MHF 4U0 | **Semester 1, 2008 - 2009**

Grade 12 Advanced Functions (University Preparation)
Unit 0 - Major Test - Review Material

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***Victim*:** Mr. Solutions

KU	APP	TIPS	COM
12 /12	24 /24	~~/18~~ 15/15	20 /20

1. *Evaluate* each of the following *expressions*. For full credit, show all work! **(12 KU)**

(a) $125^{\frac{2}{3}}$

$= \left(\sqrt[3]{125}\right)^2$

$= 5^2$

$= 25$

1K

(b) $h(-5)$, if $h(x) = \left(\frac{1}{2}\right)^{-x}$

$h(-5) = \left(\frac{1}{2}\right)^{-(-5)}$

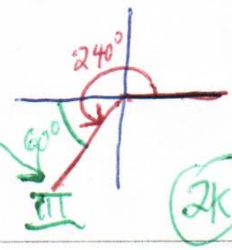
$= \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

1K

(c) $\csc 240°$

$= -\csc 60°$

$= -\frac{2}{\sqrt{3}}$



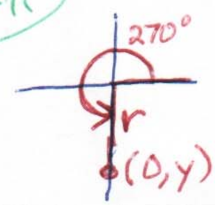
2K

(d) $\cot 270°$

$= \frac{x}{y}$
$= \frac{0}{y}$
$= 0$

see diagram

2K

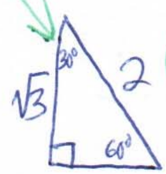


(e) t_4, if $t_n = -4(3^n)$

$t_4 = -4(3^4)$

$= -4(81)$

$= -324$



2K

(f) $g(300°)$, if $g(\theta) = 5\tan^2\theta - 2\sin\theta + \sec\theta$

$g(300°) = 5\tan^2(300°) - 2\sin 300° + \sec 300°$

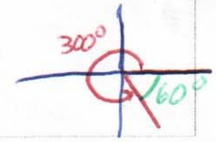
$= 5(-\tan 60°)^2 - 2(-\sin 60°) + \sec 60°$

$= 5(-\sqrt{3})^2 - 2\left(-\frac{\sqrt{3}}{2}\right) + 2$

$= 5(3) + \sqrt{3} + 2$

$= 17 + \sqrt{3}$

4K



2. *Simplify* each of the following *expressions*. For full credit, show all work! **(6 APP)**

(a) $\frac{2a}{4a^2+6a} - \frac{3}{2a+3}$

$= \frac{\cancel{2a}}{\cancel{2a}(2a+3)} - \frac{3}{2a+3}$

$= \frac{1}{2a+3} - \frac{3}{2a+3}$

$= -\frac{2}{2a+3}$

3A

(b) $\frac{\left(p^{-5}q^{-3}\right)^{-2}}{\left(p^{-2}q^{-5}\right)^4} = \frac{p^{10}q^6}{p^{-8}q^{-20}}$

$= p^{10-(-8)}q^{6-(-20)}$

$= p^{18}q^{26}$

3A

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3. ***Solve*** each of the following ***equations***. For full credit, show all work! **(8 APP)**

(a) $-3-8z=5z^2$

$\therefore 5z^2+8z+3=0$ ✓

$\therefore (5z+3)(z+1)=0$ ✓

$\therefore 5z+3=0$ or $z+1=0$ ✓

$\therefore z=-\frac{3}{5}$ or $z=-1$ ✓

4A

(b) $\frac{3}{s-1}+\frac{5}{s+3}=-\frac{1}{s-2}$

4A

$\therefore (s-1)(s+3)(s-2)\left(\frac{3}{s-1}+\frac{5}{s+3}\right)$ ✓

$=(s-1)(s+3)(s-2)\left(-\frac{1}{s-2}\right)$

$\therefore 3(s+3)(s-2)+5(s-1)(s-2)$

$=-(s-1)(s+3)$

$\therefore 3(s^2+s-6)+5(s^2-3s+2)=-(s^2+2s-3)$

$\therefore 3s^2+3s-18+5s^2-15s+10=-s^2-2s+3$ ✓

$\therefore 9s^2-10s-11=0$ ✓

$\therefore s=\frac{10\pm\sqrt{(-10)^2-4(9)(-11)}}{2(9)}$ ✓

$=\frac{10\pm\sqrt{496}}{18}=\frac{10\pm4\sqrt{31}}{18}=\frac{5\pm2\sqrt{31}}{9}$

4. ***Fully factor*** each of the following ***expressions***. For full credit, show all work! **(6 APP)**

(a) $18r^8t^{10}-21r^8t^9-9r^8t^8$

$6(-3)=-18$

$(-9)(2)=-18$

$-9+2=-7$

$=3r^8t^8(6t^2-7t-3)$

$=3r^8t^8(6t^2-9t+2t-3)$

$=3r^8t^8[3t(2t-3)+1(2t-3)]$

$=3r^8t^8(2t-3)(3t+1)$

3A

(b) $4a^2-12a+9-9x^2+24xy-16y^2$

$=4a^2-12a+9-(9x^2-24xy+16y^2)$

$=(2a-3)^2-(3x-4y)^2$

$=(2a-3+3x-4y)(2a-3-(3x-4y))$

$=(2a+3x-4y-3)(2a-3x+4y-3)$

3A

5. For which value(s) of k does $16x^2-kx+1=0$ have one real root? Use the provided grid to show what the solution(s) of the equation(s) look like graphically. **(4 APP)**

For exactly one real root,

$b^2-4ac=0$

$\therefore (-k)^2-4(16)(1)=0$

$\therefore k^2-64=0$

$\therefore k^2=64$

$\therefore k=\pm8$

$16x^2-8x+1=0$

$(4x-1)^2=0$

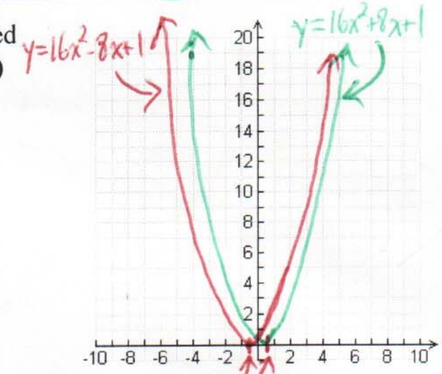
$\therefore x=\frac{1}{4}$

$16x^2+8x+1=0$

$\therefore (4x+1)=0$

$\therefore x=-\frac{1}{4}$

Rough work



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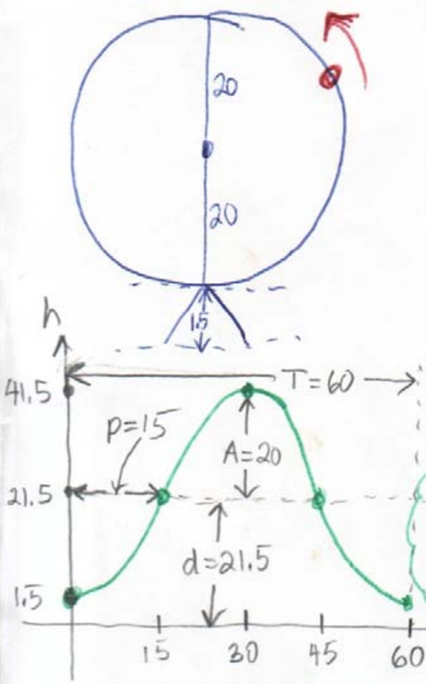
6. A Ferris wheel of radius 20 m rotates at a constant speed. It takes 60 s to complete one full rotation and the passengers get on at a point 1.5 m above the ground.

(a) Write an equation of a function that expresses the height $h(t)$ metres above the ground of a passenger at any time t seconds. Assume that the passenger gets on exactly at time zero seconds. **(6 TIPS)**



Maximum height of car above ground $= 20+20+1.5 = 41.5$ m

Minimum " " " " " $= 1.5$ m

$\therefore A = (41.5 - 1.5) \div 2 = 20$

$T = 60$s $\therefore k = \frac{360^\circ}{T} = \frac{360^\circ}{60} = 6$

$d = 21.5$ (see diagram)

$p = 15$ (shift graph of sin$t$ 15 units right)

$\therefore$ an equation for $h(t)$ is

$$h(t) = 20\sin(6(t-15)) + 21.5$$

infinitely many correct answers

(other answers:

$$h(t) = 20\cos(6(t-30)) + 21.5$$

$$h(t) = -20\cos(6t) + 21.5$$
)

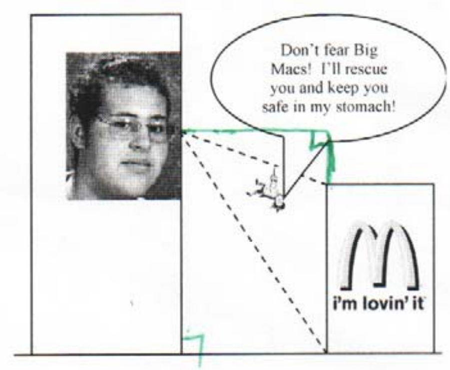
(b) How high is a passenger after 47 s? **(2 APP)**

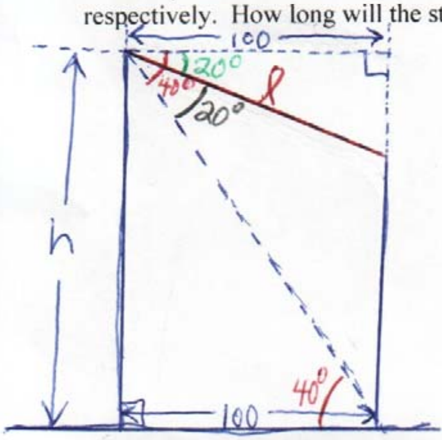
$$h(47) = 20\sin(6(47^\circ - 15^\circ)) + 21.5$$
$$= 20\sin(6(32^\circ)) + 21.5$$
$$= 20\sin(186^\circ) + 21.5$$
$$\doteq 17.3$$

Make sure calculator is in "degrees" mode.

7. Andrew lives in an apartment building that is located exactly 100 m from his favourite restaurant. To allow him to get Big Macs as quickly as possible, he wants to install a steel cable connecting his balcony to the roof of the restaurant. This would allow him to slide along the cable directly to the source of the burgers.

From his balcony, Andrew used a theodolite to measure the angles of depression to the top and to the base of the building. He found them to be 20° and 40° respectively. How long will the steel cable need to be? (~~6~~ 3 TIPS)





$\frac{100}{l} = \cos 20^\circ$

$\therefore l = \frac{100}{\cos 20^\circ}$

$\therefore l \doteq 106.4$

Andrew's cable needs to be about 110 m long.

marks reduced since this problem was so easy!

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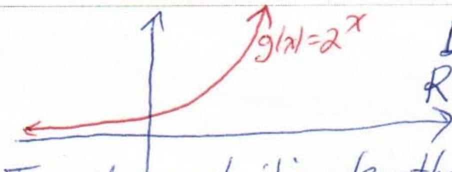
8. State whether each of the following is true or false. Provide an explanation to support each response. Keep the following points in mind:

- If a mathematical statement is said to be ***true***, it must be true in ***all possible cases***.
- A ***general proof*** is required to demonstrate that a statement is ***true***. The proof must demonstrate the truth of the statement in all possible cases! Clearly, any number of examples cannot accomplish this goal.
- To demonstrate that a statement is ***false***, it is only necessary to produce a ***single example*** that contradicts the statement. Such an example is called a ***counterexample***. **(6 TIPS)**

Statement	***True or False?***	***Proof, Counterexample or Explanation***
$(a+b)^3 = a^3 + b^3$	F	Suppose that $a=1$ and $b=1$. Then $L.S. = (1+1)^3 = 2^3 = 8$ $R.S. = 1^3+1^3 = 1+1 = 2$ $\Big\}$ $L.S. \neq R.S.$
For all functions g, $g^{-1}(x) = \dfrac{1}{g(x)}$	F	g^{-1} means the INVERSE function of g, not its reciprocal.
$\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{2}{x+y}$	F	$\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{y}{xy} + \dfrac{x}{xy} = \dfrac{x+y}{xy}$ Common denominator is needed.
The equation $3x^2 + 4y^2 - 6 = 0$ describes a parabola.	F	The graph of a quadratic function is a parabola. The general equation of a quadratic is $y = ax^2+bx+c$. There is no "y^2" term.
For the function $g(x) = 2^x$, $D = \{x \in \mathbb{R} : x > 0\}$ and $R = \mathbb{R}$. (Here D and R represent domain and range respectively.)	F	 $D = \mathbb{R}$ $R = \{y \in \mathbb{R} \mid y > 0\}$ From the graph it is clear that
Suppose that $g(x) = -6f(3x+27) - 14$. To obtain the graph of g, the following transformations must be performed to f: • Vertical stretch by a factor of –6 followed by a shift down by 14 units • Horizontal compression by a factor of 1/3 followed by a shift 27 units left.	F	$x \to \times 3 \to +27 \to 3x+27$ $3x+27 \to -27 \to \div 3 \ (\times \tfrac{1}{3}) \to x$ From the flowcharts it's clear that the shift of 27 units left should be performed before the compression.

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