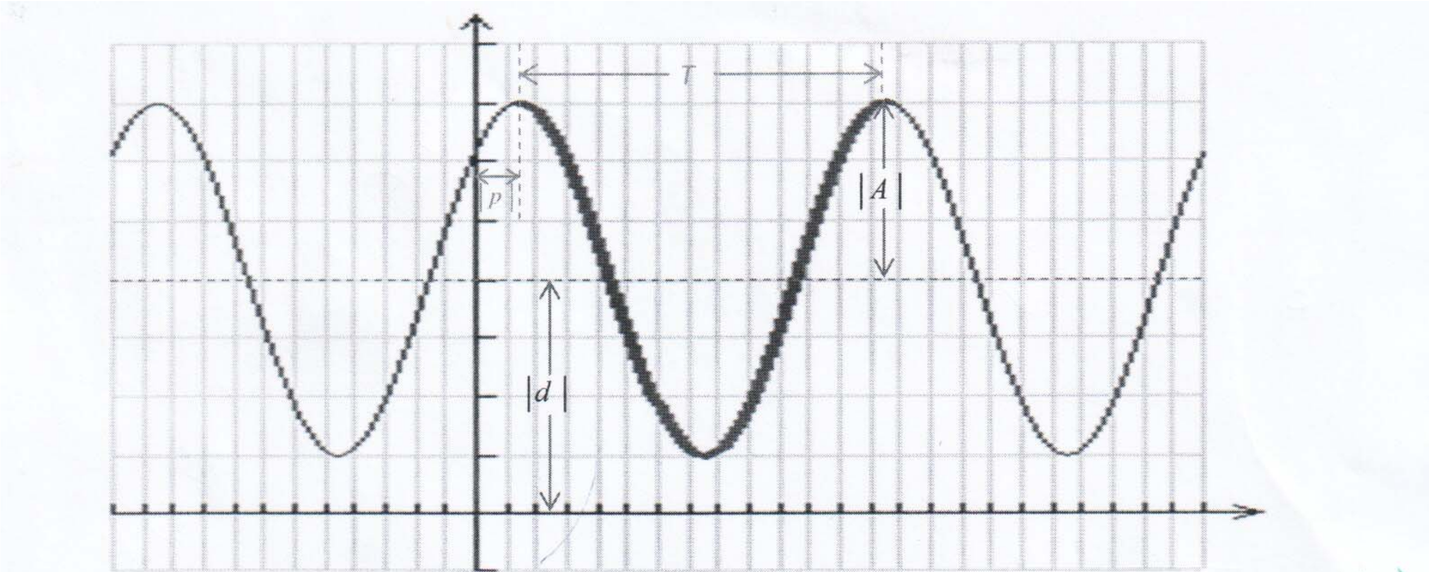
Activity 1 (Unit 2, Pages 28 – 29) – Solutions



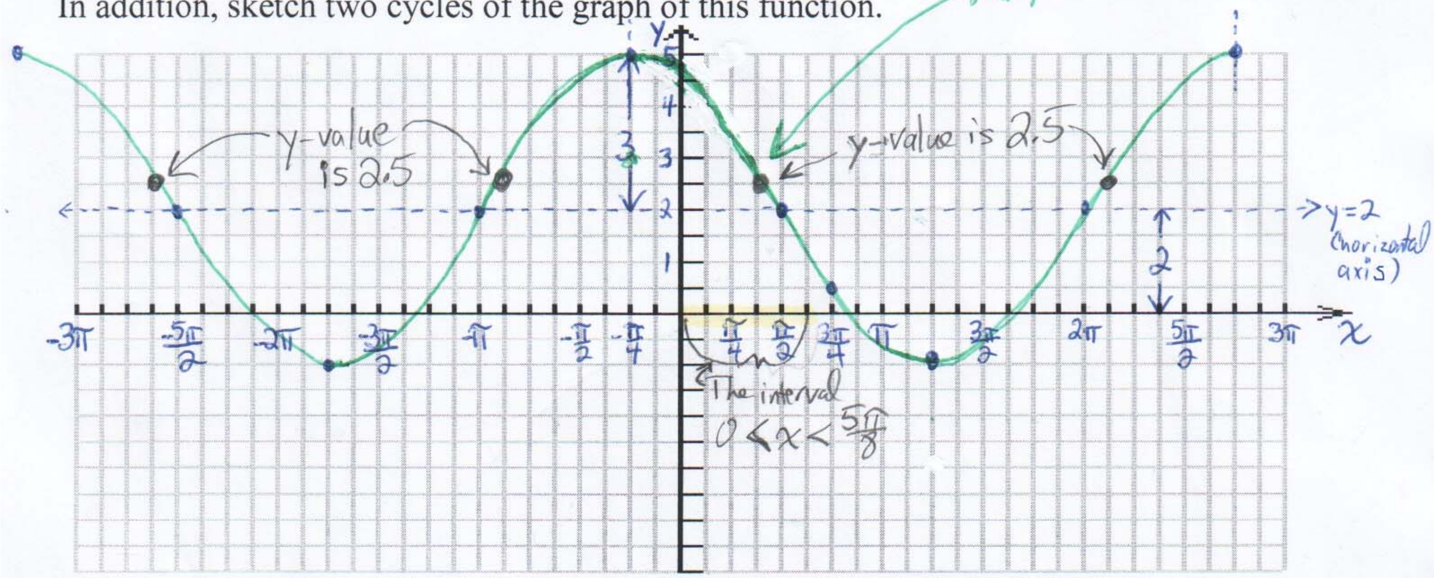
Activity 1

1. A cosine curve has an amplitude of 3 units and a period of 3π radians. The equation of the axis is $y = 2$, and a horizontal shift of $\frac{\pi}{4}$ radians to the left has been applied. Write the equation of this function. In addition, sketch two cycles of the graph of this function.

$T = 3\pi = 2\pi\left(\frac{1}{\omega}\right)$

$\therefore \omega = \frac{2\pi}{3\pi} = \frac{2}{3}$

$f(x) = 3\cos\left(\frac{2}{3}\left(x + \frac{\pi}{4}\right)\right) + 2$



2. Determine the value of the function in question 1 if $x = \frac{\pi}{2}, \frac{3\pi}{4}$, and $\frac{11\pi}{6}$.

$f\left(\frac{\pi}{2}\right) = 3\cos\left(\frac{2}{3}\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right) + 2 = 3\cos\frac{\pi}{2} + 2 = 3(0) + 2 = 2$

$f\left(\frac{3\pi}{4}\right) = 3\cos\left(\frac{2}{3}\left(\frac{3\pi}{4} + \frac{\pi}{4}\right)\right) + 2 = 3\cos\frac{2\pi}{3} + 2 = 3\left(-\frac{1}{2}\right) + 2 = \frac{1}{2}$

$f\left(\frac{11\pi}{6}\right) = 3\cos\left(\frac{2}{3}\left(\frac{11\pi}{6} + \frac{\pi}{4}\right)\right) + 2 = 3\cos\left(\frac{25\pi}{12}\right) + 2 \doteq 4.9$

3. Use your graph to estimate the x-value(s) in the domain $0 < x < 2$, where $y = 2.5$, to one decimal place.

In the interval $0<x<2$, $y=2.5$ for $x \doteq \frac{3\pi}{8}$

(Note that $\frac{5\pi}{8} \doteq 1.9635$, so the interval $0<x<2$ is just a little larger than the interval $0<x<\frac{5\pi}{8}$)

Outside the interval $0<x<2$, $y=2.5$ approximately where $x = \frac{-21\pi}{8}, -\frac{7\pi}{8}, \frac{19\pi}{8}$ (see graph on previous page)

4. The number of hours of daylight in Vancouver can be modelled by a sinusoidal function of time, in days. The longest day of the year is June 21, with 15.7 h of daylight. The shortest day of the year is December 21, with 8.3 h of daylight.

June 21

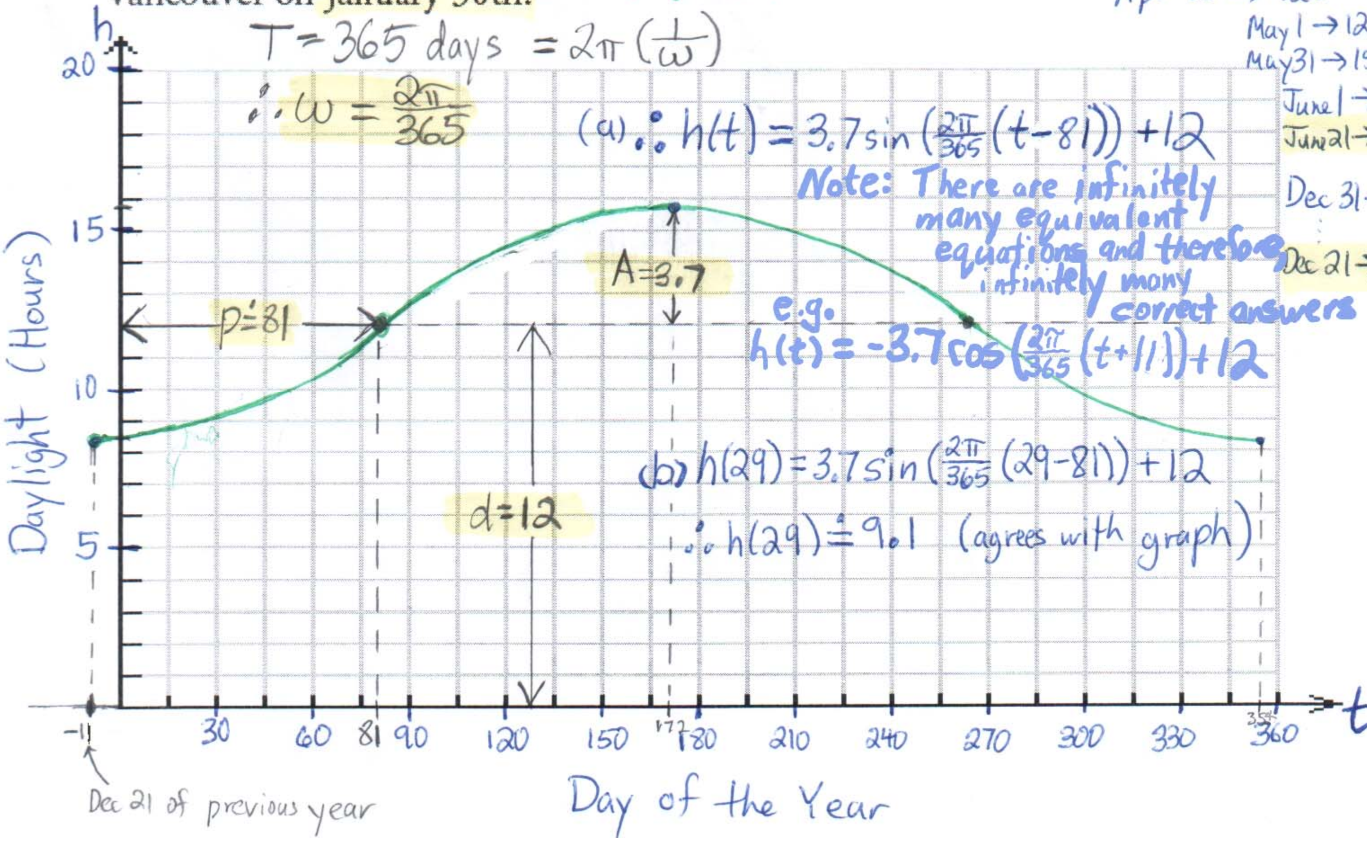
Jan 1 → 0
Jan 31 → 30
Feb 1 → 31
Feb 28 → 58
Mar 1 → 59
Mar 31 → 90
Apr 1 → 91
Apr 30 → 120
May 1 → 121
May 31 → 151
June 1 → 152
June 21 → 172
Dec 31 → 364
Dec 21 → 354

a) Find an equation for $h(t)$, the number of hours of daylight on the tth day of the year. In addition, sketch one cycle of the graph of this function.

b) Use your equation to predict the number of hours of daylight in Vancouver on January 30th. $\longrightarrow t=29$

$T = 365$ days $= 2\pi\left(\frac{1}{\omega}\right)$

$\therefore \omega = \frac{2\pi}{365}$



(a) $\therefore h(t) = 3.7\sin\left(\frac{2\pi}{365}(t-81)\right)+12$

Note: There are infinitely many equivalent equations and therefore infinitely many correct answers

e.g. $h(t) = -3.7\cos\left(\frac{2\pi}{365}(t+11)\right)+12$

(b) $h(29) = 3.7\sin\left(\frac{2\pi}{365}(29-81)\right)+12$

$\therefore h(29) \doteq 9.1$ (agrees with graph)