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Laws of Exponents

Powers

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The whole thing is called the POWER

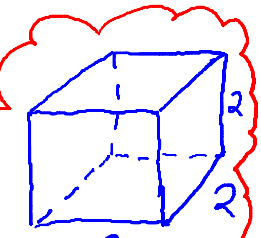
a^m — m: exponent, a: base

e.g. 2^3 → base = 2, exponent = 3

power = 2^3

Different ways of reading 2^3:

- "2 to the exponent three"
- "2 cubed"
- "the third power of 2"
- "2 to the third"
- "2 raised to the exponent 3"



$V = lwh$

$\therefore V = 2(2)(2)$

$\therefore V = 2^3$

$\therefore V = 8$ square units

The MEANING of POWERS

- a power is a short form for repeated multiplication
- e.g. Instead of writing the mass of the sun as $2000\cdots0$ kg (30 zeros), a scientist would write it as

$$2 \times 10^{30} \text{ kg}$$

This means $2 \times \underbrace{10 \times 10 \times 10 \times 10 \times \cdots \times 10}_{30 \text{ times}}$

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NEVER CONFUSE POWERS with MULTIPLICATION
If you do, then the sun would only have a mass of 600 kg.

Laws of Exponents

It is possible to SIMPLIFY expressions involving powers by making a few simple observations

e.g. • $2^3(2^4) = \underbrace{2\times2\times2}_{2^3}\times\underbrace{2\times2\times2\times2}_{2^4} = 2^7$

There are SEVEN FACTORS OF 2 altogether !!

• $x^3(x^4) = x^7$ (there are SEVEN factors of x)

• $a^6(a^{10}) = a^{16}$ (there are 16 factors of a)

• $a^6(b^{10})$ Can't be simplified because the bases are different

When writing multiplications as shown above, brackets are optional e.g. $a^6(b^{10}) = a^6b^{10}$

SUMMARY

To multiply powers with the SAME BASE keep the base and add the exponents.

i.e. $x^m x^n = x^{m+n}$

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Dividing Powers with the Same Base

e.g. Simplify $\frac{y^8}{y^5} = \frac{\cancel{y}\cancel{y}\cancel{y}\cancel{y}\cancel{y}yyy}{\cancel{y}\cancel{y}\cancel{y}\cancel{y}\cancel{y}} = \frac{y^3}{1} = y^3$

SUMMARY

To DIVIDE TWO POWERS with the SAME BASE, keep the base and subtract the exponents

i.e. $\frac{x^m}{x^n} = x^{m-n}$

BIG EXAMPLE

Simplify $\frac{4d^4w^3 \times 6dw^4}{3dw^3 \times 8dw^2}$

simplify top, simplify bottom, divide LAST!

$= \frac{4 \times 6\, d^4 d^1 w^3 w^4}{3 \times 8\, d d^1 w^3 w^2}$ ← optional step (reminds us that mult. can be done in any order.)

$= \frac{\cancel{24}\, d^5 w^7}{\cancel{24}\, d^2 w^5}$

$= \left(\frac{24}{24}\right)\left(\frac{d^5}{d^2}\right)\left(\frac{w^7}{w^5}\right)$ ← Optional Step

$= 1\, d^3 w^2$

$= d^3 w^2$

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#1, 2, 3, 6, 9a

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Raising a Power to an Exponent

e.g. $(x^2)^3 = x^2 x^2 x^2 = x^{2+2+2} = x^6$

$(y^3)^4 = y^3 y^3 y^3 y^3 = y^{3+3+3+3} = y^{12}$

$(q^4)^2 = q^4 q^4 = q^{4+4} = q^8$

SUMMARY

To raise a power to an exponent, KEEP the base and MULTIPLY the exponents.

ie. $(x^m)^n = x^{mn}$

Examples Simplify

- $(a^6)^2 = a^{12}$
- $(5x^4)^3 = (5x^4)(5x^4)(5x^4) = 5(5)(5)x^4 x^4 x^4 = 5^3 x^{12} = 125x^{12}$
- $(2pq^3)^4 = 2^4 p^4 (q^3)^4 = 16p^4 q^{12}$

Shortcut

To raise a product to an exponent, raise each factor of the product to the exponent

ie. $(ab)^n = a^n b^n$

When applied to a product of powers, this can be used as follows:

$$(x^m y^n)^p = (x^m)^p (y^n)^p = x^{mp} y^{np}$$

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Example

Simplify $(3x^2y^4)^3$ in Two different ways

LONG WAY

$$(3x^2y^4)^3 = (3x^2y^4)(3x^2y^4)(3x^2y^4)$$
$$= 3(3)(3)x^2x^2x^2y^4y^4y^4$$
$$= 3^3x^6y^{12} = 27x^6y^{12}$$

SHORT WAY

$$(3x^2y^4)^3 = 3^3(x^2)^3(y^4)^3 = 27x^6y^{12}$$

BIG EXAMPLE

Simplify $\dfrac{2ab^2(3a^3b^3)}{(4ab^2)^2}$

$$= \frac{2(3)aa^3b^2b^3}{4^2a^2(b^2)^2} \quad \text{(optional step)}$$

$$= \frac{6a^4b^5}{16a^2b^4}$$

$$= \left(\frac{6}{16}\right)\left(\frac{a^4}{a^2}\right)\left(\frac{b^5}{b^4}\right) \quad \text{(optional step)}$$

$$= \frac{3}{8}a^2b \quad \text{fully simplified expression}$$

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Homework
p. 126 - 127 C1, C2, C3, 5, 7, 8

The Distributive Law

$3(2x+4y)$ already simplified, terms are unlike

How can this be simplified? What does it mean? Can you relate it to something in the real world?

LONG WAY

$$3(2x+4y) = (2x+4y)+(2x+4y)+(2x+4y)$$
$$= 2x+4y+2x+4y+2x+4y$$
$$= 2x+2x+2x+4y+4y+4y$$
$$= 6x+12y$$

SHORT WAY

$$3(2x+4y) = 3(2x)+3(4y)$$
$$= 6x+12y$$

Multiply each term in the brackets by 3.

DISTRIBUTIVE LAW

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This is called EXPANDING using the DISTRIBUTIVE LAW

p. 166 # 2ac, 3ab

Simple Exercises to be used as a warm-up

THE DISTRIBUTIVE LAW

$a(x+y) = ax + ay$

To expand the product of a monomial and a binomial, multiply each term of the binomial by the monomial. That is, multiply each term in the brackets by a.

More Examples

Expand each of the following:

(a) $-5(x+3y) =$

(b) $-7(2a-5b) =$

(c) $x(-3x^2-y^2) =$

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Understanding WHY the Distributive Law Works

Consider the following box of chocolates.

Let x represent one  → 8x in one box

Let y represent one  → y in one box

Let z represent one  → 2z in one box

Let w represent one  → 5w in one box

Let u represent one  → 2u in one box

Let v represent one  → 2v in one box

① Using the variables x, y, z, w, u and v, write an algebraic expression that describes the total # chocolates in one box.

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② Now suppose that there are 100 boxes of these chocolates in a warehouse. Write an algebraic expression that describes the total # of chocolates in the warehouse.

③ Now use the distributive law to expand your expression. Does your answer make sense?

More Examples

Simplify each expression first. Evaluate if possible.

(a) $-3^7 \div 3^5 \times 3$ (b) $-3(4a^3b)^6$ (c) $(4xy^2)(-3x^2y)^3$

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(d) $\dfrac{4pq^3 \times (-16p^2q^5)}{32p^4q^6}$

(e) $-3(2x-4y+z)$

(f) $-5(3x^2-7x)+4x^2-12x$

(g) $-3x(-2y-7)-4(2x-xy)$