MHF 4U0

Semester 2, 2009 – 2010

Grade 12 Advanced Functions (University Preparation)
Unit 1 – Exponential and Logarithmic Functions – Quest on 8.1 – 8.6

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Victim: Mr. Solutions

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1. *Complete* each of the following *statements*. **(8 KU)**

(a) $\log_6 1296 =$ 4 because $6^{4} = 1296$

(b) $\log_3 \frac{1}{243} =$ -5 because $3^{-5} = \frac{1}{243}$

(c) $\log_{\frac{1}{5}} 25 = -2$ because $\left(\frac{1}{5}\right)^{-2} = 25$

(d) $\log_{\sqrt{3}} 27 = 6$ because $\sqrt{3}^{6} = 27$

2. Rewrite

(a) $-3a^2b = \log_y(7a-5b)^2$ in exponential form. **(2 KU)**

$y^{-3a^2b} = (7a-5b)^2$

(b) $\left(\frac{3t}{2s}\right)^t = r^2$ in logarithmic form. **(2 KU)**

$\log_r\left(\frac{3t}{2s}\right)^t = 2$ OR $\log_{\frac{3t}{2s}} r^2 = t$

3. Use the laws of logarithms to *write* the following *expression* as a *single logarithm*. *Simplify* fully! **(4 APP)**

$-3\log\left[3w(z-2)^7\right] + 5\log\left[w^3(z-3)^5\right] - \frac{1}{2}\log\left[w^6(z-2)\right]$

$= \log\left[3w(z-2)^7\right]^{-3} + \log\left[w^3(z-3)^5\right]^5 - \log\left[w^6(z-2)\right]^{\frac{1}{2}}$

$= \log\left(\frac{\left[3^{-3}w^{-3}(z-2)^{-21}\right]\left[w^{15}(z-3)^{25}\right]}{w^3(z-2)^{\frac{1}{2}}}\right)$

$= \log\left(\frac{w^9(z-3)^{25}}{27(z-2)^{\frac{43}{2}}}\right)$

4. Evaluate without a calculator. Show all work! **(3 APP)**

$\log_3\left(\frac{\sqrt[3]{243}}{243}\right)$

$= \log_3 243^{\frac{1}{3}} - \log_3 243$

$= \frac{1}{3}\log_3 243 - \log_3 243$

$= \frac{1}{3}(5) - 5$

$= \frac{5}{3} - \frac{15}{3}$

$= -\frac{10}{3}$

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5. Solve each of the following equations.

(a) $5^{2x} = 11^{3x-1}$ **(3 APP)**

$\therefore \log 5^{2x} = \log 11^{3x-1}$ ✓

$\therefore 2x \log 5 = (3x-1)\log 11$ ✓

$\therefore 2x \log 5 = 3x \log 11 - \log 11$

$\therefore 2x \log 5 - 3x \log 11 = -\log 11$

$\therefore x(2\log 5 - 3\log 11) = -\log 11$

$\therefore x = \dfrac{-\log 11}{2\log 5 - 3\log 11} \doteq 0.6033$ ✓

(b) $8^{2w+3} - 8^{2w} = 1022$ **(4 APP)**

$\therefore 8^{2w}8^3 - 8^{2w} = 1022$

$\therefore 8^{2w}(8^3 - 1) = 1022$ ✓

$\therefore 8^{2w} = \dfrac{1022}{511} = 2$

$\therefore (2^3)^{2w} = 2$ ✓

$\therefore 2^{6w} = 2^1$ ✓

$\therefore 6w = 1$

$\therefore w = \dfrac{1}{6}$ ✓

(c) $2\log_3 a - \log_3 4 = 3\log_3 4$ **(3 APP)**

$\therefore \log_3 a^2 - \log_3 4 = \log_3 4^3$ ✓

$\therefore \log_3 \dfrac{a^2}{4} = \log_3 4^3$ ✓

$\therefore \dfrac{a^2}{4} = 4^3$

$\therefore a^2 = 4^4$

$\therefore a = \pm 4^2 = \pm 16$ ✓

Note: $a = 16$ satisfies the original equation BUT $a = -16$ DOES NOT ($\log_3(-16)$ is undefined).

$\therefore a = 16$

(d) $\log_2 z + \log_2 (z-2) = 3$ **(4 APP)** , $z > 2$

$\therefore \log_2[z(z-2)] = 3$ ✓

$\therefore z(z-2) = 2^3$ ✓

$\therefore z(z-2) = 8$

$\therefore z^2 - 2z - 8 = 0$

$\therefore (z-4)(z+2) = 0$

$\therefore z = 4$ or $z = -2$ ✓

But $z > 2$ for the logs in the original equation to be defined ✓

$\therefore z = 4$

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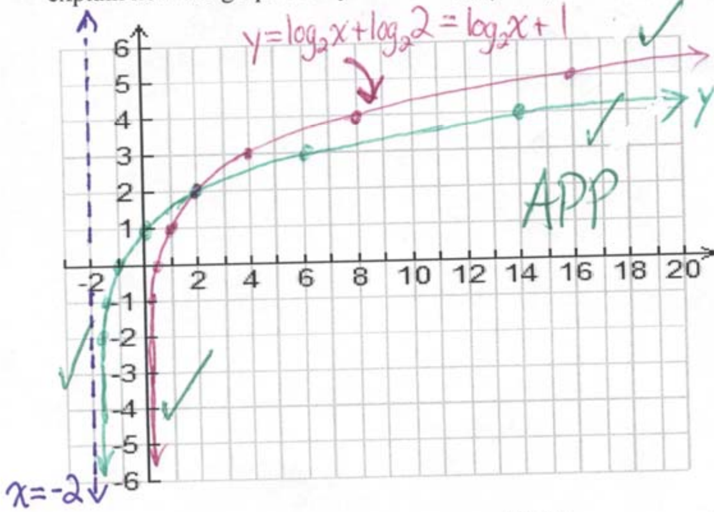
6. Use the grid shown below to sketch the graphs of the functions $f(x)=\log_2(x+2)$ and $g(x)=\log_2 x+\log_2 2$. Then explain how the graphs tell you that $\log_2(x+2)\neq\log_2 x+\log_2 2$. **(4 APP, 2 COM)**

$\log_2 2 = 1$



Graph $y=\log_2(x+2)$ and $y=\log_2 x+1$

As we can see from the graphs, $y=\log_2 x+\log_2 2$ and $y=\log_2(x+2)$ only intersect at one point. If $\log_2(x+2)$ and $\log_2 x+\log_2 2$ were equivalent expressions, the graphs would be identical. COM

7. Five hundred dollars is invested at a rate of 5.2% per annum ***compounded weekly***. How long would it take for the value of the investment to triple? **(5 APP)**

r = annual rate = 0.052
i = weekly rate = $0.052 \div 52 = 0.001$

t	$V(t)$
0	$500(1.001)^0$
1	$500(1.001)^1$
2	$500(1.001)^2$
3	$500(1.001)^3$
⋮	⋮
t	$500(1.001)^t$

Let t represent the time in weeks and $V(t)$ represent the value of the investment, in dollars, after t weeks. Then,

$$V(t)=500(1.001)^t$$

For the investment to triple,

$$V(t)=3(500)$$

$$\therefore 500(1.001)^t=3(500)$$

$$\therefore (1.001)^t=3 \quad \text{(take log of both sides)}$$

$$\therefore t\log 1.001=\log 3$$

$$\therefore t=\frac{\log 3}{\log 1.001}\doteq 1099$$

It would take about 1099 weeks (approximately 21.14 years) for the value of the investment to triple.

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