MHF 4U0 — **Semester 2, 2008 - 2009**

Grade 12 Advanced Functions (University Preparation)
Unit 3 – Major Test on Rational Functions

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Victim: Mr. Solutions Well done Mr. S. !!

KU	APP	TIPS	COM
14/14	22/22	17/17	10/10

1. Match each function with its graph. **(4 KU)**

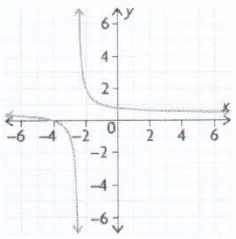
a) $h(x) = \dfrac{x+4}{2x+5}$ VA: $x = -\frac{5}{2}$, HA: $y = \frac{1}{2}$

b) $m(x) = \dfrac{2x-4}{x-2} = \dfrac{2(x-2)}{x-2} = 2,\ x \neq 2$ (hole)

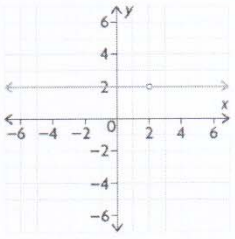
c) $f(x) = \dfrac{3}{x-1}$ VA: $x=1$, HA: $y=0$

d) $g(x) = \dfrac{2x-3}{x+2}$ VA: $x=-2$, HA: $y=2$

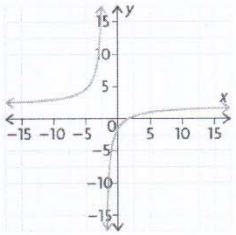
A



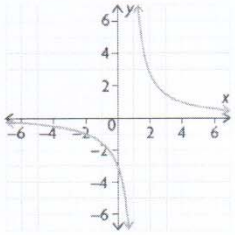
C



B



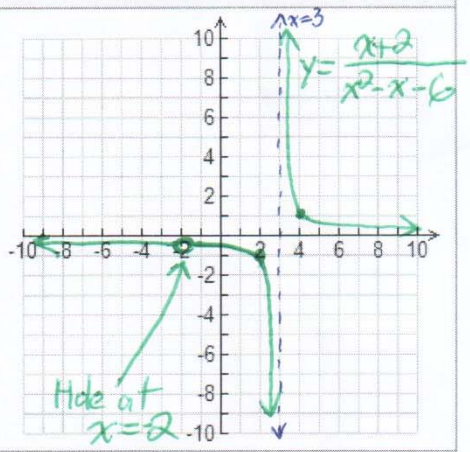
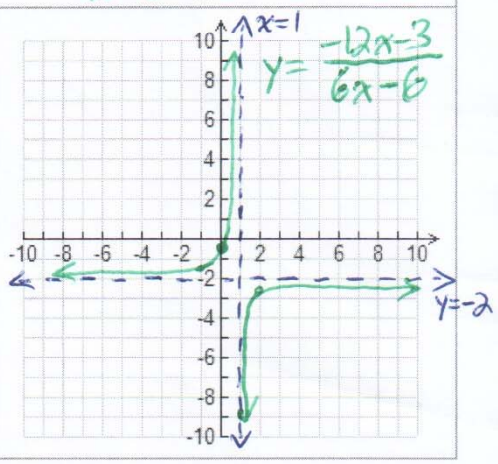
D



a) A b) C

c) D d) B

2. Complete the following table. **(10 KU, 6 APP)**

Equation of Function	*Characteristics of Function*	*Graph*
$f(x) = \dfrac{x+2}{x^2-x-6}$ $= \dfrac{(x+2)}{(x+2)(x-3)}$ $= \dfrac{1}{x-3},\ x \neq -2,\ x \neq 3$	Zeros: none y-intercept: $-\frac{1}{3}$ Domain: $\{x \in \mathbb{R};\ x \neq -2$ and $x \neq 3\}$ Range: $\{y \in \mathbb{R};\ y \neq 0\}$ Asymptote(s): $x=3$ $y=0$ As $x \to \infty$, $f(x) \to 0$ As $x \to -\infty$, $f(x) \to 0$ As $x \to 3^-$, $f(x) \to -\infty$ As $x \to 3^+$, $f(x) \to +\infty$ As $x \to -1^-$, $f(x) \to -\frac{1}{4}$ As $x \to -1^+$, $f(x) \to -\frac{1}{5}$	
$f(x) = \dfrac{-12x-3}{6x-6}$	Zeros: $-\frac{1}{4}$ y-intercept: $\frac{1}{2}$ Domain: $\{x \in \mathbb{R};\ x \neq 1\}$ Range: $\{y \in \mathbb{R};\ y \neq -2\}$ Asymptote(s): $x=1$, $y=-2$ As $x \to \infty$, $f(x) \to -2$ As $x \to -\infty$, $f(x) \to -2$ As $x \to 1^-$, $f(x) \to +\infty$ As $x \to 1^+$, $f(x) \to -\infty$	

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3. Solve.

(a) $\frac{-b}{b-1}=\frac{-3}{b+7}$ **(5 APP)** (Multiply B.S. by $(b-1)(b-7)$)

$\therefore -b(b+7)=-3(b-1)$

$\therefore -b^2-7b=-3b+3$

$\therefore -b^2-7b+3b-3=0$

$\therefore -b^2-4b-3=0$

$\therefore b^2+4b+3=0$

$\therefore (b+1)(b+3)=0$

$\therefore b+1=0$ or $b+3=0$

$\therefore b=-1$ or $b=-3$

(b) $\frac{2}{c+5}>\frac{3c}{c+10}$ **(6 APP)**

Don't multiply B.S. by $(c+5)(c+10)$ because a number of cases would need to be considered to decide if inequality needs to be reversed.

$\therefore \frac{2}{c+5}-\frac{3c}{c+10}>0$

$\therefore \frac{2(c+10)}{(c+5)(c+10)}-\frac{3c(c+5)}{(c+5)(c+10)}>0$

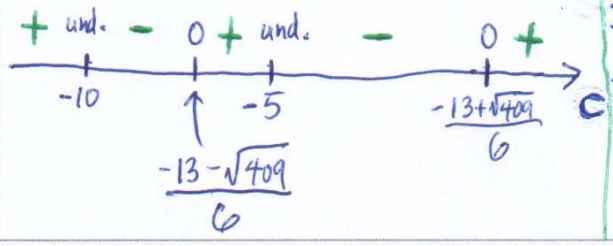
$\therefore \frac{2c+20-(3c^2+15c)}{(c+5)(c+10)}>0$

$\therefore \frac{-3c^2-13c+20}{(c+5)(c+10)}>0$

$\therefore \frac{3c^2+13c-20}{(c+5)(c+10)}<0$ (Both sides multiplied by -1)

The solution set is

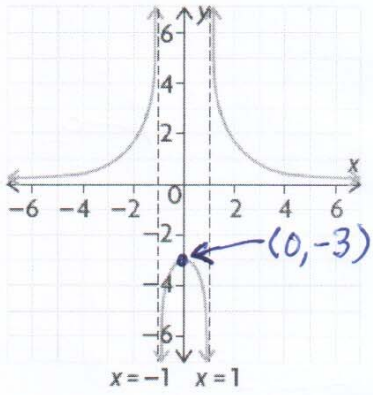
$\{c\in\mathbb{R} \mid -10<c<\frac{-13-\sqrt{409}}{6}$ or $-5<c<\frac{-13+\sqrt{409}}{6}\}$



$3c^2+13c-20=0$

$\therefore c=\frac{-13\pm\sqrt{409}}{6}$

4. The graph of the rational function f is shown at the right. If the equation of f has the form $f(x)=\frac{m}{x^2+kx+n}$, determine the values of k, m and n. **(5 TIPS)**



Since f has vertical asymptotes $x=-1$ and $x=1$

$\therefore x^2+kx+n=(x+1)(x-1)$

$\therefore x^2+kx+n=x^2-1$

$\therefore k=0$ and $n=-1$

$\therefore f(x)=\frac{m}{x^2-1}$

Since $(0,-3)$ lies on the graph of f, $f(0)=-3$

$\therefore \frac{m}{0^2-1}=-3$

$\therefore \frac{m}{-1}=-3$

$\therefore m=3$

Therefore, $k=0$, $n=-1$ and $m=3$

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5. As the date of the MHF4U0 exam drew nearer, Manavjot and Niroj became increasingly nervous. To help relieve their stress, they decided to walk along a straight line on which Mr. Nolfi had marked an origin.

At the time t seconds,

Niroj's distance from the origin is given by the function $s_N(t) = \frac{7t}{t^2+1}$ and

Manavjot's distance from the origin is given by the function $s_M(t) = t + \frac{5}{t+2}$.

(a) At what time(s) do Niroj and Manavjot collide? **(4 TIPS)**

Niroj and Manavjot collide when their distances from the origin are the same at a particular time. That is, they collide whenever

$$S_N(t) = S_M(t)$$

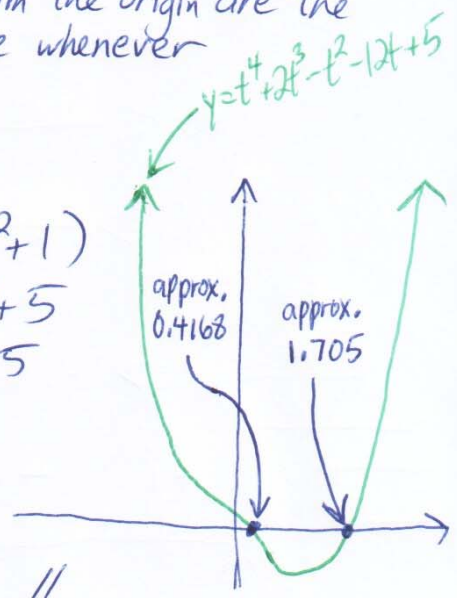
$$\therefore \frac{7t}{t^2+1} = t + \frac{5}{t+2}$$

$$\therefore 7t(t+2) = t(t+2)(t^2+1) + 5(t^2+1)$$

$$\therefore 7t^2+14t = t(t^3+2t^2+t+2) + 5t^2+5$$

$$\therefore 7t^2+14t = t^4+2t^3+t^2+2t+5t^2+5$$

$$\therefore t^4+2t^3-t^2-12t+5 = 0$$



By using a graphing calculator or graphing software, we find that $t \doteq 0.4168$ or $t \doteq 1.705$. Therefore, Manavjot and Niroj collide at about 0.42s and 1.7s. //

(b) When is Niroj closer to the origin than Manavjot? **(4 TIPS)**

Niroj is closer to the origin than Manavjot whenever

$$\frac{7t}{t^2+1} < t + \frac{5}{t+2}$$

Since $t^2+1 > 0$ for all $t \in \mathbb{R}$ and $t+2 > 0$ for all $t > 0$, we can multiply both sides by $(t+2)(t^2+1)$ without reversing the inequality. ($t+2 > 0$ since negative times are meaningless in this context). Following the same steps as in 5(a), we obtain

$$t^4+2t^3-t^2-12t+5 < 0.$$

From the graph in 5(a), we can conclude that t must lie between about 0.4168s and 1.705s. Therefore, Niroj is closer to the origin than Manavjot roughly between 0.4s and 1.7s.

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6. Three employees work at a shipping warehouse. Paco can fill an order two minutes faster than Tom but Carl fills an order one minute slower than Tom. When Tom and Paco work together they can fill an order in one minute and twenty seconds. When Paco and Carl work together, they take one minute and thirty seconds to fill an order.

(a) How long does each person take to fill an order? **(5 APP)**

80 s

$t \rightarrow$ time that it takes Tom to fill an order (in seconds)

$t-120 \rightarrow$ " " " " Paco to " " " (" ")

$t+60 \rightarrow$ " " " " Carl to " " " (" ")

fraction of order completed by Tom when working alone (in one second)

fraction of order completed by Paco in one second when working alone

fraction of order completed in one second (by Tom and Paco together)

$$\therefore \frac{1}{t} + \frac{1}{t-120} = \frac{1}{80}$$

$$\therefore 80(t-120) + 80t = t(t-120)$$

$$\therefore 160t - 9600 = t^2 - 120t$$

$$\therefore t^2 - 280t + 9600 = 0$$

$$\therefore (t-240)(t-40) = 0$$

$$\therefore t = 240 \text{ or } t = 40$$

inadmissable because this would mean that it Paco could fill an order in $40-120 = -80$ s.

Therefore, it takes Tom 4 minutes, Paco 2 minutes and Carl 5 minutes to fill an order.

(b) How long would it take all three of them working together to fill an order? **(4 TIPS)**

In one minute,

Tom completes $\frac{1}{4}$ of an order

Paco " $\frac{1}{2}$ of an order

Carl " $\frac{1}{5}$ of an order

Working together, the three of them can complete

$$\frac{1}{4} + \frac{1}{2} + \frac{1}{5} = \frac{5}{20} + \frac{10}{20} + \frac{4}{20} = \frac{19}{20} \text{ of an order}$$

in one minute.

Therefore, it takes them $\frac{20}{19} \doteq 1.05$ minutes to complete an order when working together. //

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