-Interactive)

$$= -0.025 \text{ h/day}.$$

On day 200, the number of hours of daylight is decreasing at a rate of about 0.025 h/day.

7. Use TI-Interactive or a graphing calculator to estimate the *instantaneous rate of change* of number of hours of daylight with respect to time for day 200. (Explain how you did this.) In addition, use the grid in question 5 to sketch a straight line whose slope equals the required instantaneous rate of change. **(3 KU)**

By using the tangent line tool in TI-Interactive and setting x to 200, the equation of the tangent line is found to be $y = -0.02496x + 19.9011$

Therefore, the instantaneous rate of change of daylight hours with respect to time on day 200 is approximately -0.02496 h/day

8. Compare your answers to questions 6 and 7. Which of these answers is the *better* estimate of the number of hours of daylight with respect to time for day 200? Explain. **(3 TIPS)**

The tangent tool in TI-Interactive should give the better estimate because the x-value of the point of tangency can be set to exactly 200. Nonetheless, the estimate using a small centred interval turned out to be very close to the TI-Interactive estimate. Thus, we can be fairly confident that our estimates are reasonable.

9. According to your mathematical model, approximately on what day will the number of hours of daylight be the highest? On what day will the number of hours of daylight be the lowest? Answer this question using a *graphical approach*. Explain how you arrived at your answer. **(3 APP)**

By using the "Maximum..." tool in the "Calculate" menu, the maximum # of daylight hours is found to occur on day 170.71 (ie. at some time between day 170 and day 171). Using the same tool, the minimum number of daylight hours occurs on day 358.2 (ie. at some time between day 358 and day 359).

10. According to your mathematical model, what will be instantaneous rate of change of number of hours of daylight with respect to time on day 743? Answer this question using a *graphical approach*. Explain how you arrived at your answer. **(3 TIPS)**

First of all, the scale on the x-axis should be changed so that the graph is visible in the neighbourhood of day 743. Then the tangent line tool should be used to construct a tangent line exactly at day 743. When this is done, the slope is found to be about 0.0086. This means that on day 743, the # of daylight hours is increasing at a rate of about 0.0086 h/day.