**MHF 4U0** **Semester 2, 2008 - 2009**

**Grade 12 Advanced Functions (University Preparation)**
**Unit 0 – Quest 2 - Review Material, Problems**

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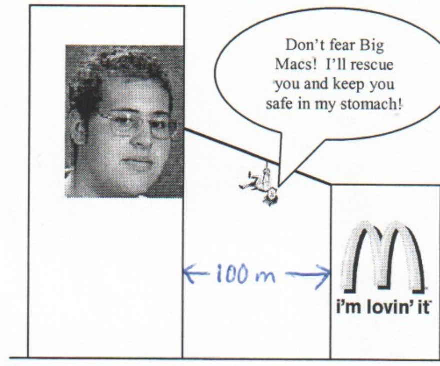
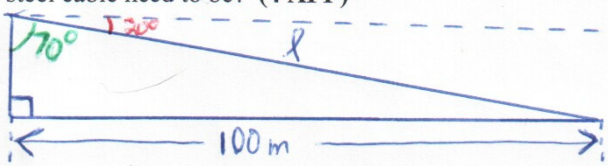
***Victim*:** Mr. Solutions Super work Mr. S.!

| APP | TIPS | COM |
|---|---|---|
| 16 /16 | 12 /12 | 10 /10 |

1. Andrew lives in an apartment building that is located exactly 100 m from his favourite restaurant. To allow him to get Big Macs as quickly as possible, he wants to install a steel cable connecting his balcony to the roof of the restaurant. This would allow him to slide along the cable directly to the source of the burgers.




From his balcony, Andrew uses a theodolite to measure the ***angle of depression*** to the top of the building and finds it to be 20°. How long does the steel cable need to be? **(4 APP)**




Let ℓ represent the length of the cable

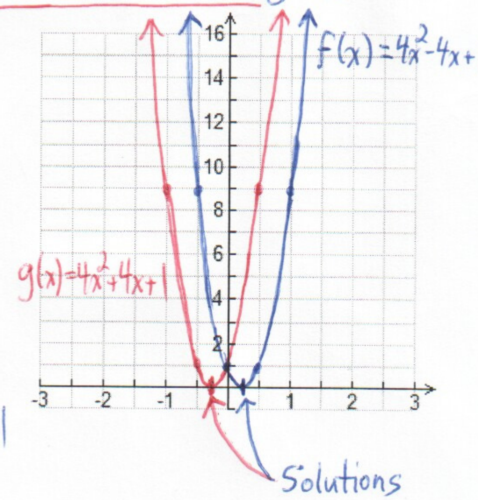
$$\frac{100}{\ell} = \sin 70^\circ$$

$$\therefore \ell = \frac{100}{\sin 70^\circ}$$

$$\therefore \ell \doteq 106.4$$

The cable should be about 110 m long (round up to include a little slack since Andrew will surely cause the cable to sag).

2. For which value(s) of $k$ does $4x^2 - 2kx + 1 = 0$ have one and only one real root? Use the provided grid to show what the solution(s) of the equation(s) look like graphically. **(4 APP)**




The quadratic equation $ax^2+bx+c=0$ has exactly one real root if $b^2-4ac=0$. For the given equation, $a=4$, $b=-2k$, $c=1$

$$\therefore (-2k)^2 - 4(4)(1) = 0$$

$$\therefore 4k^2 - 16 = 0$$

$$\therefore 4k^2 = 16$$

$$\therefore k^2 = 4$$

$$\therefore k = \pm 2$$

$f(x) = 4x^2 - 4x + 1 = (2x-1)^2$

$g(x) = 4x^2 + 4x + 1 = (2x+1)^2$

$k=+2$: $4x^2 - 4x + 1 = 0$

$k=-2$: $4x^2 + 4x + 1 = 0$

| KU | APP | TIPS | COM |
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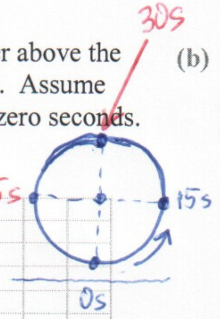
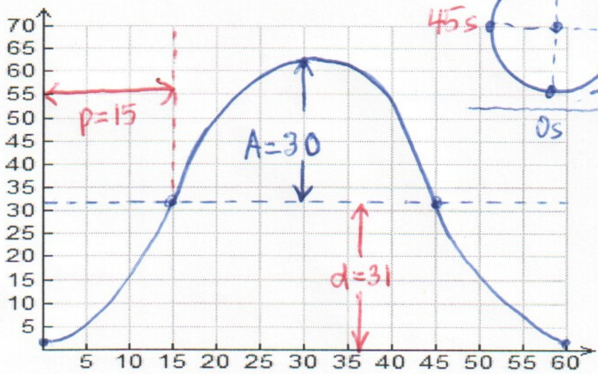
3. A Ferris wheel of radius 30 m rotates at a constant speed. It takes 60 s to complete one full rotation and the passengers get on at a point 1 m above the ground.

(a) Sketch a graph of the height of a passenger above the ground, in metres, versus time, in seconds. Assume that the passenger gets on exactly at time zero seconds. **(3 APP)**




(b) Let $h(t)$ represent the height of a passenger above the ground, in metres, at time $t$ seconds. Write an equation for $h(t)$. **(3 APP)** From the graph, it's clear that Amplitude = A = 30, Phase Shift = p = 15, Vertical Displacement = d = 31 and Period = T = 60

$\because T = \frac{360°}{K} \rightarrow K = \frac{360°}{T} = \frac{360°}{60} = 6$

$\therefore h(t) = 30\sin(6(t-15)) + 31$ is an equation that describes the given situation.

Other Correct Answers

$h(t) = 30\cos(6(t-30)) + 31$

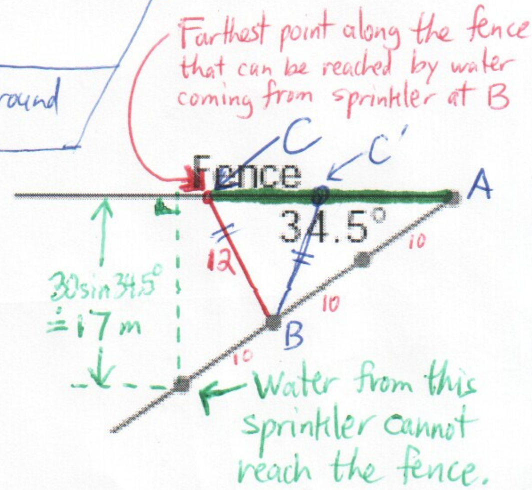
$h(t) = -30\cos(6t) + 31$

(c) How high would a passenger be after 47 s? **(2 APP)**

$$h(47) = 30\sin(6(47-15)) + 31$$
$$= 30\sin 192° + 31$$
$$\doteq 24.8$$

The passenger would be about 24.8 m above the ground after 47 s.

4. An underground water sprinkler system is laid at an angle of 34.5° to a fence. The sprinkler jets are 10 m apart and have a range of 12 m. Determine the length of the fence that gets wet from the sprinklers. (**Hint:** One of the sprinklers shown in the diagram is too far from the fence.) **(6 TIPS)**




AC is the section of the fence that gets wet

By the Law of Cosines (applied to $\Delta ABC$),

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$

$\therefore 12^2 = 20^2 + AC^2 - 2(20)(AC)\cos 34.5°$

$\therefore 144 = 400 + AC^2 - 40\cos 34.5°(AC)$

$\therefore AC^2 - 40\cos 34.5°(AC) + 256 = 0$ (quadratic equation in AC)

$\therefore AC = \frac{+40\cos 34.5° \pm \sqrt{(40\cos 34.5°)^2 - 4(1)(256)}}{2}$

$\therefore AC \doteq 20.4$ m or $AC \doteq 12.5$ m

correct answer

This value is the length obtained if C were located at C' instead.

The sprinklers wet about 20.4 m of the fence.//

| KU | APP | TIPS | COM |
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5. State whether each of the following is true or false. Provide an explanation to support each response. Keep the following points in mind:

- If a mathematical statement is said to be *true*, it must be true in *all possible cases*.
- A *general proof* is required to demonstrate that a statement is *true*. The proof must demonstrate the truth of the statement in all possible cases! Clearly, any number of examples cannot accomplish this goal.
- To demonstrate that a statement is *false*, it is only necessary to produce a *single example* that contradicts the statement. Such an example is called a *counterexample*. **(6 TIPS)**

| *Statement* | *True or False?* | *Proof, Counterexample or Explanation* |
|---|---|---|
| $(a+b)^4 = a^4 + b^4$ | F | Let $a=b=1$.<br>Then, $L.S. = (1+1)^4 = 2^4 = 16$<br>$R.S. = 1^4 + 1^4 = 2$<br>$\therefore L.S. \neq R.S.$ |
| For all functions $p$, $p^{-1}(x) = \dfrac{1}{p(x)}$ | F | The notation $p^{-1}(x)$ is used to indicate the INVERSE of $p(x)$, not its reciprocal. |
| $\dfrac{x}{p} + \dfrac{y}{q} = \dfrac{x+y}{p+q}$ | F | Let $x=y=p=q=1$. Then,<br>$L.S. = \frac{1}{1} + \frac{1}{1} = 1+1 = 2$<br>$R.S. = \frac{1+1}{1+1} = \frac{2}{2} = 1$<br>$\therefore L.S. \neq R.S.$ |
| The equation $4x^2 + 9y^2 - 36 = 0$ describes a parabola. | F | The given equation has both an $x^2$ term and a $y^2$ term. An equation of a parabola should have an $x^2$ term but instead of $y^2$, it should have a $y$ term. |
| For the function $g(x) = 10^x$, $D = \{x \in \mathbb{R} : x > 0\}$ and $R = \mathbb{R}$.<br>(Here $D$ and $R$ represent domain and range respectively.) | F | $10^x$ can be evaluated for any value of $x$.<br>For any $x \in \mathbb{R}$, $10^x > 0$.<br>$\therefore D = \mathbb{R}$, $R = \{y \in \mathbb{R} \mid y > 0\}$ |
| Suppose that $g(x) = -4f(2x+18) - 7$. To obtain the graph of $g$, the following transformations must be performed to $f$:<br>• Vertical stretch by a factor of 4, reflection in the $x$-axis, followed by a shift down by 7 units ✓<br>• Horizontal compression by a factor of $1/2$ followed by a shift 18 units left. | F | $g(x) = -4f(2x+18) - 7 = -4f(2(x+9)) - 7$<br>Vertical transformations are correct<br>Horizontal should be<br>1. Compress by factor of $\frac{1}{2}$<br>2. Shift 9 units left<br>OR<br>1. Shift 18 units left<br>2. Compress by a factor of $\frac{1}{2}$ |