Name: ________________________ Class: ___________________ Date: __________ ID: A

Trig Identities, Solving Trig Equations

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Simplify the expression $\dfrac{\tan\frac{5\pi}{8} - \tan\frac{3\pi}{8}}{1 + \tan\frac{5\pi}{8}\tan\frac{3\pi}{8}}$.

a. 0
b. 1
c. -1
d. undefined

____ 2. Which of the following is a simplification of $\sin 60° \cos 15° - \sin 15° \cos 60°$

a. $\dfrac{\sqrt{2}}{2}$
b. $\dfrac{1}{2}$
c. $\dfrac{\sqrt{3}}{2}$
d. $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$

____ 3. Which of the following expressions is NOT equivalent to $\sin 75°$?

a. $\sin 105°$
b. $\sin(45° + 30°)$
c. $\sin 45° + \sin 30°$
d. $\sin 45° \cos 30° + \sin 30° \cos 45°$

____ 4. Which expression is equivalent to $\cos 135° \cos 30° - \sin 135° \sin 30°$?

a. $-\cos 15°$
b. $\cos 105°$
c. $\cos(105° + 30°)$
d. $-\cos 105°$

____ 5. Which expression is equivalent to $-\cos\left(\dfrac{7\pi}{12}\right)$?

a. $\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$
b. $\cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$
c. $-\cos\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{6}\sin\frac{\pi}{4}$
d. $\sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$

____ 6. For an acute angle, θ, of a right triangle, $\sin\theta = \sin 60° \cos 45° - \sin 45° \cos 60°$. Which measurement is a possible size for θ?

a. 45°
b. 15°
c. 60°
d. 105°

____ 7. If $\tan x = \dfrac{6}{8}$, what is $\cos 2x$, given that $0 < x < \dfrac{\pi}{2}$?

a. $\dfrac{8}{5}$
b. $\dfrac{7}{25}$
c. $-\dfrac{7}{25}$
d. $-\dfrac{8}{5}$

____ 8. Determine the value of θ in the equation $\cos 2\theta + \sin^2\theta = \cos^2\theta + 3\theta - 6$.

a. 6
b. θ
c. $\cos 2\theta$
d. 2

____ 9. Peyton thinks that $\sin\theta = 1 - \cos\theta$ is an identity because $\sin 0° = 1 - \cos 0° = 0$. Which of the following is a counterexample to his claim?

a. 2π
b. $\dfrac{\pi}{2}$
c. π
d. $\sin\theta = 1 + \cos\theta$ is an identity.

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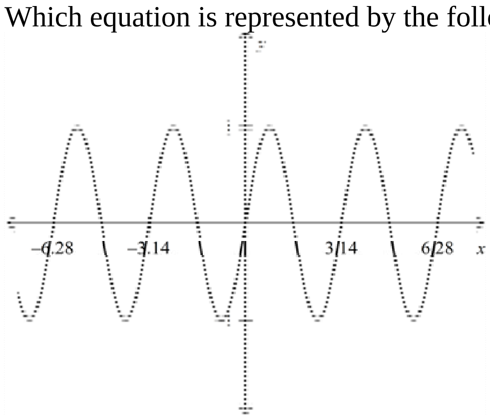
____ 10. Which equation is represented by the following graph?



a. $y = \sqrt{1 - \cos^2 x}$

c. $2\sin(\pi - x)$

b. $y = \dfrac{2\sqrt{1 - \cos^2 x}}{\sec x}$

d. $y = \cos^2 x - \sin^2 x$

____ 11. Which expression is NOT equivalent to $\csc 2x$?

a. $\dfrac{1}{2}\sec x \csc x$

c. $\dfrac{1}{2\sin x}$

b. $\dfrac{1}{2\cos x \sin x}$

d. $\dfrac{1}{\sin 2x}$

____ 12. Which value for x is a solution to $\cos x = \dfrac{\sqrt{2}}{2}$?

a. $-\dfrac{9\pi}{4}$

c. $-\dfrac{3\pi}{4}$

b. $\dfrac{3\pi}{4}$

d. $\dfrac{4\pi}{3}$

____ 13. How many solutions does the equation $\sin 2x = \sec x$ have for $0 \le x \le 2\pi$?

a. 3

c. 5

b. 4

d. 6

____ 14. Determine the related acute angle to the solutions of $\dfrac{\csc x}{5} - 2 = 0$ accurate to two decimal places.

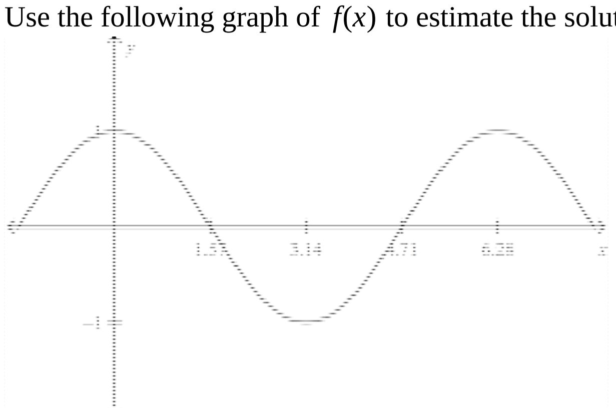
a. 5.74°

c. 6.00°

b. 95.74°

d. No solution

____ 15. Use the following graph of $f(x)$ to estimate the solution(s) of $f(x) = 0$ for $0 \le x \le 6.28$.



a. 1.57, 4.71

c. 0, 6.28

b. 3.14

d. -1.57

____ 16. Factor the expression $81 - 16\sin^2\theta$.

a. $3(27 - 5\sin^2\theta)$

c. $(9 - 4\sin\theta)^2$

b. $(9 + 4\sin\theta)(9 - 4\sin\theta)$

d. $(3 - 2\sin^2\theta)^4$

____ 17. Which of the following is NOT a solution to the equation $4\sin^2 x = 1$ for $0 \le x \le 2\pi$?

a. 30°

c. 150°

b. 210°

d. 120°

____ 18. Which of the following is a solution for the equation $\dfrac{1}{485}\tan^2 x = 0$?

a. π

c. 485π

b. $\dfrac{\pi}{2}$

d. no solution

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____ 19. Which is a solution for the equation $2(1-\cos^2 x) = \frac{3}{2}$?

a. 240° c. 30°
b. 130° d. 330°

____ 20. Which is a solution to the equation $(\sqrt{3}\tan\theta - 3)(8\cos x + 8) = 0$?

a. 360° c. 330°
b. 60° d. 90°

Short Answer

21. Calculate $\tan\left(\frac{11\pi}{12}\right)$.

22. For the expression $\frac{\tan f + \tan g}{1 - \tan f \tan g}$, assume that $f + g = 90°$. Why are the exact values of f and g irrelevant?

23. Calculate $\cos x \cos y + \sin x \sin y$ for $x - y = \frac{\pi}{4}$.

24. Rewrite $2\cos^2(4x) - 1$ as a single trigonometric ratio.

25. Sketch the graph $y = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$.

26. Sketch the graph $y = 2\sin 2x \cos 2x$.

27. Determine the value of $\cos 2\theta$ when $\tan\theta = \frac{3}{4}$ and $\pi < \theta < \frac{3\pi}{2}$.

28. Determine the value of $\tan 2\theta$ when $\sin\theta = \frac{12}{13}$ and $\frac{\pi}{2} < \theta < \pi$.

29. Determine $\cos 2x$ if $\sin^4 x = \frac{25}{36}$, where $0 < x < \frac{\pi}{2}$.

30. Determine $\cos(2k)$ when $\cos^2 k = \frac{2}{5}$, where $0 < k < \frac{\pi}{2}$.

31. Determine the single trigonometric ratio that $\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ is equivalent to.

32. Determine an identity for $\csc\theta$ in terms of $\sec\theta$.

33. Determine the solutions to the equation $2\sin x - \cos^2 x = \sin^2 x$ for $0 \le x \le 2\pi$.

34. Determine the solutions to the equation $\frac{\cos x}{\sin 2x} = \frac{5}{7}$ for $0 \le x \le 2\pi$ accurate to two decimal places.

35. The average number of customers, c, at a 24-hour sandwich shop per hour is modelled roughly by the equation $c(h) = -5\cos\left(\frac{\pi h}{12}\right) + 12$, with $h = 0$ representing midnight. How many hours per day are there at least 13 customers per hour, to the nearest hour? What time of day is peak business?

36. What quadrants contain the solutions to the equation $5\cos x = -\frac{1}{2}$?

37. What is the related acute angle to the solutions of the equation $\frac{1}{2}\sin x = \frac{\sqrt{3}}{4}$?

38. Solve the equation $\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2} = 0$ where $0 \le x \le 2\pi$.

39. Solve the equation $\sec^2 x + 3\sec x - 15 = 3$ to the nearest hundredth, where $0° \le x \le 360°$.

40. Solve the equation $\sin^2 x + 1 = -2\sin x$ for $0 \le x \le 2\pi$.

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Problem

41. Using a compound angle formula, demonstrate that $\sin\frac{2\pi}{3}$ is equivalent to $\sin\frac{\pi}{3}$.

42. Use a compound angle formula to demonstrate that $\sin(2\pi - x) = -\sin(x)$.

43. θ and α are each acute angles in standard position. $\sin\theta = \frac{3}{5}$ and $\cos\alpha = \frac{12}{13}$.
a) Determine $\cos(\alpha + \theta)$.
b) Determine $\sin(\theta - \alpha)$.
c) Explain why the direct angle measurements are not needed to find the compound trigonometric ratios.

44. Anastasia needs to find $\sin 2x$, and she knows that $0 < x < \pi$. She also knows that $\sin x$ and $\cos x$ are $\frac{7}{25}$ and $\frac{24}{25}$, but she doesn't know which is which.

a) Why isn't it necessary for her to distinguish between the two?
b) Demonstrate that it doesn't matter by finding $\sin 2x$ for both cases.

45. Penny needs to find $\cos\frac{\pi}{12}$. If she only knows that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, how can she find $\cos\frac{\pi}{12}$ using a double angle formula? What is her answer?

46. Demonstrate that the double angle formula for $\sin 2x$ is derived from the compound angle formula for $\sin(a + b)$.

47. Prove that $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x$ is an identity.

48. Prove that $\sin x + \sin x \cot^2 x = \sec x$ is an identity.

49. Solve $\tan(x) = -1$ where $0 \le x \le 2\pi$.
a) How many solutions are possible?
b) In which quadrants would you find the solutions?
c) Determine the related angle for the equation.
d) Determine all the solutions for the equation.

50. Solve $\cos x = \frac{\sqrt{3}}{2}$ where $0 \le x \le 2\pi$.
a) How many solutions are possible?
b) In which quadrants would you find the solutions?
c) Determine the related angle for the equation.
d) Determine all the solutions for the equation.

51. Solve $10\cos x = -7$ where $0 \le x \le 2\pi$.
a) How many solutions are possible?
b) In which quadrants would you find the solutions?
c) Determine the related angle for the equation to two decimal places.
d) Determine all the solutions for the equation to two decimal places.

52. The vertical location of the tip of a clock's second hand (y) relative to the clock's center measured in centimetres is modelled by the equation $y = 3\cos\left(\frac{\pi s}{30}\right)$, where s is the number of seconds that have passed.
a) How long is the second hand, and where does it start?
b) Use transformations to explain how you could change the original equation to find the vertical location of the tip of the hour and minute hands in terms of y and s if the hour hand is 1 centimetre long and the minute hand is 4 centimetres long.
c) What is a simple way you could change the equations to represent the hands' horizontal positions? Why does this work?

53. Consider the equation $5\cos^2 x + 4\cos x = 1$.
a) Put the equation in standard quadratic trigonometric equation form.
b) Use the quadratic formula to factor the equation.
c) What are the solutions to two decimal places, where
$0° \le x \le 360°$?

54. The height (h) of the water in metres at a certain point at the wave pool over a period of seconds is modelled by the equation $h(s) = \sin^2 s + \frac{1}{2}\sin s + \frac{3}{2}$.
a) How high is the water after 2 seconds?
b) During the first 10 seconds, how many times does the wave height reach 3 metres?
c) During the first 5 seconds, at what 3 points is the water level at 2 metres?

55. The height (h) in centimetres of Shuntaro's pogo stick from the ground as he jumps on it since s seconds have passed is roughly modelled by the equation $h(s) = \sin^2 4x + 2\sin 4x + 1$.
a) How many times does Shuntaro hit the ground in the first 5 seconds?
b) How many centimetres is Shuntaro off the ground after 2 seconds?
c) During the first second, when is Shuntaro at 2.5 centimetres to two decimal places?