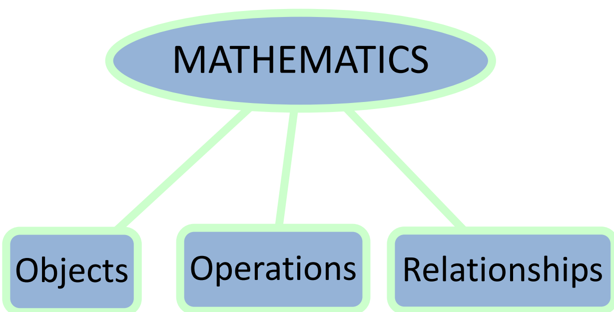
MPM1D0 Unit 0 – Review of Elementary School Mathematics Summary Notes

General Concepts

As shown below, mathematics can be reduced to ***three basic concepts***:

1. Mathematical Objects
   (e.g. ***numbers*** are mathematical objects)
2. Mathematical Operations
   (e.g. +, −, ×, ÷, exponents, $\sqrt{\ }$, etc)
3. Mathematical Relationships
   (e.g. $c^2 = a^2 + b^2$, i.e. the Pythagorean Theorem)



LANGUAGE HAS MEANING!

↓

MATH IS A LANGUAGE!

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THEREFORE, MATH HAS MEANING!

- Always ask yourself, “What does this mean?”
- Avoid the “autopilot” state of mind!
- +, − Gains and Losses
- × Groups of (Repeated Addition)
- ÷ How many groups of? (Opposite of ×)
- exponents Repeated Multiplication

always comes last!

Highlight the terms before evaluating the expression!

Working with Operations

1 2 3 4
B E DM AS
Division/Multiplication - tied, left-to-right
Addition/Subtraction - tied, left-to-right

Terms are separated by + and - signs.

Separate each expression into terms. Then apply the operations in the correct order.

+(+) add a positive: GAIN
-(-) subtract a negative: GAIN

+(-) add a negative: LOSS
-(+) subtract a positive: LOSS

Gains more than losses: + answer
Losses more than gains: - answer

Rule for Determining the Sign of the Answer when Multiplying and Dividing

Two Numbers
Signs Same: + answer
Signs Different: - answer

More than Two Numbers
Even # of Negatives: + answer
Odd # of Negatives: - answer

Working with Fractions

Adding/Subtracting Fractions

- Express each fraction with a common denominator.
- Add/subtract the numerators.
- Keep the denominator!
- If possible, reduce to lowest terms.

$$\frac{3}{10}+\frac{8}{15}=\frac{9}{30}+\frac{16}{30}=\frac{25}{30}=\frac{5}{6}$$

Multiplying Fractions

- Multiply the numerators, multiply the denominators. If possible, reduce to lowest terms. **OR…**
- Reduce first (vertically and diagonally). Multiply the numerators, multiply the denominators.

$$\frac{3}{10}\left(\frac{8}{15}\right)=\frac{24}{150}=\frac{4}{25}$$ **OR** $$\frac{\cancel{3}^{1}}{\cancel{10}_{5}}\left(\frac{\cancel{8}^{4}}{\cancel{15}_{5}}\right)=\frac{4}{25}$$

Dividing Fractions

- ***Do not*** change the 1st fraction.
- Change ÷ to ×.
- Find the reciprocal of the 2nd fraction (i.e. "***flip***").
- **Summary:** Multiply by the reciprocal (i.e. "flip 'n multiply")

$$\frac{3}{10}\div\frac{8}{15}=\frac{3}{10}\times\frac{15}{8}=\frac{45}{80}=\frac{9}{16}$$

Measurement

The Meaning of π: The ratio of the circumference of a circle to its diameter.
(i.e. The circumference of a circle is π times, or approximately 3.14 times, greater than its diameter.)

Perimeter

- The ***distance*** around a two-dimensional shape.
- The "perimeter" of a circle is called its ***circumference***.
- Perimeter is measured in ***linear units*** such as mm, cm, m, km.

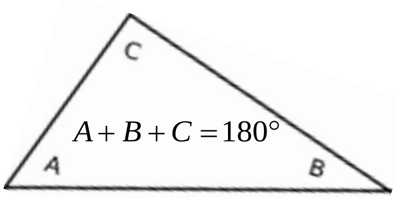
Area

- The "size" or "amount of space" inside the boundary of a two-dimensional surface, including curved surfaces.
- In the case of the surface of a three-dimensional object, the area is usually called ***surface area***.
- Area is measured in ***square units*** such as mm^2, cm^2, m^2, km^2.

Volume

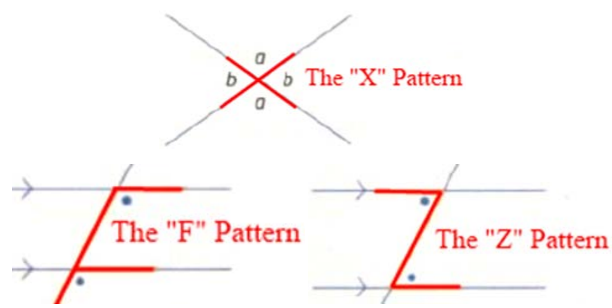
- The "amount of space" contained within the interior of a three-dimensional object. (The ***capacity*** of a 3-dimensional object.)
- Volume is measured in ***cubic units*** such as mm^3, cm^3, m^3, km^3, mL, L.
  Note: 1 mL = 1 cm^3

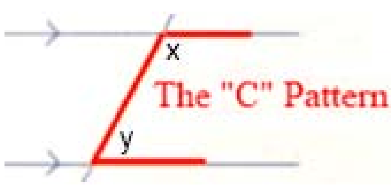
Geometry



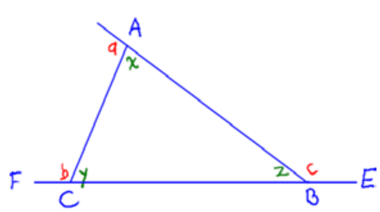
The ***sum*** of the ***interior angles*** of any ***triangle*** is 180°.

The ***sum*** of the ***exterior angles*** of ***any convex polygon*** is 360°





$$x+y=180°$$



The exterior angle at each vertex of a triangle is equal to the sum of the interior angles at the other two vertices.

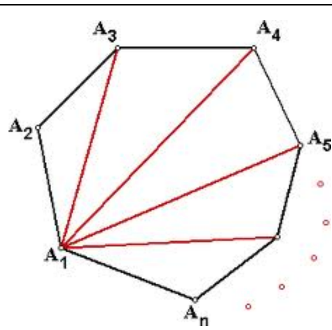
Using the diagram at the left, we can rewrite the above statement as follows:

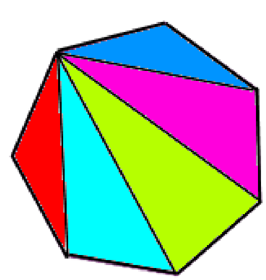
$a=y+z \qquad b=x+z \qquad c=x+y$

The ***sum*** of the ***interior angles*** of an ***n-sided convex polygon***

$=(n-2)\times180°$

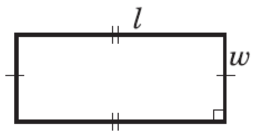
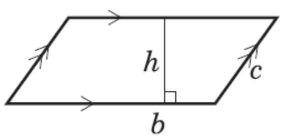
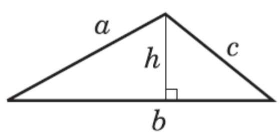
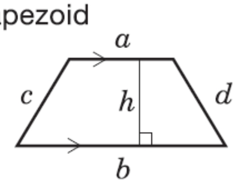
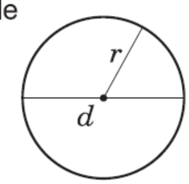
= (number of triangles) × 180°



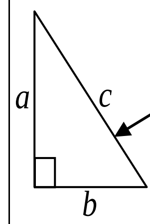


Name: Heptagon
Number of Sides: 7
Number of Triangles: 5
Sum of Interior Angles:
$5(180°)=900°$

Perimeter and Area Equations

Geometric Figure	Perimeter	Area
Rectangle 	$P = l + l + w + w$ or $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ or $P = 2(b + c)$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a + b)h}{2}$ or $A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

Pythagorean Theorem



The ***hypotenuse*** is the ***longest side*** of a right triangle. It is always found ***opposite*** the right angle.

In ***any*** right triangle, **the square of the hypotenuse** is equal to **the sum of the squares of the other two sides**.

That is,

$$c^2 = a^2 + b^2$$

By using your knowledge of rearranging equations, you can rewrite this equation as follows:

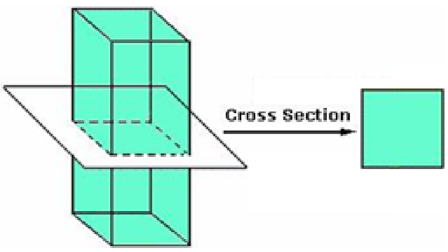
$$b^2 = c^2 - a^2$$

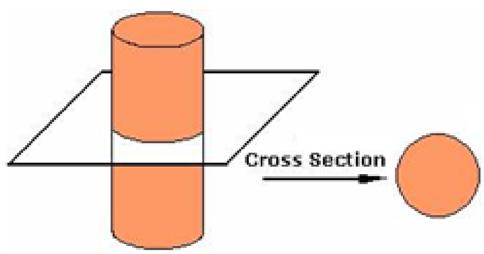
and

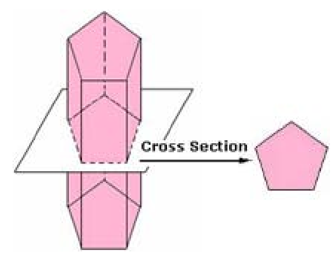
$$a^2 = c^2 - b^2$$

Volumes of Solids with a Uniform Cross-Section

A solid has a ***uniform cross-section*** if any cross-section ***parallel to the base*** is ***congruent*** to the base (i.e. has exactly the same shape and size as the base). Prisms and cylinders have a uniform cross-section. Pyramids and cones do not.



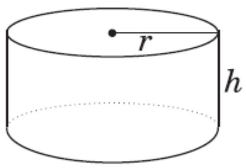
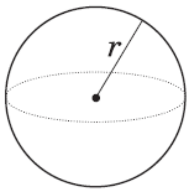
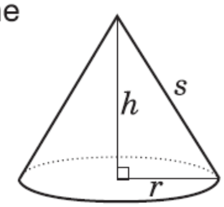
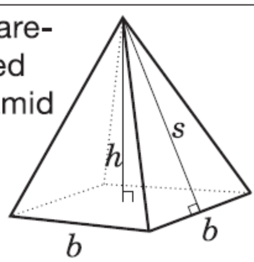
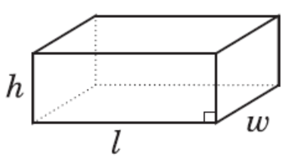
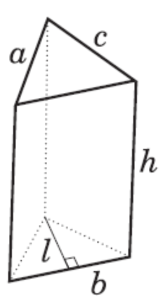




For all solids with a uniform cross-section, $V = \left(A_{\text{base}}\right)\left(\text{height}\right)$

Volume and Surface Area Equations

If you need additional help, Google "area volume solids."

Geometric Figure	Surface Area	Volume
Cylinder 	$A_{\text{base}} = \pi r^2$ $A_{\text{lateral surface}} = 2\pi rh$ $A_{\text{total}} = 2A_{\text{base}} + A_{\text{lateral surface}}$ $= 2\pi r^2 + 2\pi rh$	$V = (A_{\text{base}})(\text{height})$ This is true for all ***prisms*** and ***cylinders***. $V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$ or $V = \frac{4\pi r^3}{3}$ This is true for all ***pyramids*** and ***cones***.
Cone 	$A_{\text{lateral surface}} = \pi rs$ $A_{\text{base}} = \pi r^2$ $A_{\text{total}} = A_{\text{lateral surface}} + A_{\text{base}}$ $= \pi rs + \pi r^2$	$V = \frac{(A_{\text{base}})(\text{height})}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$
Square-based pyramid 	$A_{\text{triangle}} = \frac{1}{2}bs$ $A_{\text{base}} = b^2$ $A_{\text{total}} = 4A_{\text{triangle}} + A_{\text{base}}$ $= 2bs + b^2$	$V = \frac{(A_{\text{base}})(\text{height})}{3}$ $V = \frac{1}{3}b^2 h$ or $V = \frac{b^2 h}{3}$
Rectangular prism 	$A = 2(wh + lw + lh)$	$V = (\text{area of base})(\text{height})$ $V = lwh$ This is true for all ***prisms*** and ***cylinders***.
Triangular prism 	$A_{\text{base}} = \frac{1}{2}bl$ $A_{\text{rectangles}} = ah + bh + ch$ $A_{\text{total}} = A_{\text{rectangles}} + 2A_{\text{base}}$ $= ah + bh + ch + bl$	$V = (A_{\text{base}})(\text{height})$ $V = \frac{1}{2}blh$ or $V = \frac{blh}{2}$