Trig Identities, Solving Trig Equations
Answer Section

MULTIPLE CHOICE

1. ANS: B PTS: 1 REF: Knowledge and Understanding
   OBJ: 7.2 - Compound Angle Formulas
2. ANS: A PTS: 1 REF: Knowledge and Understanding
   OBJ: 7.2 - Compound Angle Formulas
3. ANS: C PTS: 1 REF: Knowledge and Understanding
   OBJ: 7.2 - Compound Angle Formulas
4. ANS: A PTS: 1 REF: Application OBJ: 7.2 - Compound Angle Formulas
5. ANS: A PTS: 1 REF: Application OBJ: 7.2 - Compound Angle Formulas
6. ANS: B PTS: 1 REF: Application OBJ: 7.2 - Compound Angle Formulas
7. ANS: B PTS: 1 REF: Application OBJ: 7.3 - Double Angle Formulas
8. ANS: D PTS: 1 REF: Application OBJ: 7.3 - Double Angle Formulas
9. ANS: C PTS: 1 REF: Knowledge and Understanding
   OBJ: 7.4 - Proving Trigonometric Identities
10. ANS: B PTS: 1 REF: Application
    OBJ: 7.4 - Proving Trigonometric Identities
11. ANS: C PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.4 - Proving Trigonometric Identities
12. ANS: A PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.5 - Solving Linear Trigonometric Equations
13. ANS: B PTS: 1 REF: Thinking
    OBJ: 7.5 - Solving Linear Trigonometric Equations
14. ANS: A PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.5 - Solving Linear Trigonometric Equations
15. ANS: A PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.5 - Solving Linear Trigonometric Equations
16. ANS: B PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.6 - Solving Quadratic Trigonometric Equations
17. ANS: D PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.6 - Solving Quadratic Trigonometric Equations
18. ANS: A PTS: 1 REF: Knowledge and Understanding
    OBJ: 7.6 - Solving Quadratic Trigonometric Equations
19. ANS: A PTS: 1 REF: Application
    OBJ: 7.6 - Solving Quadratic Trigonometric Equations
20. ANS: B PTS: 1 REF: Application
    OBJ: 7.6 - Solving Quadratic Trigonometric Equations

SHORT ANSWER

21. ANS:
    $\frac{\sqrt{3}-2}{2\sqrt{2}}$

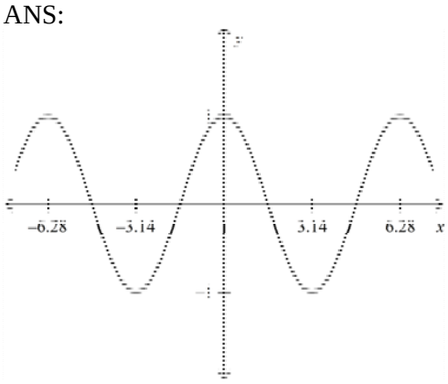
    PTS: 1 REF: Application OBJ: 7.2 - Compound Angle Formulas
22. ANS:
    Because the problem can be simplified to $\tan 90°$ by using the compound angle formula backwards.

    PTS: 1 REF: Communication OBJ: 7.2 - Compound Angle Formulas
23. ANS:
    $\frac{\sqrt{2}}{2}$

    PTS: 1 REF: Thinking OBJ: 7.2 - Compound Angle Formulas
24. ANS:
    $\cos(8x)$

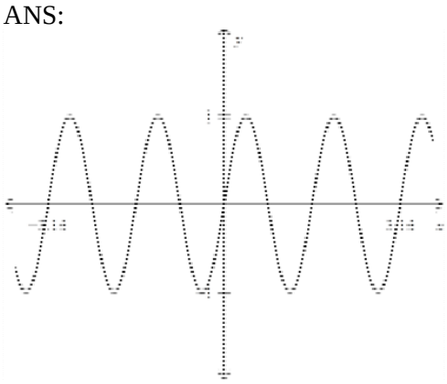
    PTS: 1 REF: Knowledge and Understanding OBJ: 7.3 - Double Angle Formulas

25. ANS:



PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

26. ANS:



PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

27. ANS:
$\frac{7}{25}$

PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

28. ANS:
$-\frac{120}{119}$

PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

29. ANS:
$-\frac{2}{3}$

PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

30. ANS:
$-\frac{1}{5}$

PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

31. ANS:
$\sin\theta$

PTS: 1 REF: Thinking OBJ: 7.4 - Proving Trigonometric Identities

32. ANS:
$\frac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$

PTS: 1 REF: Application OBJ: 7.4 - Proving Trigonometric Identities

33. ANS:
$\frac{\pi}{6}, \frac{5\pi}{6}$

PTS: 1 REF: Thinking OBJ: 7.5 - Solving Linear Trigonometric Equations

34. ANS:
44.43°, 135.57°

PTS: 1 REF: Thinking OBJ: 7.5 - Solving Linear Trigonometric Equations

35. ANS:
10 hours; noon

PTS: 1 REF: Thinking OBJ: 7.5 - Solving Linear Trigonometric Equations

36. ANS:
II and III

PTS: 1 REF: Knowledge and Understanding
OBJ: 7.5 - Solving Linear Trigonometric Equations

37. ANS:
60°

PTS: 1 REF: Knowledge and Understanding
OBJ: 7.5 - Solving Linear Trigonometric Equations

38. ANS:
$\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

PTS: 1 REF: Thinking OBJ: 7.6 - Solving Quadratic Trigonometric Equations

39. ANS:
70.53°, 99.59°, 260.41°, 289.47°

PTS: 1 REF: Thinking OBJ: 7.6 - Solving Quadratic Trigonometric Equations

40. ANS:
$\frac{3\pi}{2}$

PTS: 1 REF: Thinking OBJ: 7.6 - Solving Quadratic Trigonometric Equations

PROBLEM

41. ANS:

$$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3}$$

$$\sin(\frac{\pi}{3} + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$2\left(\frac{\sqrt{3}}{2} \cdot \frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$2\left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

PTS: 1 REF: Application OBJ: 7.2 - Compound Angle Formulas

42. ANS:

$$\sin(2\pi - x) = -\sin x$$

$$\sin 2\pi\cos x - \sin x\cos 2\pi = -\sin x$$

$$(0)\cos x - (\sin x)(1) = -\sin x$$

$$-\sin x = -\sin x$$

PTS: 1 REF: Application OBJ: 7.2 - Compound Angle Formulas

43. ANS:

a) $\frac{33}{65}$

b) $\frac{16}{65}$

c) The ratios are determined using a compound angle formula and the component ratios, without any statement of the angle measurements.

PTS: 1 REF: Thinking OBJ: 7.2 - Compound Angle Formulas

44. ANS:

a) It doesn't matter because the double angle formula needed is $\sin 2\theta = 2\sin\theta\cos\theta$, and the commutative law of multiplication allows for either number to be sine/cosine.

b) i) $\sin x = \frac{7}{25}$

$$\sin 2x = 2 \times \frac{7}{25} \times \frac{24}{25} = \frac{336}{625}$$

ii) $\cos x = \frac{7}{25}$

$$\sin 2x = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$$

PTS: 1 REF: Communication OBJ: 7.3 - Double Angle Formulas

45. ANS:

She must develop a formula for $\cos\frac{x}{2}$ in terms of $\cos x$.

$\cos 2x = 2\cos^2 x - 1$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$\frac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{\cos x + 1}{2}}$

$\cos\frac{\frac{\pi}{6}}{2} = \pm\sqrt{\frac{\cos\frac{\pi}{6} + 1}{2}}$

$\cos\frac{\pi}{12} = \pm\sqrt{\frac{\frac{\sqrt{3}}{2} + 1}{2}}$

$\cos\frac{\pi}{12} = \pm\frac{\sqrt{\sqrt{3} + 2}}{2}$

PTS: 1 REF: Thinking OBJ: 7.3 - Double Angle Formulas

46. ANS:

$\sin(a + b) = \sin a\ \cos b + \cos a\ \sin b$

$a = x$

$b = x$

$\sin(x + x) = \sin x\cos x + \cos x\sin x$

$\sin(2x) = 2\sin x\cos x$

PTS: 1 REF: Communication OBJ: 7.3 - Double Angle Formulas

47. ANS:

$$\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$$

$$\frac{1-\tan^2 x}{\sec^2 x} = \cos 2x$$

$$\cos^2 x(1-\tan^2 x) = \cos 2x$$

$$\cos^2 x - \cos^2 x \tan^2 x = \cos 2x$$

$$\cos^2 x - \frac{\cos^2 x \sin^2 x}{\cos^2 x} = \cos 2x$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos 2x = \cos 2x$$

PTS: 1 REF: Thinking OBJ: 7.4 - Proving Trigonometric Identities

48. ANS:

$$\sin x + \sin x \cot^2 x = \csc x$$

$$\sin x(1+\cot^2 x) = \csc x$$

$$\sin x \csc^2 x = \csc x$$

$$\frac{\sin x}{\sin^2 x} = \csc x$$

$$\frac{1}{\sin x} = \csc x$$

$$\csc x = \csc x$$

PTS: 1 REF: Application OBJ: 7.4 - Proving Trigonometric Identities

49. ANS:
a) 2
b) II, IV
c) $\frac{\pi}{4}$
d) $\frac{3\pi}{4}, \frac{7\pi}{4}$

PTS: 1 REF: Application OBJ: 7.5 - Solving Linear Trigonometric Equations

50. ANS:
a) 2
b) I, IV
c) $\frac{\pi}{6}$
d) $\frac{\pi}{6}, \frac{5\pi}{6}$

PTS: 1 REF: Application OBJ: 7.5 - Solving Linear Trigonometric Equations

51. ANS:
a) 2
b) II, III
c) 45.57°
d) 134.43°, 225.57°

PTS: 1 REF: Application OBJ: 7.5 - Solving Linear Trigonometric Equations

52. ANS:
a) 3 centimetres; on the 12/0 (the top of the clock)

b) Hour: Change the coefficient 3 to a 1 to shrink the highest and lowest points to 1 and -1. Multiply $\frac{1}{30}$ by $\frac{1}{3600}$ to get $\frac{1}{108\,000}$ (3600 is the number of seconds in an hour) to stretch the equation horizontally.

Minute: Change the coefficient 3 to a 5 to stretch the highest and lowest points to 4 and -4. Multiply $\frac{1}{30}$ by $\frac{1}{60}$ to get $\frac{1}{1800}$ (60 is the number of seconds in a minute) to stretch the equation horizontally.

c) Change cosine to sine in each equation. It works because sine and cosine represent the relationship of the sides of a triangle with the tip of the hands as the angle of interest.

PTS: 1 REF: Communication
OBJ: 7.5 - Solving Linear Trigonometric Equations

53. ANS:
a) $5\cos^2 x + 4\cos x - 1 = 0$

b) $\left(\cos x - \frac{1}{5}\right)(\cos x + 1) = 0$

c) 78.46°, 180°, 281.54°

PTS: 1 REF: Thinking OBJ: 7.6 - Solving Quadratic Trigonometric Equations

54. ANS:
a) 2.78 metres
b) Twice.
c) 0.52 seconds, 2.61 seconds, 4.71 seconds

PTS: 1 REF: Application OBJ: 7.6 - Solving Quadratic Trigonometric Equations

55. ANS:
a) 3 times
b) 3.96 cm
c) 0.16 seconds, 0.63 seconds

PTS: 1 REF: Application OBJ: 7.6 - Solving Quadratic Trigonometric Equations