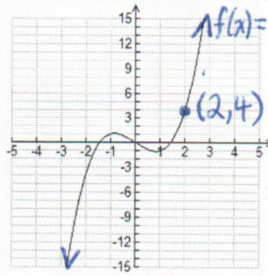
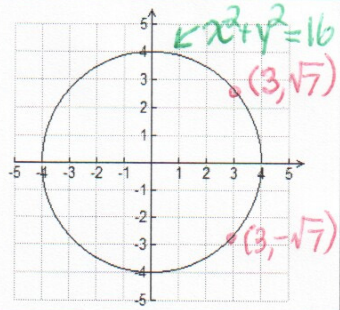
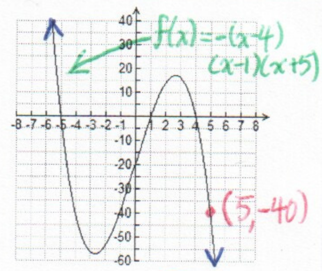
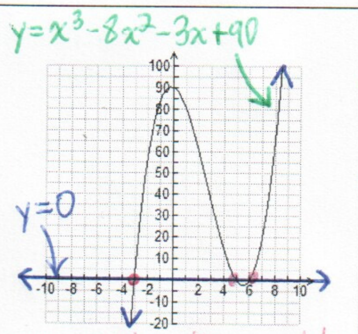
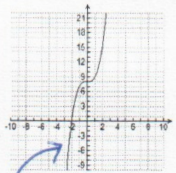
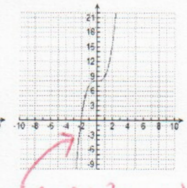
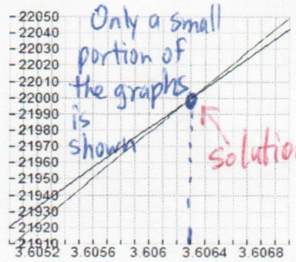
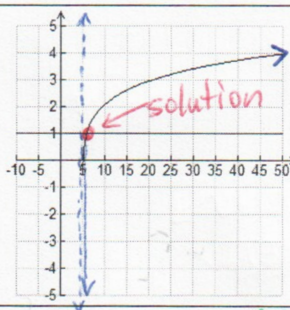
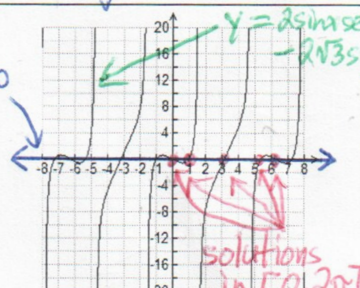
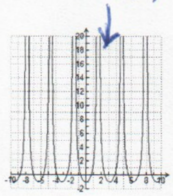
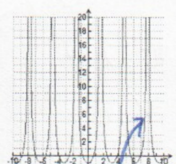
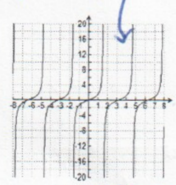
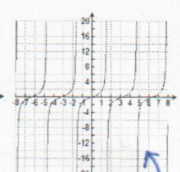
MHF4U0 Final Exam Review #2 – Equations and Graphs

1. The following is a list of types of equations that we have encountered in this course.
   (a) Classify each equation as an *identity*, an *equation to be solved* for the unknown, an *equation of a function* or an *equation of a relation*.
   (b) Give a geometric (graphical) representation of each equation.
   (c) For the equations that are identities, prove that the expression on the L.S. is *equivalent* to that on the R.S.
   (d) For the equations of functions/relations, use the equation to find a point that lies on the graph of the function/relation. (Mark that point on the graph.)
   (e) Solve the equations that are neither identities nor equations of functions/relations.

Equation	*Type of Equation*	*Geometric(Graphical) Representation*	*Proof/Solution/Evaluation to find Point on Graph*
$f(x) = x^3 - 2x$	function	 zeros are $-\sqrt{2}$, 0, $\sqrt{2}$	$f(2) = 2^3 - 2(2)$ $= 8 - 4 = 4$ $\therefore$ (2,4) lies on the graph of $f(x) = x^3 - 2x$
$x^2 + y^2 = 16$	relation		Let $x = 3$. Then, $3^2 + y^2 = 16$ $\therefore y^2 = 7$ $\therefore y = \pm\sqrt{7}$ $\therefore (3,\sqrt{7})$ and $(3,-\sqrt{7})$ lie on circle.
$f(x) = -(x-4)(x-1)(x+5)$	function	 zeros are –5, 1, 4	$f(5) = -(5-4)(5-1)(5+5)$ $= -1(4)(10) = -40$ $\therefore$ (5,-4) lies on the graph of f
Divisors of 90 are $\pm1, \pm2, \pm3, \pm5, \pm6, \pm9, \pm10, \pm15, \pm18, \pm30, \pm45, \pm90$ $x^3 - 8x^2 - 3x + 90 = 0$ If $x=-3$, $(-3)^3 - 8(-3)^2 - 3(-3) + 90$ $= -27 - 8(9) + 9 + 90 = 0$ $\therefore x+3$ is a factor $x+3 \overline{) x^3 - 8x^2 - 3x + 90}$ quotient $x^2 - 11x + 30$ $x^3 + 3x^2$ $-11x^2 - 3x$ $-11x^2 - 33x$ $30x + 90$ $30x + 90$ 0	equation to be solved	 points of intersection are solutions	$x^3 - 8x^2 - 3x + 90 = 0$ $\therefore (x+3)(x^2 - 11x + 30) = 0$ $\therefore (x+3)(x-5)(x-6) = 0$ $\therefore x+3=0,\ x-5=0$ or $x-6=0$ $\therefore x=-3,\ x=5$ or $x=6$

$a^3+b^3=(a+b)(a^2-ab+b^2)$	identity	Let $a=x$ and $b=2$   $y=x^3+8$ $\quad y=(x+2)(x^2-2x+4)$	R.S. $=(a+b)(a^2-ab+b^2)$ $=a^3-a^2b+ab^2+a^2b-ab^2+b^3$ $=a^3+b^3$ $=$ L.S. $\therefore$ L.S. $=$ R.S.
$4^{2x}=5^{2x-1}$	equation to be solved	 Only a small portion of the graphs is shown solution	$4^{2x}=5^{2x-1}$ $\therefore \log 4^{2x}=\log 5^{2x-1}$ $\therefore 2x(\log 4)=(2x-1)\log 5$ $\therefore 2x(\log 4)-2x(\log 5)=-\log 5$ $\therefore 2x(\log 4-\log 5)=-\log 5$ $\therefore x=\frac{-\log 5}{2(\log 4-\log 5)}\doteq 3.6063$
$\log_7(x+1)+\log_7(x-5)=1$	equation to be solved	 solution	$\log_7(x+1)+\log_7(x-5)=1$ $\therefore \log_7[(x+1)(x-5)]=\log_7 7$ $\therefore (x+1)(x-5)=7$ $\therefore x^2-4x-5=7$ $\therefore x^2-4x-12=0$ $\therefore (x-6)(x+2)=0$ $\therefore x=6$ ✓ or $\cancel{x=-2}$ inadmissable
$2\sin x\sec x-2\sqrt{3}\sin x=0$ (find all solutions in $[0,2\pi]$)	equation to be solved	$y=2\sin x\sec x-2\sqrt{3}\sin x$ $y=0$  solutions in $[0,2\pi]$	$2\sin x\sec x-2\sqrt{3}\sin x=0$ $\therefore 2\sin x(\sec x-\sqrt{3})=0$ $\therefore \sin x=0$ or $\sec x=\sqrt{3}$ $\therefore \sin x=0$ or $\cos x=\frac{1}{\sqrt{3}}$ $\therefore x=0,\pi,2\pi$ or $x\doteq 0.96$, or $x\doteq 2\pi-0.96\doteq 5.32$
$\tan^2 x-\cos^2 x=\frac{1}{\cos^2 x}-1-\cos^2 x$	identity	$y=\tan^2 x-\cos^2 x$   $y=\frac{1}{\cos^2 x}-1-\cos^2 x$	L.S. $=\sec^2 x-1-\cos^2 x$ $=\frac{1}{\cos^2 x}-1-\cos^2 x$ $=$ R.S. $\therefore$ L.S. $=$ R.S. Note: $\tan^2 x+1=\sec^2 x$ $\therefore \tan^2 x=\sec^2 x-1$
$\frac{2\tan 2x-\sec^2 x\tan 2x}{2}=\tan x$	identity	$y=\tan x$   $y=\frac{2\tan 2x-\sec^2 x\tan 2x}{2}$	L.S. $=\frac{1}{2}[\tan 2x(2-\sec^2 x)]$ $=\frac{1}{2}\left(\frac{2\tan x}{1-\tan^2 x}\right)(2-(1+\tan^2 x))$ $=\frac{1}{\cancel{2}}\left(\frac{\cancel{2}\tan x}{\cancel{1-\tan^2 x}}\right)\left(\frac{\cancel{1-\tan^2 x}}{1}\right)$ $=\tan x$ $=$ R.S. $\therefore$ L.S. $=$ R.S.

2. Match each equation with the most suitable graph. Explain your reasoning.

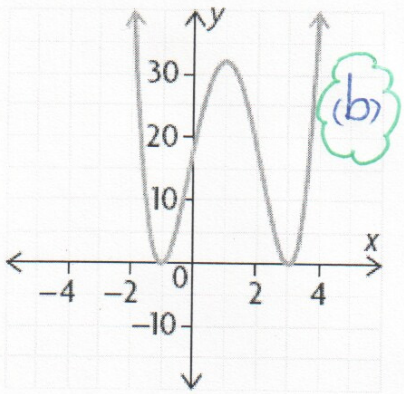
a) $f(x) = 2(x+1)^2(x-3)$

b) $f(x) = 2(x+1)^2(x-3)^2$

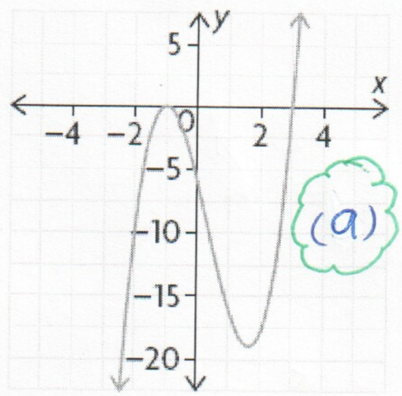
c) $f(x) = -2(x+1)(x-3)^2$

d) $f(x) = x(x+1)(x-3)(x-5)$

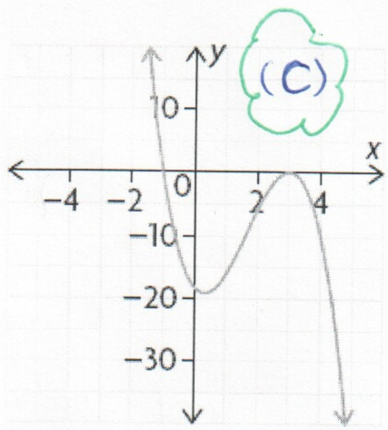
A



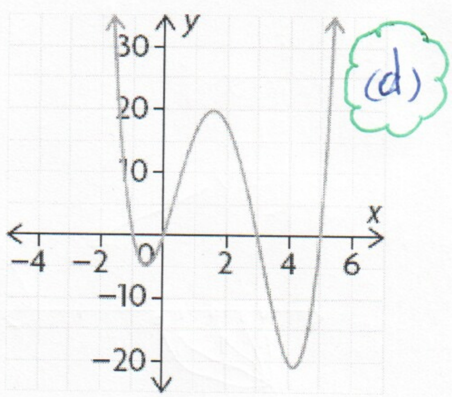
C



B



D



a, c → degree 3 → only possible graphs are B, C because of opposite end behaviours

→ zeros of B and C are the same, which does not help make a choice

→ (c) must match B because when expanded, the coefficient of x^3 would be negative

b, d → degree 4 → only possible graphs are A and D (same end behaviours)

→ (d) must match D because the zeros match