**MHF 4U0** | **Semester 2, 2009 - 2010**

**Grade 12 Advanced Functions (University Preparation)**
**Unit 0 – Diagnostic Test on Review Material**

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***Victim*:** Mr. Solutions

Wow! This was a pleasure to mark!!

| KU | APP | TIPS | COM |
|---|---|---|---|
| 32/32 | 20/20 | 12/12 | 18/18 |

**1.** ***Evaluate*** each of the following ***expressions***. For full credit, show all work! **(12 KU)**

**(a)** $576^{\frac{3}{2}}$

$= \left(\sqrt{576}\right)^3$

$= 24^3$

$= 13824$

**(b)** $h(-2)$, if $h(x) = 5\left(\frac{2}{3}\right)^{-x}$

$h(-2) = 5\left(\frac{2}{3}\right)^{-(-2)}$

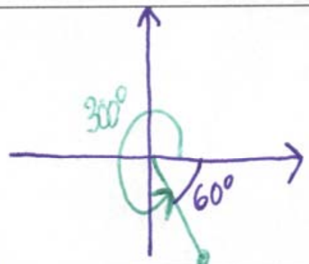
$= 5\left(\frac{2}{3}\right)^2$

$= \left(\frac{5}{1}\right)\left(\frac{4}{9}\right) = \frac{20}{9}$

**(c)** $\csc 300°$

$= -\csc 60°$

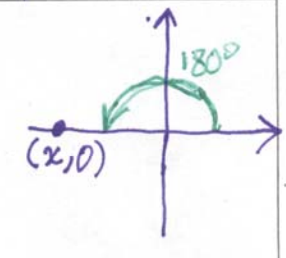
$= -\frac{2}{\sqrt{3}}$




**(d)** $\cot 180°$

is undefined

$\left(\cot\theta = \frac{x}{y}\right.$ and $y = 0$ when $\left.\theta = 180°\right)$



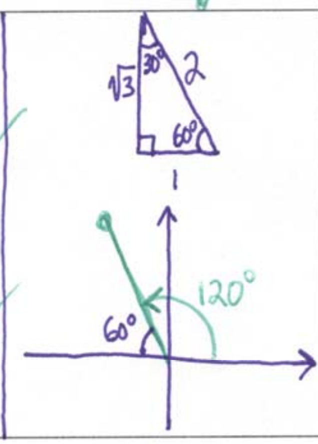

**(e)** $t_6$, if $t_n = -3\left(2^{n+1}\right)$

$t_6 = -3\left(2^{6+1}\right)$

$= -3(2^7)$

$= -3(128)$

$= -384$




**(f)** $g(120°)$, if $g(\theta) = 5\tan^2\theta - 2\sin\theta + \sec\theta$

$g(120°) = 5\tan^2(120°) - 2\sin 120° + \sec 120°$

$= 5(-\tan 60°)^2 - 2\sin 60° - \sec 60°$

$= 5(-\sqrt{3})^2 - 2\left(\frac{\sqrt{3}}{2}\right) - 2$

$= 5(3) - \sqrt{3} - 2$

$= 15 - \sqrt{3} - 2$

$= 13 - \sqrt{3}$

**2.** ***Simplify*** each of the following ***expressions***. For full credit, show all work! **(6 KU)**

**(a)** $\frac{-2a}{2a+b} - \frac{3b}{2b+a}$

$= \frac{-2a(2b+a)}{(2a+b)(2b+a)} - \frac{3b(2a+b)}{(2a+b)(2b+a)}$

$= \frac{-4ab - 2a^2 - (6ab + 3b^2)}{(2a+b)(2b+a)}$

$= \frac{-10ab - 2a^2 - 3b^2}{(2a+b)(2b+a)}$

**(b)** $\frac{\left(a^{-2}b^{-4}\right)^{-3}}{\left(a^{-1}b^{-3}\right)^7} = \frac{a^6 b^{12}}{a^{-7} b^{-21}}$

$= a^{13} b^{33}$

| KU | APP | TIPS | COM |
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3. *Solve* each of the following *equations*. For full credit, show all work! **(8 KU)**

**(a)** $6+z=5z^2$

$\therefore 5z^2-z-6=0$

$\therefore (5z-6)(z+1)=0$

$\therefore 5z-6=0$ or $z+1=0$

$\therefore z=\frac{6}{5}$ or $z=-1$

**(b)** $\frac{4}{s+1}-\frac{5}{s+2}=\frac{3}{s}$

$\therefore \left(\frac{s(s+1)(s+2)}{1}\right)\left(\frac{4}{\cancel{s+1}}\right)-\left(\frac{s(s+1)(s+2)}{1}\right)\left(\frac{5}{\cancel{s+2}}\right)=\left(\frac{s(s+1)(s+2)}{1}\right)\left(\frac{3}{\cancel{s}}\right)$

$\therefore 4s(s+2)-5s(s+1)=3(s+1)(s+2)$

$\therefore 4s^2+8s-5s^2-5s=3(s^2+3s+2)$

$\therefore -s^2+3s=3s^2+9s+6$

$\therefore 4s^2+6s+6=0$

$\therefore 2(2s^2+3s+3)=0$

$\therefore 2s^2+3s+3=0$

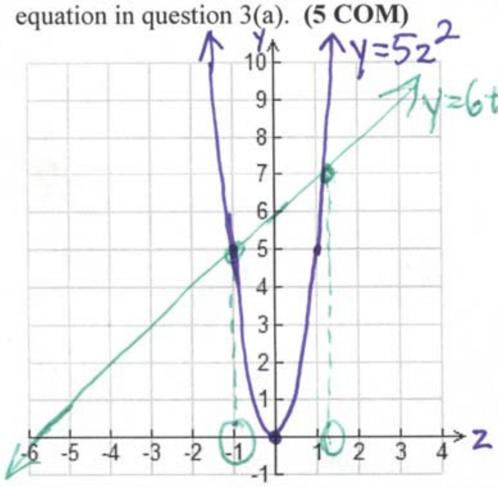
Now $b^2-4ac=3^2-4(2)(3)$

$=9-24$

$=-15<0$

$\therefore$ the equation has no real roots

4. Give a graphical representation of the equation in question 3(a). **(5 COM)**




5. Shown below is the solution to a simple quadratic equation. Explain the *logic* that allows us to reach the conclusion in the circled step. **(3 COM)**

$x^2+3x-4=0$

$\therefore (x-1)(x+4)=0$ ?

$\therefore x-1=0$ or $x+4=0$

$\therefore x=1$ or $x=-4$

Why does the statement $(x-1)(x+4)=0$ allow us to conclude that $x-1=0$ or $x+4=0$?

$(x-1)(x+4)=0$ means that $x-1$ TIMES $x+4$ gives a result of zero. The only way that the product of two numbers can be zero is if one of the numbers is zero. Therefore, $x-1$ is zero or $x+4$ is zero

6. *Fully factor* each of the following *expressions*. For full credit, show all work! **(6 KU)**

**(a)** $18w^8z^{10}-21w^8z^9-9w^8z^8$

$=3w^8z^8(6z^2-7z-3)$

$=3w^8z^8[6z^2-9z+2z-3]$

$=3w^8z^8[3z(2z-3)+1(2z-3)]$

$=3w^8z^8(2z-3)(3z+1)$

**(b)** $(2w-3)^2-u^2+2uv-v^2$

$=(2w-3)^2-(u^2-2uv+v^2)$

$=(2w-3)^2-(u-v)^2$

$=(2w-3+u-v)(2w-3-(u-v))$

$=(2w+u-v-3)(2w-u+v-3)$

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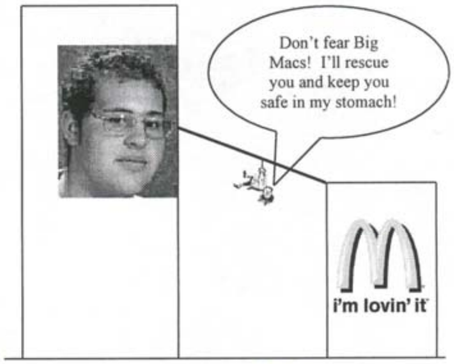
7. Andrew lives in an apartment building that is located exactly 75 m from his favourite restaurant. To allow him to get Big Macs as quickly as possible, he wants to install a steel cable connecting his balcony to the roof of the restaurant. This would allow him to slide along the cable directly to the source of the burgers.

From his balcony, Andrew uses a theodolite to measure the ***angle of depression*** to the top of the building and finds it to be 30°. How long does the steel cable need to be? **(5 APP)**



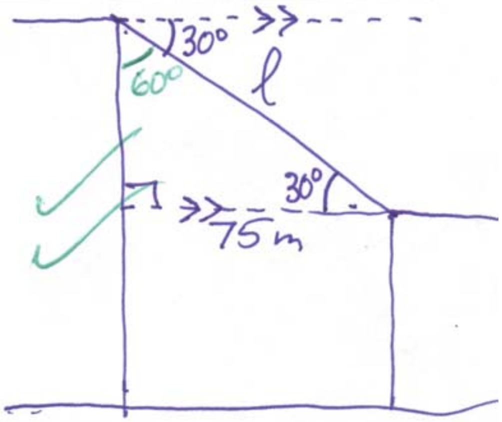

Let $\ell$ represent the length of the cable

Then, $\frac{75}{\ell} = \cos 30°$

$\therefore \frac{75}{\ell} = \frac{\sqrt{3}}{2}$

$\therefore \sqrt{3}\ell = 150$

$\therefore \ell = \frac{150}{\sqrt{3}} \doteq 86.6$




Andrew needs a cable that is about 90 m long.

8. For which value(s) of $k$ does $4x^2 - 2kx + 1 = 0$ have one and only one real root? Use the provided grid to show what the solution(s) of the equation(s) look like graphically. **(5 APP)**

$a=4, \ b=-2k, \ c=1$

For the quadratic equation $ax^2+bx+c=0$ to have only ONE real root, $b^2-4ac=0$

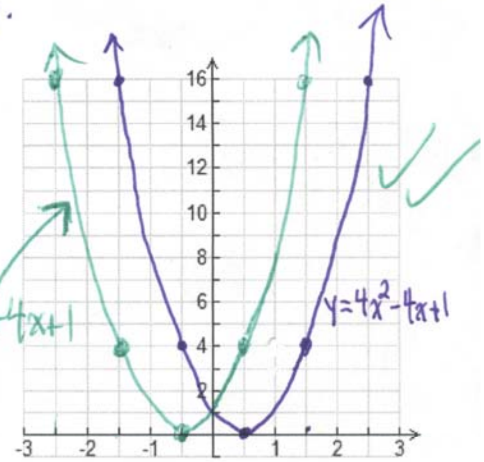
$\therefore (-2k)^2 - 4(4)(1) = 0$

$\therefore 4k^2 - 16 = 0$

$\therefore 4k^2 = 16$

$\therefore k^2 = 4$

$\therefore k = \pm 2$




$k=2$: $4x^2-4x+1=0$

$\therefore (2x-1)^2=0$

$\therefore x=\frac{1}{2}$

$k=-2$: $4x^2+4x+1=0$

$\therefore (2x+1)^2=0$

$\therefore x=-\frac{1}{2}$

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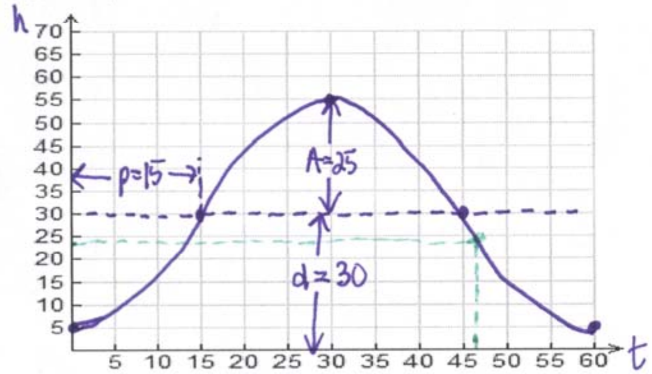
**9.** A Ferris wheel of radius 25 m rotates at a constant speed. It takes 60 s to complete one full rotation and the passengers get on at a point 5 m above the ground.

**(a)** Sketch a graph of the height of a passenger above the ground, in metres, versus time, in seconds. Assume that the passenger gets on exactly at time zero seconds. **(4 APP)**




**(b)** Let $h(t)$ represent the height of a passenger above the ground, in metres, at time $t$ seconds. Write an equation for $h(t)$. **(4 APP)**

$T = 60\text{s} = \frac{360^\circ}{k} \longrightarrow k = 6$

Using $f(t) = \sin t$ as the base function, we can write an equation for $h(t)$ as follows:

$h(t) = 25 \sin 6(t-15) + 30$

(Infinitely many correct answers)

**(c)** How high would a passenger be after 47 s? **(2 APP)**

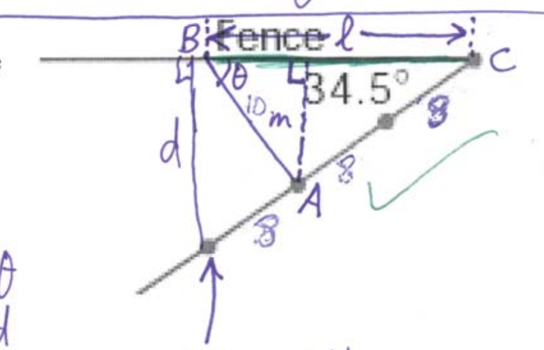
$h(47) = 25 \sin 6(47-15) + 30$

$= 25 \sin 192^\circ + 30$

$\doteq 24.8$ (agrees with graph)

After 47s, a passenger would be about 24.8 m above the ground.

**10.** An underground water sprinkler system is laid at an angle of 34.5° to a fence. The sprinkler jets are 8 m apart and have a range of 10 m. Determine the length of the fence that gets wet from the sprinklers. (**Hint:** One of the sprinklers shown in the diagram is too far from the fence.) **(6 TIPS)**




This sprinkler is too far from the fence

$\frac{d}{24} = \sin 34.5^\circ$

$\therefore d = 24 \sin 34.5^\circ$

$\doteq 13.6 > 10$

Let $\ell$ represent the length of the fence that gets wet (see diagram) and let $\theta$ represent the angle between the fence and line segment $AB$ (see diagram)

By the Law of Sines (using $\triangle ABC$),

$\frac{\sin\theta}{AC} = \frac{\sin 34.5^\circ}{AB}$

$\therefore \frac{\sin\theta}{16} = \frac{\sin 34.5^\circ}{10}$

$\therefore \sin\theta = \frac{16 \sin 34.5^\circ}{10}$

$\therefore \theta \doteq 65^\circ$

$\therefore \angle BAC \doteq 180^\circ - 34.5^\circ - 65^\circ = 80.5^\circ$

By the Law of Cosines

$\ell^2 \doteq 10^2 + 16^2 - 2(10)(16)\cos 80.5^\circ$

$\therefore \ell \doteq 17.4$

About 17.4 m of the fence will get wet.

| KU | APP | TIPS | COM |
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**11.** State whether each of the following is true or false. Provide an explanation to support each response. Keep the following points in mind:

- If a mathematical statement is said to be *true*, it must be true in ***all possible cases***.
- A ***general proof*** is required to demonstrate that a statement is *true*. The proof must demonstrate the truth of the statement in all possible cases! Clearly, any number of examples cannot accomplish this goal.
- To demonstrate that a statement is ***false***, it is only necessary to produce a ***single example*** that contradicts the statement. Such an example is called a ***counterexample***. **(6 TIPS)**

| ***Statement*** | ***True or False?*** | ***Proof, Counterexample or Explanation*** |
|---|---|---|
| $(a+b)^4 = a^4 + b^4$ | F | Let $a=b=1$. Then, <br> L.S. $=(1+1)^4 = 2^4 = 16$ <br> R.S. $= 1^4 + 1^4 = 1+1 = 2$ <br> L.S. $\neq$ R.S. <br> $\therefore (a+b)^4 \neq a^4+b^4$ |
| For all functions $p$, $p^{-1}(x) = \dfrac{1}{p(x)}$ | F | $p^{-1}(x)$ means the inverse function of $p$ evaluated at $x$ <br> $\dfrac{1}{p(x)}$ is the reciprocal of $p(x)$ |
| $\dfrac{x}{p} + \dfrac{y}{q} = \dfrac{x+y}{p+q}$ | F | Let $x=y=p=q=1$. Then, <br> L.S. $= \frac{1}{1} + \frac{1}{1} = 1+1 = 2$ <br> R.S. $= \frac{1+1}{1+1} = \frac{2}{2} = 1$ <br> L.S. $\neq$ R.S. $\therefore \frac{x}{p} + \frac{y}{q} \neq \frac{x+y}{p+q}$ <br> Note: To add fractions, a common denominator is required. |
| $\dfrac{\sin\theta}{\theta} = \dfrac{\sin\cancel{\theta}}{\cancel{\theta}} = \sin$ | F | "sin" is a function, NOT a number <br> $\therefore \frac{\sin\theta}{\theta} \neq \sin$ |
| $\dfrac{a^2+2a+1}{a+1} = \dfrac{a^2+2\cancel{a}+\cancel{1}}{\cancel{a}+\cancel{1}} = a^2+2$ | F | Let $a=1$. Then <br> L.S. $= \frac{1^2+2(1)+1}{1+1} = \frac{4}{2} = 2$ <br> R.S. $= 1^2+2 = 3$ <br> L.S. $\neq$ R.S. $\longrightarrow \frac{a^2+2a+1}{a+1} \neq a^2+2$ |
| Suppose that $g(x) = -4f(2x+18) - 7$. To obtain the graph of $g$, the following transformations must be performed to $f$: <br> • Vertical stretch by a factor of 4, reflection in the $x$-axis, *followed by* a shift down by 7 units <br> • Horizontal compression by a factor of 1/2 *followed by* a shift 18 units left. | F | • Without factoring $2x+18$, order would be 1. Shift 18 Left <br> 2. Compress horiz. by $\frac{1}{2}$ <br> • With factoring, $2x+18 = 2(x+9)$ <br> 1. Compress horiz. by factor of $\frac{1}{2}$ <br> 2. Shift 9 to the left |

wrong order $x \to \times 2 \to +18 \to 2x+18$

$x \leftarrow \div 2 \leftarrow -18 \leftarrow 2x+18$

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