Answers to Logarithm Practice Test

Multiple Choice Questions

1. d	**2.** c	**3.** a	**4.** d	**5.** b
6. d	**7.** b	**8.** a	**9.** d	**10.** c
11. d	**12.** d	**13.** c	**14.** a	**15.** c
16. c	**17.** c	**18.** a	**19.** b	**20.** b

Short Answer Questions

21. $D=\{x\in\mathbb{R}:x<5\}$
$R=\mathbb{R}$

22. $x=-4$

23. Change 1.5 to 6. The curve is already to the left of the vertical asymptote.

24. 4.11

25. 164

26. 9

27. $\log_{20}200$, $\log 100$, $\log_5 40$, $\log_3 30$, $\log_2 16$

28. $\log_a(xy)=\log_a x+\log_a y$
$a^x a^y=a^{x+y}$

29. log 32

30. $2^x=\frac{1}{\sqrt{8}}$

31. 7.7 years

32. 2

33. 0.53

34. $-\frac{5}{4}$

35. $x\geq\sqrt{5}$

36. 16

37. 8

38. 4.6 years

39. $f(x)=a(b-1)^x, 1<b<2$

40. $C(t)=C_0(1.023)^t$

41. *Solution*

The transformed function $f(x)=\log(-2x)+3$ has the same range as the parent function, since the range of all transformed logarithmic functions have a range of all real numbers. The *y*-intercept is the vertical asymptote of both the parent and transformed functions.

The transformed function curves to the left, the original function curve to the right. The two functions will have different *x*-intercepts, the intercepts being reflected over the *y*-axis.

42. *Solution*

The vertical asymptote helps define the domain of a function. The vertical asymptote changes when a horizontal translation is applied.
The vertical asymptote of $f(x)=3\log_{10}(x-5)+2$ is $x=5$.
The vertical asymptote of $f(x)=3\log_{10}[-(x+5)]+2$ is $x=-5$.
The graph of the first function curves to the right of the asymptote. The domain of $f(x)=3\log_{10}(x-5)+2$ is $\{x\in\mathbf{R}\mid x>5\}$.
Since the expression $(x+5)$ is multiplied by -1, the graph is reflected in the *y*-axis and curves to the left of the asymptote. The domain of $f(x)=3\log_{10}[-(x+5)]+2$ is $\{x\in\mathbf{R}\mid x<-5\}$.

43. *Solution*

The equation for relating the amount of radium, *r*, in grams and the amount of time, *t*, in years is

$$r=12\times\left(\frac{1}{2}\right)^{(t\div 1620)}.$$

Substituting 8 in for *r* gives $8=12\times(\frac{1}{2})^{(t\div 1620)}$

$$\frac{2}{3}=\left(\frac{1}{2}\right)^{(t\div 1620)}$$

Using guess and check gives $\frac{t}{1620}=0.59$

44. *Solution*

Using the laws of logarithms, $\log_4 x^3 - \log_4 8$ can be rewritten as the single logarithm $3\log_4\left(\frac{1}{2}x\right)$ by first applying the quotient law and then the product law of logarithms. Comparing the new form of $g(x)$ to $f(x)$ produces a vertical stretch by a factor of 3 and a horizontal stretch by a factor of 2.

45. *Solution*

$$\frac{1}{3}\log_a x + \frac{1}{2}\log_a 2y - \frac{1}{6}\log_a 4z$$
$$= \log_a \sqrt[3]{x} + \log_a \sqrt{2y} - \log_a \sqrt[6]{4z}$$
$$= \log_a \frac{\sqrt[3]{x}\,\sqrt{2y}}{\sqrt[6]{4z}}$$

46. *Solution*

We use the fact that when two exponential expressions with the same base are equal their exponents are equal to set the exponents equal to one another and solve. If $a^m = a^n$, then $m = n$.
When we have two exponential expressions with different bases set equal to each other, we use the fact that taking the log of equal expressions maintains their equality to start the solution process. If $M = N$, then $\log M = \log N$. Given that M and N are powers, we use the power rule to continue the solution process.

47. *Solution*

$$\log\left(\frac{x-y}{3}\right) = \frac{1}{2}(\log x + \log y)$$
$$2\log\left(\frac{x-y}{3}\right) = \log x + \log y$$
$$2\log\left(\frac{x-y}{3}\right) = \log xy$$
$$\log\left(\frac{x-y}{3}\right)^2 = \log xy$$
$$\left(\frac{x-y}{3}\right)^2 = xy$$
$$\frac{x^2 - 2xy + y^2}{9} = xy$$
$$x^2 - 2xy + y^2 = 9xy$$
$$x^2 + y^2 = 11xy$$

48. *Solution*

$$1000 = 400(1.01)^{12t}$$
$$2.5 = (1.01)^{12t}$$
$$\log 2.5 = \log(1.01)^{12t}$$
$$\log 2.5 = 12t\log 1.01$$
$$0.3979 = 12t(0.00432)$$
$$t = 7.7 \text{ years}$$

49. *Solution*

$$S(d) = 86\log d + 112$$
$$S(d) = 86\log 49.9 + 112 \qquad S(d) = 86\log 50.1 + 112$$
$$= 258.0366 \qquad = 258.1860$$

Instantaneous Rate of Change

$$\frac{258.1860 - 259.0366}{50.1 - 49.9} = 0.747 \text{ mph/mi}$$

50. *Solution*

The graph of $f(x) = ab^x$ is constantly increasing when a is positive and b is greater than 1. The graph rise slowly and then more rapidly, but at no point does its direction change. Similarly, the graph of $f(x) = ab^x$ constantly decreasing when a is positive and b is between 0 and 1. The graph first decreases rapidly and t much more slowly but, again, at no point does its direction change.