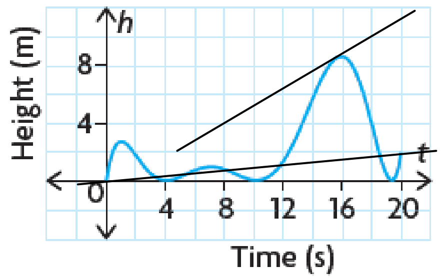
# MHF4U0 Final Exam Review #3 – Rates of Change and Modelling – Solutions

**1.**




**(a)** Approximate values given: (0,1), (4, 7), (10, 16), (19, 20)
***Positive*** instantaneous rates of change → elevation increases with time

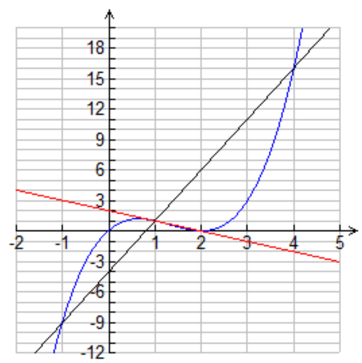
**(b)** Approximate values given: (1,4), (7, 10), (16, 19)
***Negative*** instantaneous rates of change → elevation decreases with time

**(c)** Average rate of change = slope of secant $= \frac{2-0}{20-0} = \frac{2}{20} = \frac{1}{10} = 0.1$ m/s
(secant line passes through (0,0) and (0,20))

**(d)** Sketch a tangent line as carefully as possible at $t = 15.5$. Then find its slope:
Instantaneous rate of change = slope of tangent $\doteq \frac{10-4}{18-8} = \frac{6}{10} = 0.6$ m/s (remember that this is only a rough estimate)

**(e)** ***speeding up***: same as (a) ***slowing down***: same as(b)

**2.** Consider the function $f(x) = x^3 - 4x^2 + 4x$.




**(a)** See graph at right

**(b)** Average rate of change = slope of secant $= \frac{16-(-9)}{4-(-1)} = \frac{25}{5} = 5$
(secant line passes through (-1,-9) and (4,16))

**(c)** Sketch a tangent line as carefully as possible at $x = 1$. Then find its slope:
Instantaneous rate of change = slope of tangent $\doteq \frac{0-1}{2-1} = -1$
(using calculus we can confirm that the slope of the tangent at $x = 1$ is exactly $-1$)

**(d)** **positive**: $(-\infty, 0.5)$, $(2, \infty)$ **negative**: $(0.5, 2)$ **zero**: $x \doteq 0.5$, $x \doteq 2$

**3.** The following table shows the monthly average number hours of sunshine for Toronto.

| Month | J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Average Monthly Sunshine (h) | 95.5 | 112.6 | 150.5 | 187.7 | 229.7 | 254.9 | 278.0 | 244.0 | 184.7 | 145.7 | 82.3 | 72.6 |

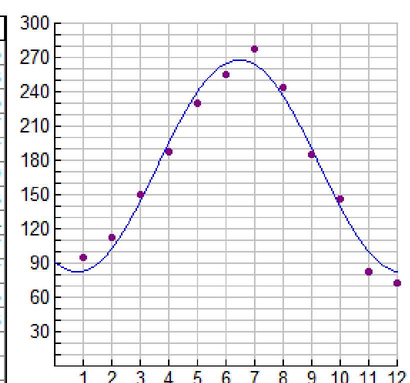
*Source:* Environment Canada

Since weather patterns are similar from year-to-year, a sinusoidal regression is probably the best choice.

**Minimum:** Some time in January
**Maximum:** Some time in June or July

Since there is a great deal of variation in weather patterns, we can't expect our model to be a perfect fit. Nonetheless, our model is useful as long as we realize its limitations. That is, we must be aware of the fact that it can only be used to make very rough predictions.

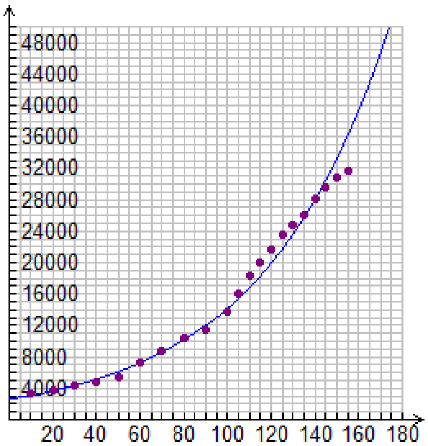
| L1 | L2 |
|---|---|
| 1 | 95.5 |
| 2 | 112.6 |
| 3 | 150.5 |
| 4 | 187.7 |
| 5 | 229.7 |
| 6 | 254.9 |
| 7 | 278 |
| 8 | 244 |
| 9 | 184.7 |
| 10 | 145.7 |
| 11 | 82.3 |
| 12 | 72.6 |

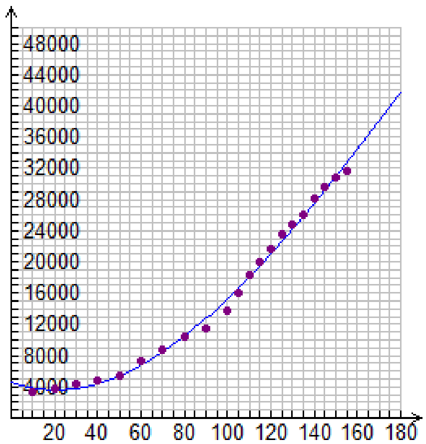



Sinusoidal Regression
regEQ(x) = 92.6752*sin(.554409*x+-2.01206)+175.036

**4. Horizontal Axis**: years since 1851 **Vertical Axis**: thousands of people

| L1 | L2 |
|---|---|
| 10 | 3230 |
| 20 | 3689 |
| 30 | 4325 |
| 40 | 4833 |
| 50 | 5371 |
| 60 | 7207 |
| 70 | 8788 |
| 80 | 10377 |
| 90 | 11507 |
| 100 | 13648 |
| 105 | 16081 |
| 110 | 18238 |
| 115 | 20015 |
| 120 | 21568 |
| 125 | 23450 |
| 130 | 24820 |
| 135 | 26101 |
| 140 | 28031 |
| 145 | 29672 |
| 150 | 30755 |
| 155 | 31613 |




Exponential Regression
regEQ(x) = 2609.1*1.01711^x

Cubic Regression
regEQ(x) = -.004531x^3 + 2.50689x^2 + -98.2742x + 4597.03

**Exponential Model**: predicts a population of about 43 million for 2016

**Cubic Model**: predicts a population of about 36 million for 2016

Canada's population (as of 2009) is about 33.5 million. To fulfil the prediction of the exponential model, Canada's population would have to increase by over 9 million people in a span of only 7 years, requiring an average growth rate of more than 1 million people/year. Since the current growth rate is much lower than 1 million people/year, it appears that the cubic model gives a more realistic prediction.

Use TI-Interactive to sketch a tangent line at the point on the cubic function at which $t = 149$ (corresponds to the year 2000). The slope at this point is about 347. This means that the population growth rate is 347000 people/year.