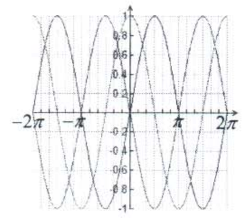
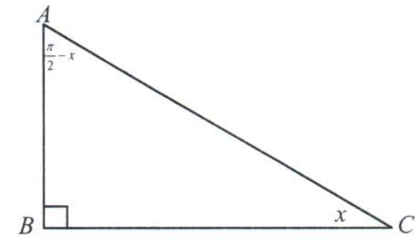
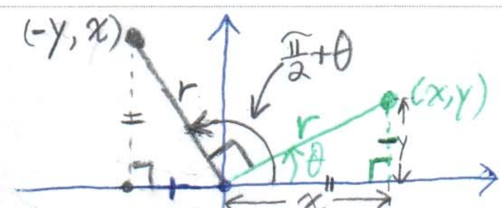
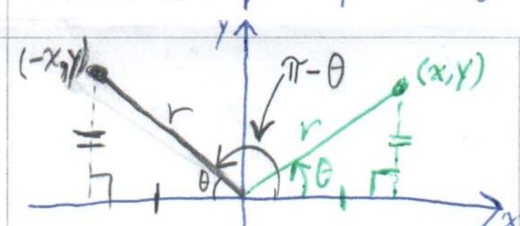
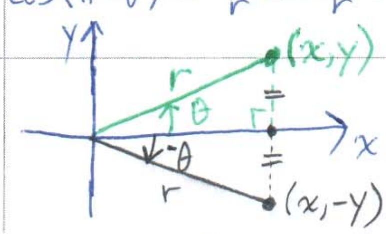
Solutions: Exercises on Equivalence of Trigonometric Expressions

Complete the following table. The first row is done for you.

Identity	*Graphical Justification*	*Justification using Right Triangle or Angle of Rotation*
$\sin(\frac{\pi}{2}-x)=\cos x$	Since $\sin\left(\frac{\pi}{2}-x\right)=\sin\left(-1\left(x-\frac{\pi}{2}\right)\right)$, the graph of $y=\sin\left(\frac{\pi}{2}-x\right)$ can be obtained by reflecting $y=\sin x$ in the y-axis, followed by a shift to the right by $\frac{\pi}{2}$. Once these transformations are applied, lo and behold, the graph of $y=\cos x$ is obtained! 	 $\cos x=\frac{BC}{AC}$ $\sin\left(\frac{\pi}{2}-x\right)=\frac{BC}{AC}$ $\therefore \cos x=\sin\left(\frac{\pi}{2}-x\right)$
$\cos(\frac{\pi}{2}-x)=\sin x$	$y=\cos\left(\frac{\pi}{2}-x\right)=\cos\left(-1\left(x-\frac{\pi}{2}\right)\right)$ The graph of $y=\cos\left(\frac{\pi}{2}-x\right)$ can be obtained by taking the graph of $y=\cos x$, reflecting in the y-axis and then shifting $\frac{\pi}{2}$ right. Doing this produces the graph of $y=\sin x$	Use the same triangle given above. $\sin x=\frac{AB}{AC}$ $\cos\left(\frac{\pi}{2}-x\right)=\frac{AB}{AC}$ $\therefore \cos\left(\frac{\pi}{2}-x\right)=\sin x$
$\cos(\frac{\pi}{2}+\theta)=-\sin\theta$	$y=\cos\left(\frac{\pi}{2}+\theta\right)=\cos\left(\theta+\frac{\pi}{2}\right)$. When the graph of $y=\cos\theta$ is shifted $\frac{\pi}{2}$ to the left, the graph of $y=-\sin\theta$ is obtained.	 $\cos\left(\frac{\pi}{2}+\theta\right)=\frac{-y}{r}=-\frac{y}{r}=-\sin\theta$
$\sin(\pi-\theta)=\sin\theta$	$y=\sin(\pi-\theta)=\sin(-1(\theta-\pi))$ $y=\sin\theta \rightarrow$ reflect in y-axis then shift π to the right The graph of $y=\sin\theta$ is obtained!	 $\sin(\pi-\theta)=\frac{y}{r}=\sin\theta$
$\cos(\pi-\theta)=-\cos\theta$	$y=\cos(\pi-\theta)=\cos(-1(\theta-\pi))$ $y=\cos\theta \rightarrow$ reflect in y-axis then shift π to the right The graph of $y=-\cos\theta$ is obtained.	Using the same diagram as in the previous row, we have $\cos(\pi-\theta)=\frac{-x}{r}=-\frac{x}{r}=-\cos\theta$
$\sin(-\theta)=-\sin\theta$	$y=\sin(-\theta)=\sin(-1\theta)$ $y=\sin\theta \rightarrow$ reflect in y-axis The graph of $y=-\sin\theta$ is obtained.	 $\sin(-\theta)=\frac{-y}{r}=-\frac{y}{r}=-\sin\theta$
$\cos(-\theta)=\cos\theta$	$y=\cos(-\theta)=\cos(-1\theta)$ $y=\cos\theta \longrightarrow$ reflect in y-axis. The graph of $y=\cos\theta$ is obtained!	Using the same diagram as in the previous row, $\cos(-\theta)=\frac{x}{r}=\cos\theta$