MHF 4U0 **Semester 1, 2008 - 2009**

Grade 12 Advanced Functions (University Preparation)
Unit 2 – Solving Trig Equations, Unit 3 – Polynomial Functions

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Victim: Mr. Solutions

KU	APP	TIPS	COM
/11	/16	/5	/5

1. Solve the trigonometric equation $\cos 2\theta = -\frac{\sqrt{3}}{2}$ for the interval $[0, 2\pi]$. In addition to providing an algebraic solution, give a graphical representation of the equation and show the solutions as angles of rotation.

Algebraic Solution **(5 APP)**

Hint: You don't need any identities!

$\cos 2\theta = -\frac{\sqrt{3}}{2}$

$\therefore 2\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\therefore 2\theta = \pi - \frac{\pi}{6}$ or $2\theta = \pi + \frac{\pi}{6}$

$\therefore 2\theta = \frac{5\pi}{6}$ or $2\theta = \frac{7\pi}{6}$

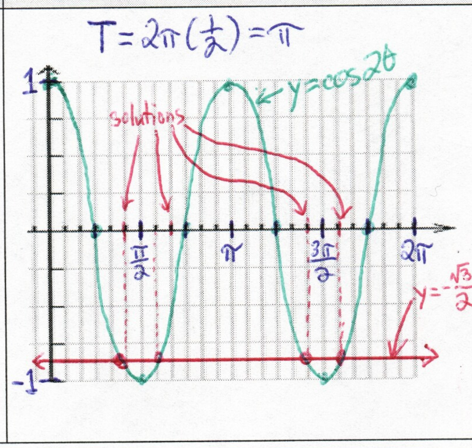
$\therefore \theta = \frac{5\pi}{12}$ or $\theta = \frac{7\pi}{12}$

Since period of $y = \cos 2\theta$ is π, $\frac{5\pi}{12} + \pi = \frac{17\pi}{12}$ and $\frac{7\pi}{12} + \pi = \frac{19\pi}{12}$ are also solutions (see graph)

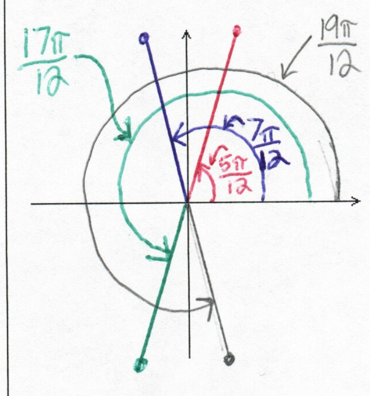
$\therefore \theta = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}$ or $\frac{19\pi}{12}$

Graphical Representation **(3 KU)**

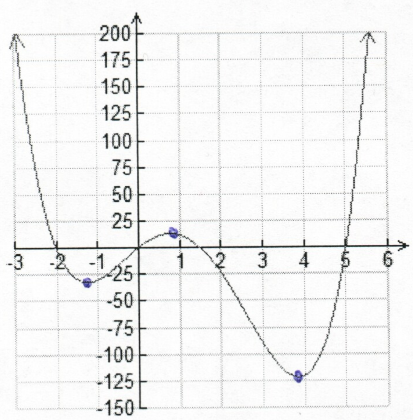
$T = 2\pi\left(\frac{1}{2}\right) = \pi$



Angles of Rotation **(3 KU)**



2. Given below is the graph of the function $p(x) = 2x^4 - 9x^3 - 11x^2 + 30x$. Determine each of the following. **(5 KU)**



(a) *End Behaviours*	(b) *Number of Turning Points* (Mark the turning points on the graph)
As $x \to \infty$, $y \to \infty$ As $x \to -\infty$, $y \to \infty$	3
(c) *Intervals of Increase* (approximate)	**(d)** *Intervals of Decrease* (approximate)
$(-1.2, 0.8)$ $(3.9, \infty)$	$(-\infty, -1.2)$ $(0.8, 3.9)$

3. Given the polynomial function $q(x) = -3x^5 - 9x^4 + 4x^2 + 7x - 2$, determine each of the following. **(5 APP)**

(a) *End Behaviours*

As $x \to \infty$, $y \to -\infty$

As $x \to -\infty$, $y \to \infty$

(b) *Number of Possible…*

Zeros 1, 2, 3, 4 or 5 (odd degree polynomial must have at least one zero)

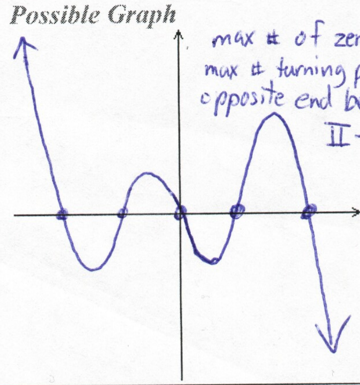
Turning Points 0, 2 or 4 (odd degree polynomial must have an even # of turning points)

(c) *Absolute Max, Min or Neither? Why?*

$q(x)$ is an odd degree polynomial $\therefore$ it has opposite end behaviours $\therefore$ there cannot be any absolute min or max points

(d) *Possible Graph*

max # of zeros = 5
max # turning points = 4
opposite end behaviour
II → IV



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4. Solve the equation $-6x^2+11x=-x^3+6$ **(6 APP)**

$\therefore x^3-6x^2+11x-6=0$

Let $f(x)=x^3-6x^2+11x-6$

Then $f(1)=1^3-6(1)^2+11(1)-6=0$.

Therefore, by the remainder theorem, when $f(x)$ is divided by $x-1$, the remainder is 0.

Thus, $x-1$ is a factor of $f(x)$.

$$
\begin{array}{r}
x^2-5x+6 \\
x-1\overline{)\,x^3-6x^2+11x-6} \\
\underline{x^3-x^2} \\
-5x^2+11x \\
\underline{-5x^2+5x} \\
6x-6 \\
\underline{6x-6} \\
0
\end{array}
$$

$\therefore x^3-6x^2+11x-6=(x-1)(x^2-5x+6)$

Continuing,

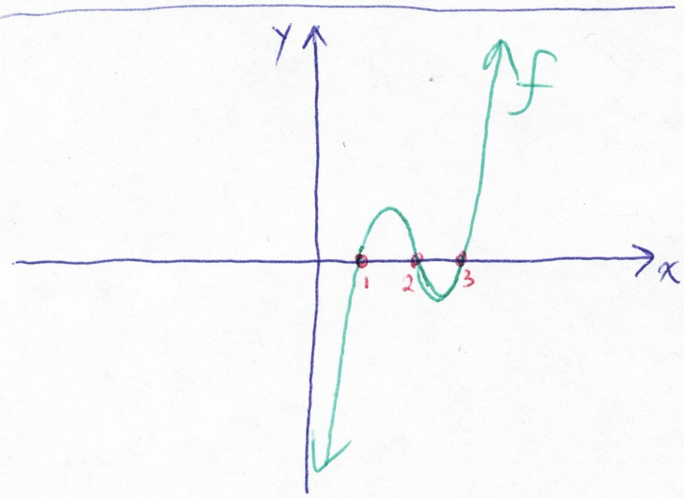
$x^3-6x^2+11x-6=0$

$\therefore (x-1)(x^2-5x+6)=0$

$\therefore (x-1)(x-2)(x-3)=0$

$\therefore x-1=0$ or $x-2=0$ or $x-3=0$

$\therefore x=1$ or $x=2$ or $x=3$ //



5. Explain why the polynomial equation $2x^{10}+13x^8+5x^6+x^4+19x^2+1=0$ has ***no real roots***. In addition, sketch a possible graph of the polynomial function $f(x)=2x^{10}+13x^8+5x^6+x^4+19x^2+1$.
(5 TIPS)

The polynomial $f(x)$ consists only of even powers of x.

For any power of x

x^n, where n is even,

$x^n \geq 0$ for all $x \in \mathbb{R}$

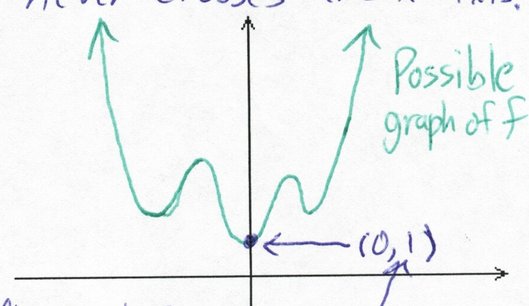
(i.e. whether x be negative or positive, x^n is positive, $x^n=0$ only if $x=0$)

$\therefore$ for all $x \in \mathbb{R}$

$2x^{10}+13x^8+5x^6+x^4+19x^2 \geq 0$

$\therefore 2x^{10}+13x^8+5x^6+x^4+19x^2+1 \geq 1$

$\therefore$ the polynomial $f(x)$ has an absolute minimum value of 1, which means that it never crosses the x-axis.



- As $x \to \pm\infty$, $f(x) \to +\infty$
- The max # of turning points is 9

absolute minimum point

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