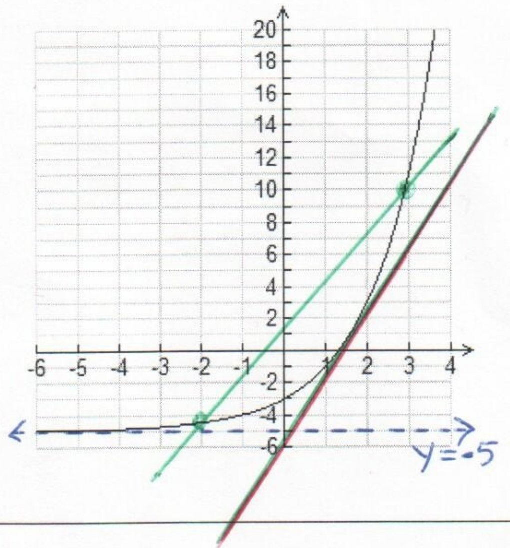
MHF4U0 Final Exam Review #1 – Characteristics and Behaviour of Functions



Equation of Function: $f(x) = 2^{x+1} - 5$ (**Hint:** $a = 2$)

Domain: $\mathbb{R}$

Range: $\{y \in \mathbb{R} \mid y > -5\}$

Equation of Horizontal Asymptote: $y = -5$ (Sketch asymptote on grid)

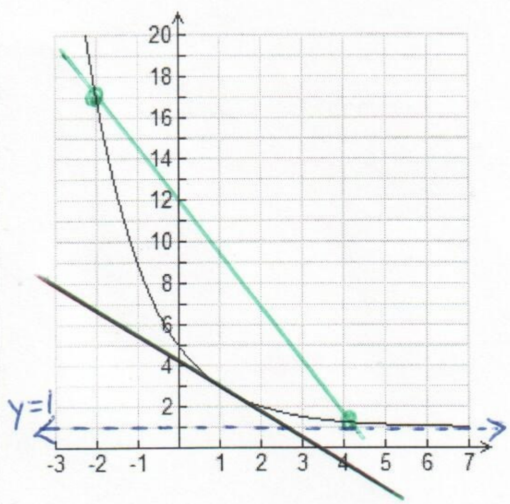
As $x \to \infty$, $f(x) \to$ $+\infty$

As $x \to -\infty$, $f(x) \to$ -5

Average Rate of Change (slope of secant) over any interval is positive

Instantaneous Rate of Change (slope of tangent) at any point is positive

Interval(s) of Increase: $(-\infty, \infty)$ Interval(s) of Decrease: none



Equation of Function: $f(x) = \left(\frac{1}{2}\right)^{x-2} + 1$ (**Hint:** $a = \frac{1}{2}$)

Domain: $\mathbb{R}$

Range: $\{y \in \mathbb{R} \mid y > 1\}$

Equation of Horizontal Asymptote: $y = 1$ (Sketch asymptote on grid)

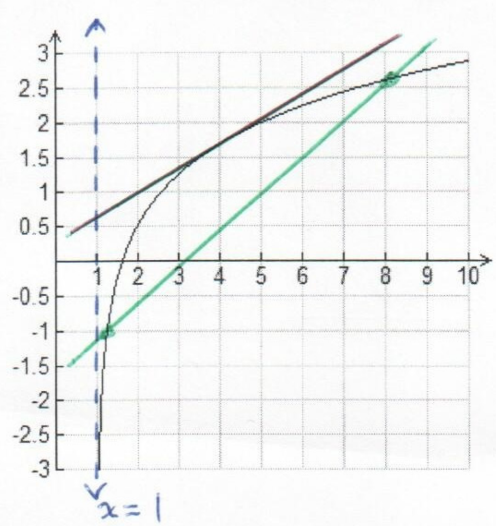
As $x \to \infty$, $f(x) \to$ 1

As $x \to -\infty$, $f(x) \to$ $+\infty$

Average Rate of Change (slope of secant) over any interval is negative

Instantaneous Rate of Change (slope of tangent) at any point is negative

Interval(s) of Increase: none Interval(s) of Decrease: $(-\infty, \infty)$



Equation of Function: $f(x) = 1.5\log_4(0.25(x-1)) + 2$ (**Hint:** $a = 4$)

Domain: $\{x \in \mathbb{R} \mid x > 1\}$

Range: $\mathbb{R}$

Equation of Vertical Asymptote: $x = 1$ (Sketch asymptote on grid)

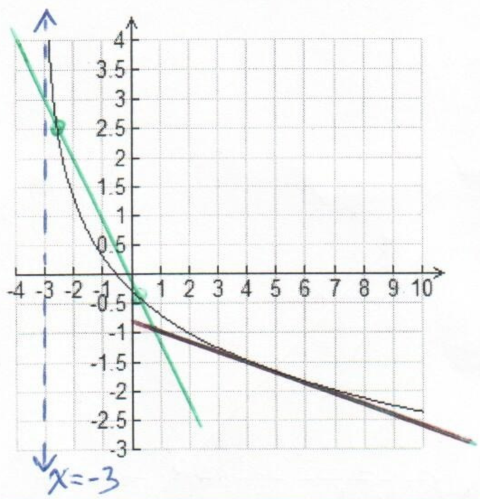
As $x \to \infty$, $f(x) \to$ $+\infty$

As $x \to 1^+$, $f(x) \to$ $-\infty$

Average Rate of Change (slope of secant) over any interval is positive

Instantaneous Rate of Change (slope of tangent) at any point is positive

Interval(s) of Increase: $(1, \infty)$ Interval(s) of Decrease: none



Equation of Function: $f(x)=2\log_{\frac{1}{4}}(0.1(x+3))-2$ (**Hint:** $a = 1/4$)

Domain: $\{x\in\mathbb{R} \mid x>-3\}$

Range: $\mathbb{R}$

Equation of Vertical Asymptote: $x=-3$ (Sketch asymptote on grid)

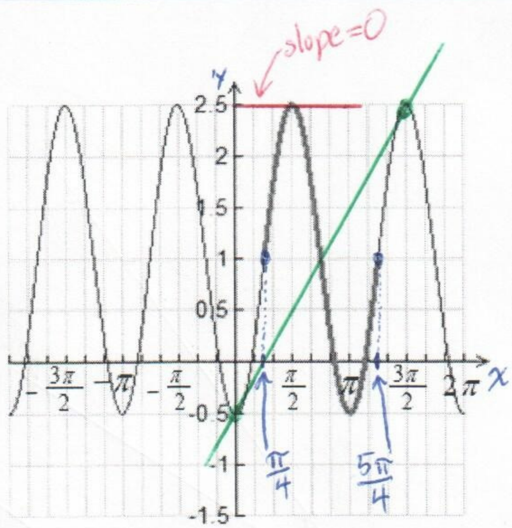
As $x\to\infty$, $f(x)\to -\infty$

As $x\to -3^+$, $f(x)\to +\infty$

Average Rate of Change (slope of secant) over any interval is negative

Instantaneous Rate of Change (slope of tangent) at any point is negative

Interval(s) of Increase: none Interval(s) of Decrease: $(-3,\infty)$



Amplitude: 1.5 Vertical Displacement: 1 (up)

Period: $\frac{5\pi}{4}-\frac{\pi}{4}=\pi$ Phase Shift: $\frac{\pi}{4}$ (right)

Equation of Function: $f(x)=1.5\sin\left(2\left(x-\frac{\pi}{4}\right)\right)+1$

Domain: $\mathbb{R}$

Range: $\{y\in\mathbb{R} \mid -0.5\le y\le 2.5\}$

Average Rate of Change (slope of secant) from $(0,-0.5)$ to $\left(\frac{3\pi}{2},2.5\right)$:

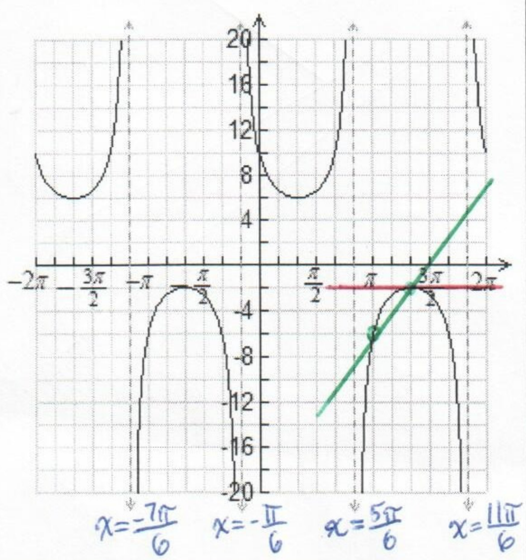
$$\frac{\Delta y}{\Delta x}=\frac{f(\frac{3\pi}{2})-f(0)}{\frac{3\pi}{2}-0}=\frac{2.5-(-0.5)}{(\frac{3\pi}{2})}=\frac{2}{\pi}$$

Instantaneous Rate of Change (slope of tangent) at $x=\frac{\pi}{2}$ is 0

Interval(s) of Increase: $\left(n\pi,\frac{(2n+1)\pi}{2}\right)$ for all $n\in\mathbb{Z}$

Interval(s) of Decrease: $\left(\frac{(2n-1)\pi}{2},n\pi\right)$ for all $n\in\mathbb{Z}$

Turning Points: $\left(\frac{(2n+1)\pi}{2},2.5\right)$, $(n\pi,-0.5)$ for all $n\in\mathbb{Z}$



Vertical Stretch Factor: 4 Vertical Displacement: +2 (up)

Period: 2π Phase Shift: $\frac{\pi}{3}$ (right)

Equation of Function: $f(x)=4\sec\left(x-\frac{\pi}{3}\right)+2$

Domain: $\{x\in\mathbb{R} \mid x\ne\frac{(6n+5)\pi}{6}, n\in\mathbb{Z}\}$

Range: $\{y\in\mathbb{R} \mid y\ge 6 \text{ or } y\le -2\}$

Average Rate of Change (slope of secant) from $(\pi,-6)$ to $\left(\frac{4\pi}{3},-2\right)$:

$$\frac{\Delta y}{\Delta x}=\frac{f(\frac{4\pi}{3})-f(\pi)}{\frac{4\pi}{3}-\pi}=\frac{-2-(-6)}{(\frac{\pi}{3})}=\frac{12}{\pi}$$

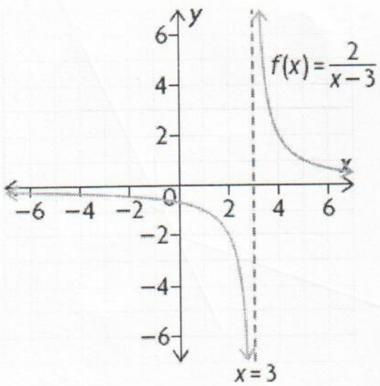
Instantaneous Rate of Change (slope of tangent) at $x=\frac{4\pi}{3}$ is 0

Interval(s) of Increase: $\left(\frac{(6n+1)\pi}{3},\frac{(12n+5)\pi}{6}\right)$, $\left(\frac{(12n-7)\pi}{6},\frac{(6n-2)\pi}{3}\right)$

Interval(s) of Decrease: $\left(\frac{(6n-2)\pi}{3},\frac{(12n-1)\pi}{6}\right)$, $\left(\frac{(6n+4)\pi}{3},\frac{(12n+11)\pi}{6}\right)$

Turning Points: $\left(\frac{(6n+1)\pi}{3},6\right)$, $\left(\frac{(6n-2)\pi}{3},-2\right)$

for all $n\in\mathbb{Z}$



Domain: $\{x \in \mathbb{R} \mid x \neq 3\}$

Range: $\{y \in \mathbb{R} \mid y \neq 0\}$

Given that the parent function is $g(x) = \frac{1}{x}$, $f(x)$ is obtained by stretching f vertically by a factor of 2 and translating 3 units to the right.

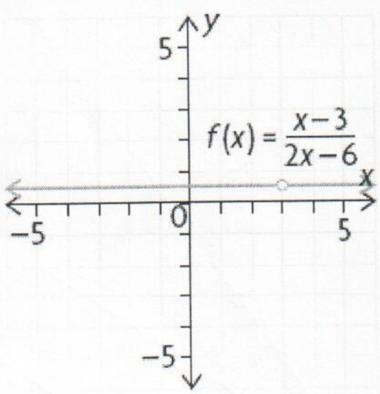
Equation of Vertical Asymptote: $x = 3$

Equation of Horizontal Asymptote: $y = 0$

As $x \to \infty$, $f(x) \to$ 0   As $x \to -\infty$, $f(x) \to$ 0

As $x \to 3^-$, $f(x) \to$ $-\infty$ As $x \to 3^+$, $f(x) \to$ $+\infty$

Interval(s) of Increase: none   Interval(s) of Decrease: $(-\infty, 3)$, $(3, \infty)$



There is a "hole" in f at $x = 3$ because division by zero is undefined:

$$f(x) = \frac{x-3}{2x-6} = \frac{x-3}{2(x-3)} = \frac{1}{2}, x \neq 3$$

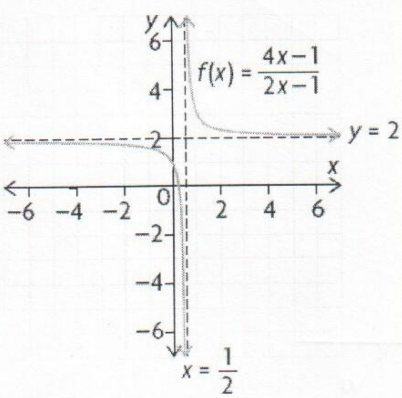
$f(x) = \frac{1}{2}$, $x \neq 3$

Domain: $\{x \in \mathbb{R} \mid x \neq \frac{1}{2}\}$

Range: $\{y \in \mathbb{R} \mid y = \frac{1}{2}\} = \{\frac{1}{2}\}$

As $x \to \infty$, $f(x) \to$ $\frac{1}{2}$ As $x \to -\infty$, $f(x) \to$ $\frac{1}{2}$

Interval(s) of Increase: none   Interval(s) of Decrease: none



Domain: $\{x \in \mathbb{R} \mid x \neq \frac{1}{2}\}$

Range: $\{y \in \mathbb{R} \mid y \neq 2\}$

Equation of Vertical Asymptote: $x = \frac{1}{2}$

Equation of Horizontal Asymptote: $y = 2$

As $x \to \infty$, $f(x) \to$ 2   As $x \to -\infty$, $f(x) \to$ 2

As $x \to \frac{1}{2}^-$, $f(x) \to$ $-\infty$ As $x \to \frac{1}{2}^+$, $f(x) \to$ $+\infty$

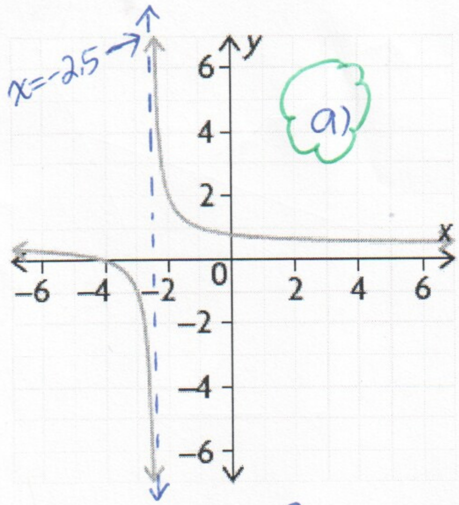
Interval(s) of Increase: none   Interval(s) of Decrease: $(-\infty, \frac{1}{2})$, $(\frac{1}{2}, \infty)$

1. Match each function with its graph.

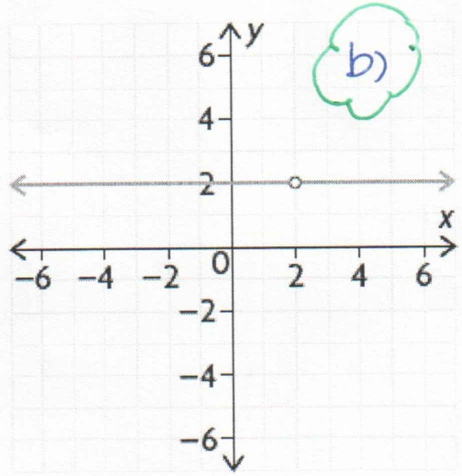
a) $h(x) = \dfrac{x+4}{2x+5}$ c) $f(x) = \dfrac{3}{x-1}$

b) $m(x) = \dfrac{2x-4}{x-2}$ d) $g(x) = \dfrac{2x-3}{x+2}$

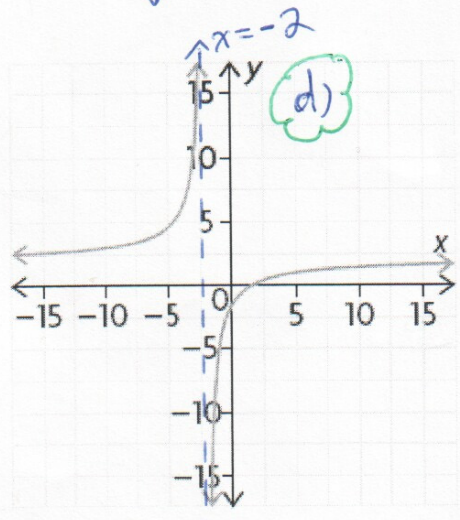
A



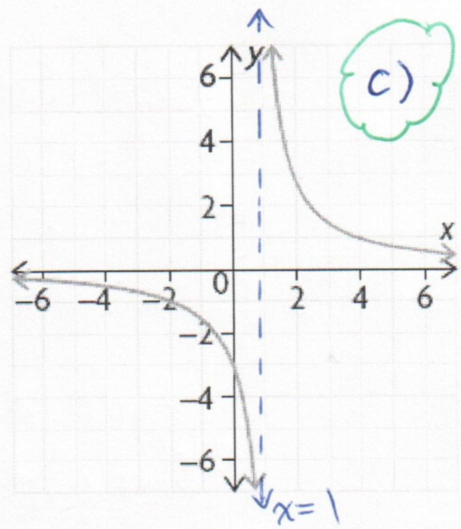
C



B



D



2. Consider the function $f(x) = \dfrac{3}{x-2}$.

a) State the equation of the vertical asymptote. $x=2$

b) Use a table of values to determine the behaviour(s) of the function near its vertical asymptote. As $x \to 2^-$, $f(x) \to -\infty$. As $x \to 2^+$, $f(x) \to +\infty$

c) State the equation of the horizontal asymptote. $y=0$

d) Use a table of values to determine the end behaviours of the function near its horizontal asymptote. As $x \to -\infty$, $f(x) \to 0$. As $x \to \infty$, $f(x) \to 0$

e) Determine the domain and range. $D = \{x \in \mathbb{R} \mid x \neq 2\}$, $R = \{y \in \mathbb{R} \mid y \neq 0\}$

f) Determine the positive and negative intervals.

f is positive on $(2, \infty)$
f is negative on $(-\infty, 2)$

g) Sketch the graph.