MPM1D0 – Unit 1 Summary and Review

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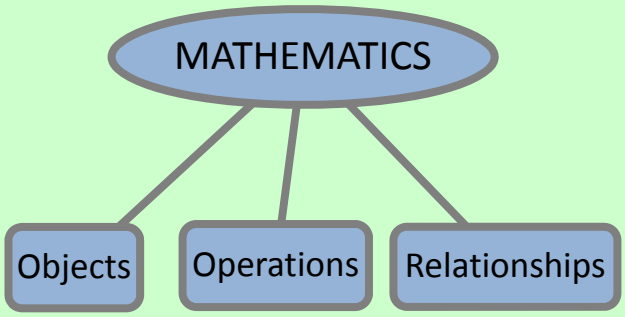
Main Ideas

Review – Framework for Understanding Mathematics

As shown below, mathematics can be reduced to ***three basic concepts***:

1. Mathematical ***Objects*** (e.g. ***numbers*** are mathematical objects)
2. Mathematical ***Operations*** (e.g. +, −, ×, ÷, exponents, $\sqrt{\ }$, etc)
3. Mathematical ***Relationships*** (e.g. $c^2 = a^2 + b^2$, i.e. the Pythagorean Theorem)

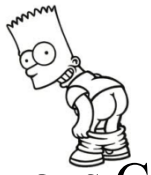


Overview of Arithmetic and Mathematical Operations

Operating with Integers, Fractions and Algebraic Expressions

Essential Ideas of Arithmetic and Algebra

BEDMAS *always* applies



Always Comes Last!

Addition and Subtraction (+ and −)

+, +(+), −(−) → GAIN
−, −(+), +(−) → LOSS

Gains greater than Losses → Positive Answer
Gains less than Losses → Negative Answer

COMMON DENOMINATOR REQUIRED

Add/Subtract **NUMERATORS ONLY**

Simplify by **COLLECTING LIKE TERMS**

Add the opposite **ONLY** **if a bracket contains two or more terms and is preceded by a negative sign.**

Use the *Distributive Property* if a polynomial with two or more terms is multiplied by a monomial.

Multiplication and Division (× and ÷)

× → Groups of
÷ → How Many Groups of?

NO Common Denominator

Even # of Negatives → Positive Answer
Odd # of Negatives → Negative Answer

Multiplication: Numerator×Numerator *OVER* Denominator×Denominator

Division: Multiply by the Reciprocal

Simplify by **REARRANGING FACTORS, LAWS OF EXPONENTS, DISTRIBUTIVE PROPERTY**

Summary of Laws of Exponents (Apply ONLY to MULTIPLICATION and DIVISION)

Name of Rule	*Algebraic Form*	*Verbal Form*	*Example Showing WHY Law Works*
Product	$a^x a^y = a^{x+y}$	To ***multiply*** two powers with the ***same base***, ***keep the base*** and ***add the exponents***.	$a^2a^4 = (a)(a)(a)(a)(a)(a) = a^6$ ***Two*** factors of a multiplied by ***four*** factors of a gives ***six*** factors of a.
Quotient	$\frac{a^x}{a^y} = a^{x-y}$	To ***divide*** two powers with the ***same base***, ***keep the base*** and ***subtract the exponents***.	$\frac{a^5}{a^2} = \frac{(a)(a)(a)(a)(a)}{(a)(a)} = a^3$ ***Five*** factors of a divided by ***two*** factors of a leaves ***three*** factors of a.
Power of a Power	$(a^x)^y = a^{xy}$	To ***raise a power*** to an exponent, ***keep the base*** and ***multiply*** the exponents.	$(a^3)^4 = (a^3)(a^3)(a^3)(a^3) = a^{12}$
Power of a Product	$(ab)^x = a^x b^x$	To ***raise a product*** to an exponent, ***raise each factor in the product*** to the exponent.	$(a^3b^4c)^2 = (a^3b^4c)(a^3b^4c) = a^3a^3b^4b^4cc = a^6b^8c^2$

Examples

1. Evaluate.

$-5(4^2 - 12^2) - 5(4-12)^2$

B E DM AS

$= -5(16-144) - 5(-8)^2$ ①

$= -5(-128) - 5(64)$ ②

$= 640 - 320$ ③

$= 320$ LAST!!

Answer is positive because GAIN > LOSS

- **BEDMAS**
- Separate into terms.
- Follow BEDMAS within each term.
- Be careful with negative values, fractions and powers.

2. Substitute the given values for the variables.

$-3b^2 - 5a^2b$, if $a = -4$ and $b = -\frac{2}{5}$

$= -3\left(-\frac{2}{5}\right)^2 - 5(-4)^2\left(-\frac{2}{5}\right)$

$= \frac{-3}{1}\left(\frac{4}{25}\right) - 5(16)\left(-\frac{2}{5}\right)$

$= \frac{-12}{25} - \frac{80}{1}\left(\frac{-2}{5}\right)$

$= -\frac{12}{25} - \left(\frac{-160}{5}\right)$ $-(-) \rightarrow$ GAIN

$= \frac{-12}{25} + \frac{160 \times 5}{5 \times 5}$ Common Denominator Required!

$= \frac{-12}{25} + \frac{800}{25}$

$= \frac{-12+800}{25}$ Add NUMERATORS ONLY

$= \frac{788}{25}$ Leave answer in improper form

- Replace variables with empty brackets.
- Fill in the brackets with given values.
- Follow BEDMAS within each term.
- Calculate carefully as in the first example.

Rough Work: $\left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right)\left(\frac{-2}{5}\right) = \frac{2\times2}{5\times5} = \frac{4}{25}$

answer must be positive

$5(16) = 80 = \frac{80}{1}$

$(-4)^2 = (-4)(-4) = 16$

answer must be positive

3. Simplify.

$-7s^3 + 11t^5 - 9s^3 - 12t^5$

$= -7s^3 - 9s^3 + 11t^5 - 12t^5$ Collect Like Terms

$= -16s^3 - t^5$ Simplify $-7-9 = -16$, $+11-12 = -1$

Note: $t^5 = 1t^5$

REMEMBER! 2 cows + 4 cows = 6 cows; 2 boys + 4 boys = 6 boys

BUT 2 cows + 4 boys $\neq$ 6 cowboys

- **Addition & Subtraction**
- Collect Like Terms
- Gains/Losses
- To avoid careless arithmetic errors, use a calculator to check answers.

4. Simplify.

No operation between brackets ∴ understood to be MULTIPLICATION!

$$(-7s^3)(+11t^5)(-9s^3)(-2t^5)$$

$$=(-7)(+11)(-9)(-2)s^3s^3t^5t^5$$

$$=-1386s^6t^{10}$$

Since × can be performed in any order

$t^{5+5}=t^{10}$

$s^{3+3}=s^6$

factors can be rearranged in any order

Like terms should NOT enter your mind because there is no add/subtract

- **MULTIPLICATION!**
- Although the expression resembles the one in example 3, it is ***very different***!
- Simplify by ***rearranging factors*** and using ***laws of exponents***.

5. Simplify.

$$\frac{-52x^{10}y^7}{13x^3y^6}$$

$$=\left(\frac{-52}{13}\right)\left(\frac{x^{10}}{x^3}\right)\left(\frac{y^7}{y^6}\right)$$

$$=-4x^7y$$

- Rewrite as a product of "fractions."
- Simplify each "fraction."

Like terms should NOT enter your mind because there is no add/subtract

- **MULTIPLICATION AND DIVISION!**
- Rewrite as a product of "fractions."
- To divide two powers with the same base, ***keep*** the base and ***subtract*** the exponents.

Rough Work:

$\frac{-52}{13}=-52\div 13=-4$, $\frac{x^{10}}{x^7}=x^{10-7}=x^3$, $\frac{y^7}{y^6}=y^{7-6}=y^1=y$

6. Simplify

add opposite

$$(-7s^2-11s)-(9s^2+2s)$$

$$=-7s^2-11s+(-9s^2-2s)$$

$$=-7s^2-11s+(-9s^2)-2s$$

$$=-7s^2-11s-9s^2-2s$$

$$=-7s^2-9s^2-11s-2s$$

$$=-16s^2-13s$$

- ***Add the opposite* only** when a bracket is preceded by a "–" sign!
- Brackets can be removed if preceded by a "+"
- Collect Like Terms
- Gains/Losses

7. Simplify.

$$-3(2d^2-8d)-3d(d^2-8d)$$

$$=-6d^2+24d-3d^3+24d^2$$

$$=-6d^2+24d^2+24d-3d^3$$

$$=18d^2+24d-3d^3$$

- ***Distributive Property!***
- Do ***not*** add the opposite! This will only complicate matters!
- After multiplying, collect like terms.

PRACTICE!

Day 1

1. Evaluate.

(a) $-3\left[6^2-12(-5)^2\right]-5\left[14+2(-12)\right]^2$	**(b)** $-7a^2b-2ab^2$, if $a=-\frac{2}{7}$ and $b=-13$

2. Simplify.

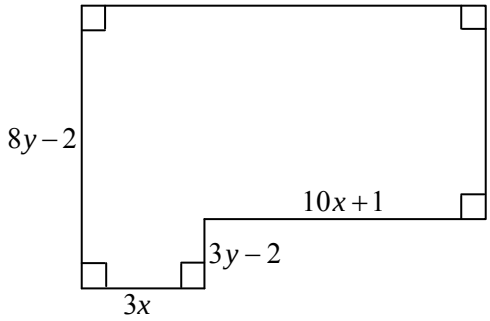
(a) $-17s^3t^4-11s^4t^3+9s^3t^4-\left(-6s^4t^3\right)$	**(b)** $\left(-17s^3t^4\right)\left(-11s^4t^3\right)\left(-9s^3t^4\right)\left(-6s^4t^3\right)$	**(c)** $\frac{-108x^{15}y^{17}}{-12x^{14}y^{13}}$

3. Simplify.

(a) $\left(-8s^2-13s\right)-\left(11s^2+12s\right)$	**(b)** $\frac{100b^9d^2\left(-bd^3\right)}{2\left(5b^4d\right)^2}$	**(c)** $-3d\left(2d^2-8d\right)-3d^2\left(d^2-8d\right)$

4. Write ***fully simplified*** algebraic expressions for…

(a) …the ***perimeter*** of the figure at the right.



(b) …the ***area*** of the figure at the right.

5. The *KatContrarian Loudspeaker Company* is producing a new ultra-loud speaker for P.A. systems. Each speaker costs $50.00 to make but there is also a $3000 set-up charge for the machinery used to make them.

(a) Write an algebraic expression that represents the ***total cost*** of ***manufacturing*** n speakers. (n represents the number of speakers manufactured)

(b) *KatContrarian Loudspeaker* sells each speaker for $200.00. Write an algebraic expression that represents the ***total amount of money obtained*** for ***selling*** n speakers. (n represents the number of speakers sold)

(c) How many speakers does *KatContrarian Loudspeaker* need to sell to make a ***profit***?

Day 2

1. Evaluate.

(a) $-5\left[7^2+3(-2)^5\right]-5\left[1+2(-1)\right]^5$	**(b)** $-5a^3b-4ab^3$, if $a=-\frac{2}{3}$ and $b=-3$

2. Simplify.

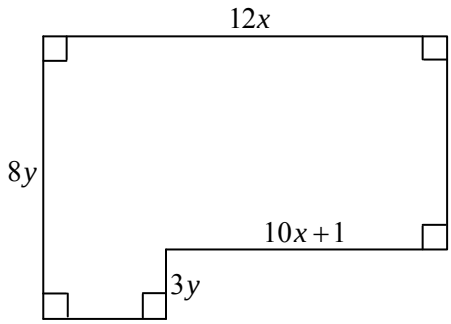
(a) $11xy-8xy^2+(-11xy)-3xy^2$	**(b)** $(11xy)(-8xy^2)(-11xy)(-3xy^2)$	**(c)** $\frac{-108a^5b^7}{-24a^4b^7}$

3. Simplify.

(a) $-(-8s^2-13s)+(11s^2-12s)$	**(b)** $\frac{48b^9d^7(-2b^7d^3)}{2(4b^4d^3)^2}$	**(c)** $-3d(-2d^2+8d)-3d^2(-d^2+8d)$

4. Write ***fully simplified*** algebraic expressions for…

(a) …the ***perimeter*** of the figure at the right.



(b) …the ***area*** of the figure at the right.

5. The *AvJo* toy company is planning to produce a new toy truck. Each one costs $3.00 to make and there is a $300 set-up charge for the machinery to make them.

(a) Write an expression that represents the ***total cost*** of ***manufacturing*** *n* trucks.

(b) *AvJo* sells each truck for $5.00. Write an expression that represents the ***total money obtained*** for ***selling*** *n* trucks.

(c) How many trucks does *AvJo* need to sell to make a profit?

Day 3

1. Evaluate.

(a) $-2\left[\left(-\frac{1}{4}\right)^2-12(-1)^2\right]-5\left[24+2(-12)\right]^2$	**(b)** $-a^2b^2-3a^3b^2$, if $a=-\frac{1}{3}$ and $b=-1$

2. Simplify.

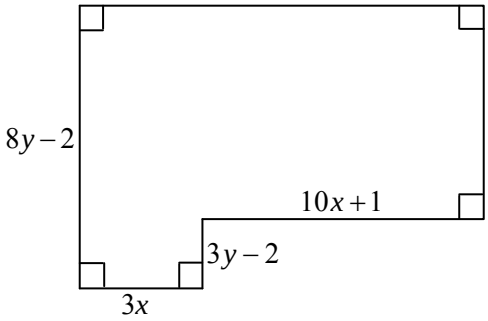
(a) $-17s^3t^4-11s^4t^3+9s^3t^4-\left(-6s^4t^3\right)$	**(b)** $\left(-17s^3t^4\right)\left(-11s^4t^3\right)\left(-9s^3t^4\right)\left(-6s^4t^3\right)$	**(c)** $\frac{-108x^{15}y^{17}}{-12x^{14}y^{13}}$

3. Simplify.

(a) $\left(8ab^2-21a^2b\right)-\left(-8ab^2+21a^2b\right)$	**(b)** $\frac{-100u^9v^2\left(-u^3v\right)}{2\left(5u^4v\right)^3}$	**(c)** $-3m\left(2m^2-5n\right)-3m^2\left(m-n\right)$

4. Write ***fully simplified*** algebraic expressions for…

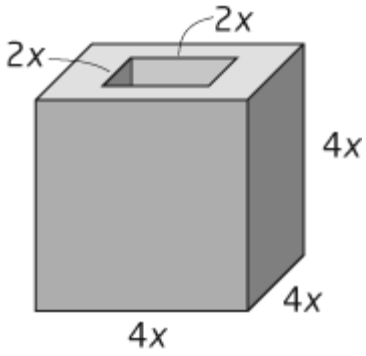
(a) …the ***perimeter*** of the figure at the right.



(b) …the ***area*** of the figure at the right.

5. A square hole is cut all the way through a cube.

(a) Find the volume of the shape that remains.



(b) Find the surface area of the shape.

Day 4

1. Evaluate.

(a) $-3\left[4-2(-5)\right]^2-5\left[4^3+5(-2)^3\right]$	**(b)** $-7a^2b-2ab^2$, if $a=-3$ and $b=-\frac{5}{6}$

2. Simplify.

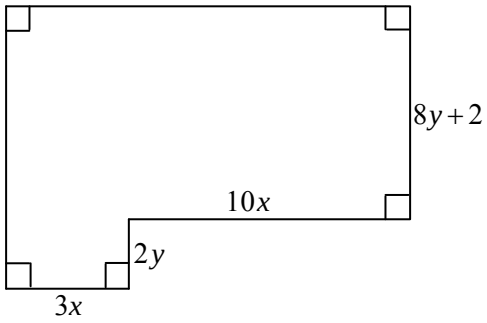
(a) $-17cd-11d+9cd-(-6d)$	**(b)** $(-17cd)(-11d)(-9cd)(-6d)$	**(c)** $\frac{108m^{15}n^{17}}{-72m^{10}n^{16}}$

3. Simplify.

(a) $(-8s-13s^2)-(12s^2-11s)$	**(b)** $\frac{100b^9d^2(-5b^4d^8)}{2(5b^4d)^3}$	**(c)** $-3d(8d-2d^2)-3d^2(8d-d^2)$

4. Write ***fully simplified*** algebraic expressions for…

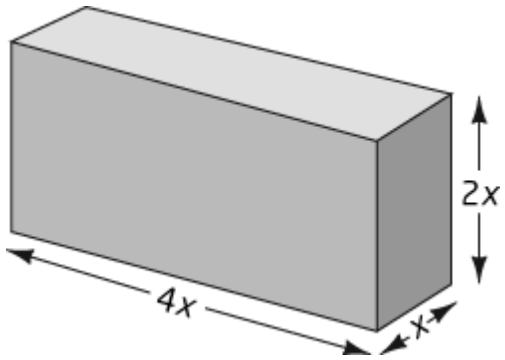
(a) …the ***perimeter*** of the figure at the right.



(b) …the ***area*** of the figure at the right.

5. Consider the rectangular prism shown at the right.

(a) Write a fully simplified algebraic expression that represents the ***volume*** of the prism.



(b) Write a fully simplified algebraic expression that represents the ***surface area*** of the prism.

(c) What is the ***edge length*** of the ***cube*** that has the ***same volume*** as this rectangular prism? Explain your reasoning.

Answers

Day 1

1. (a) 292 (b) 104

2. (a) $-8s^3t^4 - 5s^4t^3$

(b) $10098s^{14}t^{14}$

(c) $9xy^4$

3. (a) $-19s^2 - 25s$

(b) $-2b^2d^3$

(c) $18d^3 + 24d^2 - 3d^4$

4. (a) $P = 26x + 16y - 2$

(b) $A = 74xy - 6x + 5y$

5. (a) $50n + 3000$

(b) $200n$

(c) The profit is given by the expression $150n - 3000$. This expression must have a positive value for the company to make a profit. If n is 21 or higher, the expression yields a positive value.

Day 2

1. (a) 240 (b) $-\frac{688}{9}$

2. (a) $-11xy^2$

(b) $-2904x^4y^6$

(c) $\frac{9}{2}a$ or $\frac{9a}{2}$

3. (a) $19s^2 + s$

(b) $-3b^8d^4$

(c) $-18d^3 - 24d^2 + 3d^4$

4. (a) $P = 24x + 16y$

(b) $A = 66xy - 3y$

5. (a) $3n + 300$

(b) $5n$

(c) The profit is given by the expression $2n - 300$. This expression must have a positive value for the company to make a profit. If n is 151 or higher, the expression yields a positive value.

Day 3

1. (a) $\frac{191}{8}$ (b) 0

2. (a) $-8s^3t^4 - 5s^4t^3$

(b) $10098s^{14}t^{14}$

(c) $9xy^4$

3. (a) $16ab^2 - 42a^2b$

(b) $\frac{2}{5}$

(c) $-9m^3 + 15mn + 3m^2n$

4. (a) $P = 26x + 16y - 2$

(b) $A = 74xy - 6x + 5y$

5. (a) $V = (4x)^3 - (4x)(2x)^2$

$= 4^3x^3 - (4x)(2^2x^2)$

$= 64x^3 - (4x)(4x^2)$

$= 64x^3 - 16x^3$

$= 48x^3$

(b) $A = 6(4x)^2 + 4(4x)(2x) - 2(2x)^2$

$= 6(16x^2) + 32x^2 - 2(4x^2)$

$= 96x^2 + 32x^2 - 8x^2$

$= 120x^2$

Day 4

1. (a) -708 (b) $\frac{170}{3}$

2. (a) $-8cd - 5d$

(b) $10098c^2d^4$

(c) $-\frac{3}{2}m^5n$ or $-\frac{3m^5n}{2}$

3. (a) $3s - 25s^2$

(b) $-2bd^7$

(c) $-18d^3 - 24d^2 + 3d^4$

4. (a) $P = 26x + 20y + 4$

(b) $A = 110xy + 26x$

5. (a) $V = x(2x)(4x) = 8x^3$

(b) $A = 2(x)(2x) + 2(2x)(4x) + 2(x)(4x)$

$= 4x^2 + 16x^2 + 8x^2$

$= 28x^2$

(c) $2x$ because $(2x)(2x)(2x) = (2x)^3 = 8x^3$