MHF 4U0 — Semester 1, 2009 - 2010

Grade 12 Advanced Functions (University Preparation)
Unit 2 – Identities Test

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Victim: Mr. Solutions

Superb work Mr. S!!

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8/8	19/19	12/12	10/10

1. Use the three methods indicated below to demonstrate that the equation $\sin(\pi+\theta) = -\sin\theta$ is an identity.

Compound Angle Identity (3 KU)

$\sin(\pi+\theta)$

$= \sin\pi\cos\theta + \cos\pi\sin\theta$

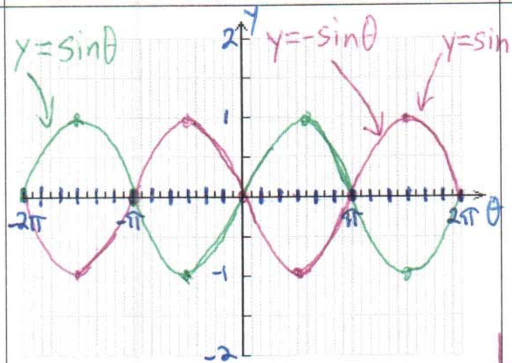
$= 0(\cos\theta) + (-1)\sin\theta$

$= 0 - \sin\theta$

$= -\sin\theta$

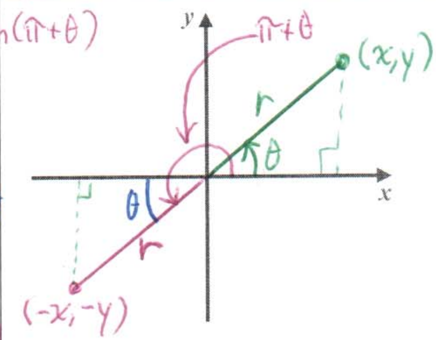
$\therefore \sin(\pi+\theta) = -\sin\theta$ IS an identity.

Graphical (Transformations) (3 KU)



Consider the graph of $y=\sin(\theta+\pi)$. It can be obtained by translating the graph of $y=\sin\theta$ π units to the left. Doing so produces the graph of $y=-\sin\theta$. (The graphs are coincident)

Angles of Rotation (3 TIPS)



$\sin(\pi+\theta) = \frac{-y}{r}$

$-\sin\theta = -\frac{y}{r} = \frac{-y}{r}$

$\therefore \sin(\pi+\theta) = -\sin\theta$

2. Evaluate the following trig ratios without using a calculator. Exact values are required!

(a) $\sin 15°$ **(4 APP)**

$= \sin(45° - 30°)$

$= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$ } optional

(b) $\cos\frac{17\pi}{12}$ **(4 APP)**

$= \cos\left(\frac{9\pi}{12} + \frac{8\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$

$= -\frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$ } optional

(c) $\sin 22.5°$ **(4 TIPS)**

$\cos 2\theta = 1 - 2\sin^2\theta$

Let $\theta = \frac{x}{2}$

$\therefore \cos x = 1 - 2\sin^2\frac{x}{2}$

$\therefore 2\sin^2\frac{x}{2} = 1 - \cos x$

$\therefore \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\therefore \sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Let $x = 45°$

$\therefore \sin 22.5° = +\sqrt{\frac{1-\cos 45°}{2}}$ (22.5° is in quadrant I)

$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$

$= \sqrt{\frac{1}{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)}$

$= \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

$= \sqrt{\frac{2-\sqrt{2}}{4}}$

$= \frac{\sqrt{2-\sqrt{2}}}{2}$ } optional

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3. Using the *three* methods listed below, demonstrate that the equation $\sin\left(x+\frac{\pi}{2}\right)=\sin x+\sin\frac{\pi}{2}$ *is not* an identity.

(a) *Counterexample* **(3 APP)**

Let $x=\frac{\pi}{2}$

Then, $L.S. = \sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=\sin\pi=0$

$R.S. = \sin\frac{\pi}{2}+\sin\frac{\pi}{2}$

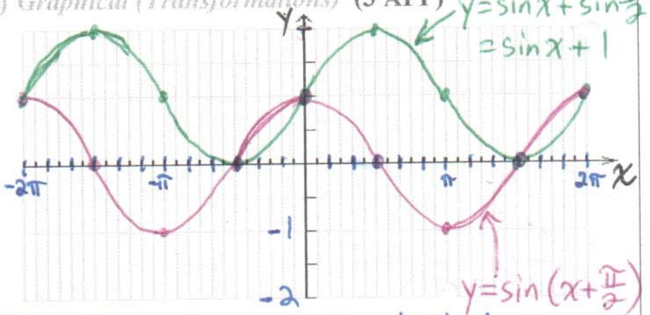
$= 1+1$

$= 2$

$\therefore L.S. \neq R.S.$

$\therefore$ the given equation is NOT an identity

(b) *Graphical (Transformations)* **(3 APP)**



Since the graphs are NOT coincident, the equation is NOT an identity.

(c) *Order of Operations* (2 KU)

L.S. → First, the sum of x and $\frac{\pi}{2}$ is found. Then sin is applied to the result. (First add, then apply sin)

R.S. → First, sin is applied to both x and $\frac{\pi}{2}$. Then the two resulting values are added. (First apply sin, then add)

Since the operations are applied in a different order, we cannot expect the results to be the same.

4. Prove that the following equations *are* identities.

(a) $\cos 4\theta = 1-8\sin^2\theta+8\sin^4\theta$ **(5 APP)**

$L.S. = \cos 4\theta$

$= \cos(2(2\theta))$

$= 1-2\sin^2 2\theta$

$= 1-2(2\sin\theta\cos\theta)^2$

$= 1-8\sin^2\theta\cos^2\theta$

$= 1-8\sin^2\theta(1-\sin^2\theta)$

$= 1-8\sin^2\theta+8\sin^4\theta$

$= R.S.$

$\therefore L.S. = R.S.$

(b) $\dfrac{\cos x\cos 2x}{\cos x+\sin x}+2\cos x\sin\frac{x}{2}\cos\frac{x}{2}=1-\sin^2 x$ **(5 TIPS)**

$L.S. = \dfrac{\cos x(\cos^2 x-\sin^2 x)}{\cos x+\sin x}+\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\cos x$

$= \dfrac{\cos x(\cos x-\sin x)\cancel{(\cos x+\sin x)}}{\cancel{\cos x+\sin x}}+\sin\left(2\left(\frac{x}{2}\right)\right)\cos x$

$= \cos^2 x-\sin x\cos x+\sin x\cos x$

$= \cos^2 x$

$= 1-\sin^2 x$

$= R.S.$

$\therefore L.S. = R.S.$

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