MHF 4U0 **Semester 2, 2008 - 2009**

Grade 12 Advanced Functions (University Preparation)
Unit 3 – Essential Features of Polynomial Functions

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***Victim*:** ______________________________

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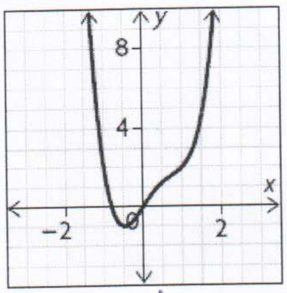
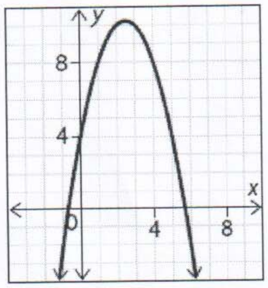
1. Use end behaviours, turning points and zeros to match each graph to the most likely polynomial equation. **(4 KU)**

a) $y = -x^2 + 5x + 4$ b) $y = 2x^3 - 3x^2 + x - 2$ c) $y = -3x^4 + 5x^3 + x^2 - 6x + 7$ d) $y = 2x^4 - 3x^3 + 3x$

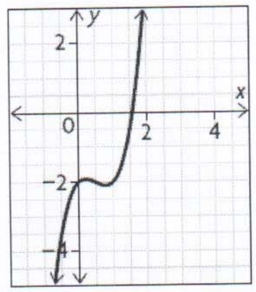
e) $y = x^2 + 5x + 4$ f) $y = 2/9(x-3)(x^2+3)$ g) $y = -7/16(5x+4)(7x-4)(x^2+1)$

A


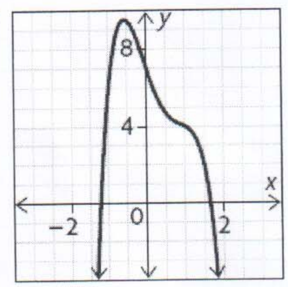
Equation: d

B


Equation: a

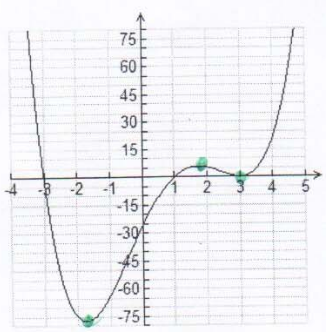
C


Equation: b

D


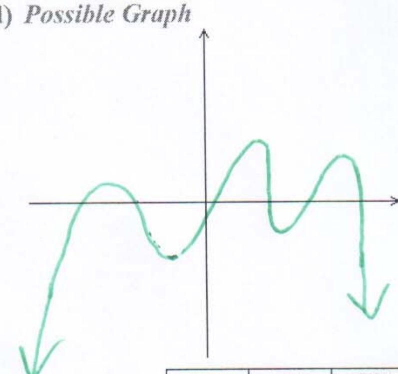
Equation: c

2. Given below is the graph of the polynomial function $p(x)$. Determine each of the following. **(6 KU)**



(a) *End Behaviours*	(b) *Number of Turning Points* (Mark the turning points on the graph)	(c) *Possible Equation of p(x)*
As $x \to \infty$, $y \to$ ∞ As $x \to -\infty$, $y \to$ ∞	3	Zeros of $p(x)$ are approximately -3, 1 and 3. Therefore, a possible equation of $p(x)$ is $p(x) = a(x+3)(x-1)(x-3)^2$, where a is some positive real number.
(d) *Intervals of Increase* (Approximate) $(-1.8, 1.8)$, $(3, \infty)$	(e) *Intervals of Decrease* (Approximate) $(-\infty, -1.8)$, $(1.8, 3)$	

3. Given the polynomial function $q(x) = -3x^6 - 9x^4 + 4x^2 + 7x - 3$, determine each of the following. **(5 KU)**

(a) *End Behaviours*	(b) *Number of Possible…*	(c) *Absolute Max, Min or Neither? Why?*	(d) *Possible Graph*
As $x \to \infty$, $y \to -\infty$ As $x \to -\infty$, $y \to -\infty$	Zeros: 0, 1, 2, 3, 4, 5, 6 Turning Points: 1, 3, 5 (e) *The y-intercept of q(x)* $q(0) = -3$ The y-intercept is -3	This function must have an absolute maximum point because it is an even degree polynomial with $y \to -\infty$ as $x \to \pm\infty$	

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4. Solve the equation $2x^4 = 9x^2 - 4$. **(6 APP)**

$\therefore 2x^4 - 9x^2 + 4 = 0$

$\therefore (x-2)(2x^3+4x^2-x-2) = 0$

$\therefore (x-2)(x+2)(2x^2-1) = 0$

$\therefore x-2=0,\ x+2=0$ or $2x^2-1=0$

$\therefore x=2,\ x=-2$ or $x = \pm \frac{1}{\sqrt{2}}$

Let $f(x) = 2x^4 - 9x^2 + 40$

Since $f(2)=0$, by the factor theorem, $x-2$ is a factor.

$$\begin{array}{r} 2x^3+4x^2-x-2 \\ x-2\,\overline{)\,2x^4+0x^3-9x^2+0x+4} \\ \underline{2x^4-4x^3} \\ 4x^3-9x^2 \\ \underline{4x^3-8x^2} \\ -x^2+0x \\ \underline{-x^2+2x} \\ -2x+4 \\ \underline{-2x+4} \\ 0 \end{array}$$

Let $g(x)=2x^3+4x^2-x-2$

Since $g(-2)$ by the factor theorem, $x+2$ is a factor of $g(x)$.

$$\begin{array}{r} 2x^2+0x-1 \\ x+2\,\overline{)\,2x^3+4x^2-x-2} \\ \underline{2x^3+4x^2} \\ 0x^2-x \\ \underline{0x^2+0x} \\ -x-2 \\ \underline{-x-2} \\ 0 \end{array}$$

Alternate Method: Let $y=x^2$. Then, the original equation can be written $2y^2-9y+4=0$.

$\therefore (2y-1)(y-4)=0$

$\therefore y=\frac{1}{2}$ or $y=4$

$\therefore x^2=\frac{1}{2}$ or $x^2=4$

$\therefore x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$ or $x=\pm 2$

5. Sketch the graph of $g(x) = -\frac{1}{8}\left(-3(x+2)\right)^3 + 1$ by applying transformations to the parent function $f(x)=x^3$. **(9 APP)**

(a) State the transformations required to obtain g from the base/parent/mother function $f(x)=x^3$.

Horizontal	*Vertical*
1. Compression by a factor of $\frac{1}{3}$, reflection in y-axis	1. Compression by a factor of $\frac{1}{8}$, reflection in x-axis.
2. Translation 2 units to the left	2. Translation up one unit

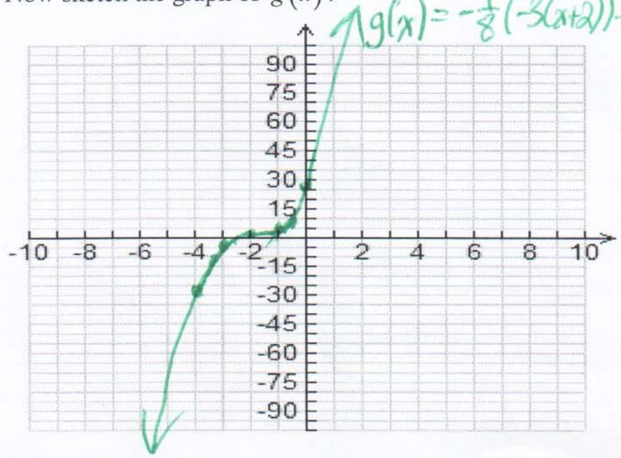
(b) Express the transformation in ***mapping notation***.

$(x, y) \to \left(-\frac{1}{3}x-2,\ -\frac{1}{8}y+1\right)$

(c) Apply the transformation to a few key points on the graph of the base function $f(x)=x^3$

Pre-image Point on $y=f(x)$	*Image Point* on $y=g(x)$
$(4,64)$	$(-\frac{10}{3}, -7)$
$(3,27)$	$(-3, -\frac{19}{8})$
$(0,0)$	$(-2, 1)$
$(-3,-27)$	$(-1, \frac{35}{8})$
$(-4,-64)$	$(-\frac{2}{3}, 9)$

(d) Now sketch the graph of $g(x)$.



Rough Work

$(6, 216)$ $(-4, -26)$

$(-6, -216)$ $(0, 29)$

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