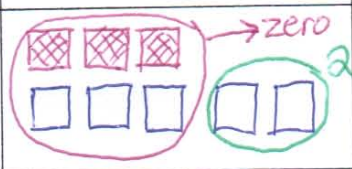
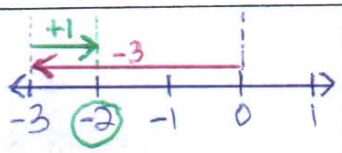
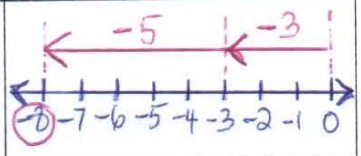
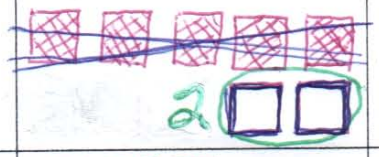
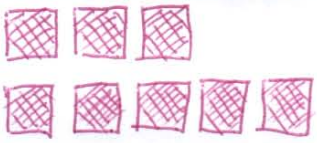
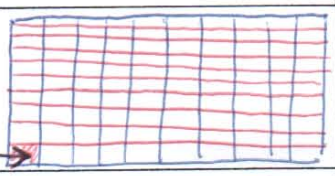
□ = 1

▨ = -1

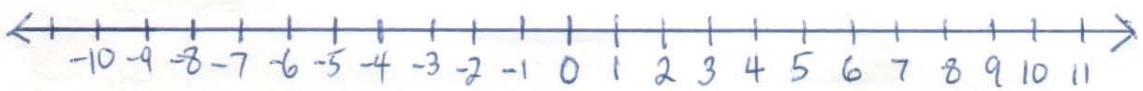
Exercises

Complete the following table.

Mathematical Expression	*Meaning*	*Picture*	*Evaluate the Expression*
$-3+5$	loss of 3 followed by a gain of 5		$-3+5$ $=2$ ✓
$-3+1$	loss of 3 followed by a gain of 1		$-3+1$ $=-2$ ✓
$-3-5$	loss of 3 followed by a loss of 5		$-3-5$ $=-8$ ✓
gain $-3-(-5)$	loss of 3 followed by a gain of 5		$-3-(-5)$ $=-3+5$ ✓ $=2$ ✓
loss $-3+(-5)$	loss of 3 followed by a loss of 5		$-3+(-5)$ $=-3-5$ ✓ $=-8$ ✓
loss $-3-(+5)$	loss of 3 followed by a loss of 5		$-3-(+5)$ $=-3-5$ ✓ $=-8$ ✓
$\frac{7}{5}-\frac{3}{10}$	three-tenths of a whole is taken away from one whole plus two-fifths of a whole		$\frac{7}{5}\times\frac{2}{2}-\frac{3}{10}$ ✓ $=\frac{14}{10}-\frac{3}{10}=\frac{11}{10}$ ✓
$\frac{7}{15}-\frac{3}{10}$	three-tenths of a whole is subtracted from seven-fifteenths of a whole	Similar to previous one	$\frac{7}{15}\times\frac{2}{2}-\frac{3}{10}\times\frac{3}{3}$ ✓ $=\frac{14}{30}-\frac{9}{30}=\frac{5}{30}=\frac{1}{6}$ ✓
$10\div 0.1$	How many groups of 0.1 can be made from 10? 100		$10\div 0.1$ $=100$ ✓
$10\div 0$	How many groups of zero can be made from 10?	No picture can be drawn because 10 cannot be divided up into groups of zero	$10\div 0$ is undefined ✓

MPM1DO – Filling in the Gaps

1. Draw a number line. Label all *integers* from −10 to 10 inclusive.



2. Evaluate each of the following expressions *without* using a calculator.

(a) $-3+5$
$=2$

(b) $-3-5$
$=-8$

(c) $-3+(-5)$
$=-3-5$
$=-8$

(d) $-3-(-5)$
$=-3+5$
$=2$

(e) $-3+5-6+1$
$=2-6+1$
$=-4+1$
$=-3$

(f) $-3-5-6+1$
$=-13$

(g) $-3-(-5)+(-6)+1$
$=-3+5-6+1$
$=-3$

(h) $3+(-5)+(-6)-(-1)$
$=3-5-6+1$
$=-7$

Explain the general principle(s) that you used to evaluate each of the expressions in question 2.

Adding and subtracting → gains and losses
add a positive +(+) } GAIN add a negative +(−) } LOSS
subtract a negative −(−) } subtract a positive −(+) }

3. Evaluate each of the following expressions *without* using a calculator.

(a) $-3(5)$
$=-15$

(b) $-3(+5)$
$=-15$

(c) $-3(-5)$
$=15$

(d) $-3(5)$
$=-15$

(e) $-3(5)(-6)(1)$
$=90$

(f) $-3(5)(-6)(-1)$
$=-90$

(g) $3(5)(6)(-1)$
$=-90$

(h) $-3(-5)(-6)(-1)$
$=90$

Explain the general principle(s) that you used to evaluate each of the expressions in question 3.

MULTIPLYING
even # of negative signs → positive answer
odd # of negative signs → negative answer

4. Evaluate each of the following expressions *without* using a calculator.

(a) $\frac{36}{-12}$
$=-3$

(b) $\frac{-36}{-12}$
$=3$

(c) $\frac{+49}{+7}$
$=7$

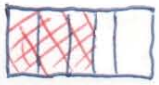
(d) $\frac{-64}{16}$
$=-4$

Explain the general principle(s) that you used to evaluate each of the expressions in question 4.

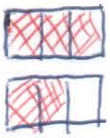
DIVIDING
(same as multiplying)

5. Draw diagrams to represent each of the following fractions.

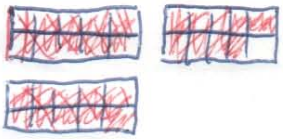
(a) $\frac{3}{5}$



(b) $\frac{5}{3}$



(c) $2\frac{9}{10}$



(d) $\frac{6}{2} = 3$

6. Evaluate each of the following expressions.

(a) $\frac{3}{5} - \frac{2}{5}$

$= \frac{1}{5}$

(b) $\frac{-5}{3} + \frac{5}{6}$ (×2, ×2)

$= \frac{-10}{6} + \frac{5}{6}$

$= \frac{-5}{6}$

(c) $-\frac{5}{14} + \left(-\frac{8}{21}\right)$ (×3, ×2, ×2)

$= \frac{-15}{42} - \frac{16}{42}$

$= \frac{-31}{42}$

(d) $-\frac{5}{14} - \left(-\frac{8}{21}\right)$

$= \frac{-15}{42} + \frac{16}{42}$

$= \frac{1}{42}$

Explain the general principle(s) that you used to evaluate each of the expressions in question 5.

Express each fraction with a common denominator
Add/subtract the numerators, KEEP the denominators
Reduce to lowest terms

7. Evaluate each of the following expressions.

(a) $\frac{3}{5}\left(-\frac{2}{5}\right)$

$= -\frac{6}{25}$

(b) $\frac{-5}{3}\left(+\frac{5}{6}\right)$

$= \frac{-25}{18}$

(c) $-\frac{5}{14}\left(-\frac{8}{21}\right)$ (4, 7)

$= \frac{20}{147}$

(d) $\frac{5}{14}\left(-\frac{8}{21}\right)$

$= -\frac{20}{147}$

Explain the general principle(s) that you used to evaluate each of the expressions in question 7.

Multiply the numerators, multiply the denominators, reduce to lowest terms
OR
Reduce first (vertically/diagonally) then multiply

8. Evaluate each of the following expressions.

(a) $\frac{3}{5} \div \left(-\frac{2}{5}\right)$

$= \frac{3}{5} \times \left(\frac{5}{-2}\right)$ (1, 1)

$= -\frac{3}{2}$

(b) $\frac{-5}{3} \div \left(+\frac{5}{6}\right)$

$= \frac{-5}{3} \times \frac{6}{5}$ (1, 2, 1)

$= \frac{-2}{1} = -2$

(c) $-\frac{5}{14} \div \left(-\frac{8}{21}\right)$ (3)

$= \frac{-5}{14} \times \left(\frac{-21}{8}\right)$ (2)

$= \frac{15}{16}$

(d) $\frac{5}{14} \div \left(-\frac{8}{21}\right)$ (3)

$= \frac{5}{14} \times \left(\frac{-21}{8}\right)$ (2)

$= \frac{-15}{16}$

Explain the general principle(s) that you used to evaluate each of the expressions in question 7.

Multiply by the reciprocal of the second fraction
("Flip 'n Multiply)

1	2	3	4
B	E	DM	AS

Division/Multiplication - tied, left-to-right
Addition/Subtraction - tied, left-to-right

Terms are separated by + and - signs.

Separate each expression into terms. Then apply the operations in the correct order.

+(+) add a positive: GAIN
-(-) subtract a negative: GAIN

+(-) add a negative: LOSS
-(+) subtract a positive: LOSS

Gains more than losses: + answer
Losses more than gains: - answer

Rule for Determining the Sign of the Answer when Multiplying and Dividing

Two Numbers
Signs Same: + answer
Signs Different: - answer

More than Two Numbers
Even # of Negatives: + answer
Odd # of Negatives: - answer

Adding/Subtracting Fractions

- Express each fraction with a common denominator.
- Add/subtract the numerators.
- Keep the denominator!
- If possible, reduce to lowest terms.

$$\frac{3}{10}+\frac{8}{15}=\frac{9}{30}+\frac{16}{30}=\frac{25}{30}=\frac{5}{6}$$

Multiplying Fractions

- Multiply the numerators and multiply the denominators.
- If possible, reduce to lowest terms.

OR

- Reduce first (vertically and diagonally).
- Multiply the numerators and multiply the denominators.

$$\frac{3}{10}\left(\frac{8}{15}\right)=\frac{24}{150}=\frac{4}{25} \quad \text{OR} \quad \frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}}\left(\frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{15}}}\right)=\frac{4}{25}$$

Dividing Fractions

- *Do not* change the 1st fraction.
- Change ÷ to ×.
- Find the reciprocal of the 2nd fraction (i.e. "*flip*").
- Summary: Multiply by the reciprocal (i.e. "flip 'n multiply")

$$\frac{3}{10}\div\frac{8}{15}=\frac{3}{10}\times\frac{15}{8}=\frac{45}{80}=\frac{9}{16}$$

9. *Evaluate* each of the following *expressions*. (The rules for working with integers and fractions are summarized above.)

(a) $-20\div\left(-4-(-8)\right)$

$=-20\div(-4+8)$
$=-20\div 4$
$=-5$

(b) $-20-4(-8)^2$

$=-20-4(64)$
$=-20-256$
$=-276$

(c) $-20+\left(-4-(-2)^3\right)$

$=-20+(-4-(-8))$
$=-20+(-4+8)$
$=-20+4=-16$

(d) $2(-7)-\dfrac{10}{2^2-3^2}+2(-3)^4$

$=-14-\dfrac{10}{4-9}+2(81)$
$=-14-\left(\dfrac{10}{-5}\right)+162$
$=-14-(-2)+162$
$=-14+2+162$
$=150$

(e) $-3\left[-2+2(6)-4(3)^3\right]^4$

$=-3[-2+12-4(27)]^4$
$=-3[10-108]^4$
$=-3(-98)^4$
$=-3(92236816)$
$=-276710448$

(f) $\dfrac{-10+5(-3)}{\left[2-(-3)\right]^2}$

$=\dfrac{-10+(-15)}{[2+3]^2}$
$=\dfrac{-10-15}{5^2}$
$=\dfrac{-25}{25}$
$=-1$

(g) $-\frac{5}{14}+\left(-\frac{8}{21}\right)\left(\frac{7}{-4}\right)$

$=-\frac{5}{14}+\frac{2}{3}$

$=\frac{-15}{42}+\frac{28}{42}$

$=\frac{13}{42}$

(h) $-\frac{5}{3}\div\frac{10}{9}+\left(-\frac{8}{21}\right)\left(\frac{3}{-4}\right)^2$

$=-\frac{5}{3}\times\frac{9}{10}+\left(-\frac{8}{21}\right)\left(\frac{9}{16}\right)$

$=-\frac{3}{2}+\left(-\frac{3}{14}\right)$

$=\frac{-21}{14}-\frac{3}{14}$

$=\frac{-24}{14}=-\frac{12}{7}$

(i) $\frac{-10+5(-3)}{[2-(-3)]^2}-\left(\frac{-5}{3}\right)\left(+\frac{5}{6}\right)$

$=\frac{-10+(-15)}{(2+3)^2}-\left(\frac{-25}{18}\right)$

$=\frac{-25}{5^2}+\frac{25}{18}$

$=\frac{-25}{25}+\frac{25}{18}$

$=-1+\frac{25}{18}$

$=\frac{-18}{18}+\frac{25}{18}=\frac{7}{18}$

10. *Simplify* each of the following *expressions*.

(a) $6a+7b-9a+4b$

$=6a-9a+7b+4b$

$=-3a+11b$

(b) $-6a+7b-9a-4b$

$=-6a-9a+7b-4b$

$=-15a+3b$

(c) $-6a-7b-9a-4b$

$=-6a-9a-7b-4b$

$=-15a-11b$

(d) $-n^2-7n-9n^2+4n$

$=-n^2-9n^2-7n+4n$

$=-10n^2+3n$

(e) $-n^2-7n-9m^2+4m$

Cannot be simplified (no like terms)

(f) $-11x^2y-7y+9x^2y+6y$

$=-11x^2y+9x^2y-7y+6y$

$=-2x^2y-y$

11. *Substitute* the given values into each of the following *expressions*.

(a) $-3a+11b$, $a=-5$, $b=-2$

$=3(-5)+11(-2)$

$=15+(-22)$

$=15-22$

$=-7$

(b) $-10n^2-3n$, $n=-5$

$=-10(-5)^2-3(-5)$

$=-10(25)-(-15)$

$=-250+15$

$=-235$

(c) $-2x^2y-y$, $x=-3$, $y=-10$

$=-2(-3)^2(-10)-(-10)$

$=-2(9)(-10)+10$

$=180+10$

$=190$

12. *Solve* each of the following *equations*.

(a) $-3a+7=13$

$\therefore -3a+7-7=13-7$

$\therefore -3a=6$

$\therefore \frac{-3a}{-3}=\frac{6}{-3}$

$\therefore a=-2$

(b) $5x-14=13$

$\therefore 5x-14+14=13+14$

$\therefore 5x=27$

$\therefore \frac{5x}{5}=\frac{27}{5}$

$\therefore x=\frac{27}{5}$

(c) $\frac{y}{6}-4=3$

$\therefore \frac{y}{6}-4+4=3+4$

$\therefore \frac{y}{6}=7$

$\therefore \frac{6}{1}\left(\frac{y}{6}\right)=6(7)$

$\therefore y=42$

add: sum, plus, increased by, more than, greater than, total of
subtract: difference, minus, decreased by, less than
multiply: product, times, of, double (x2), twice (x2), triple (x3)
divide: quotient, ratio of

13. Write each of the following as an *algebraic expression*.

(a) The difference of a number and 5: $x-5$

(b) A number increased by 3: $n+3$

(c) The quotient of a number and 7: $\frac{d}{7}$

(d) The product of a number and 5: $5y$

(e) A number decreased by 3: $m-3$

(f) The sum of a number and 9: $p+9$

14. Write each of the following as a *verbal expression*.

(a) $\frac{x}{2}$: Half of a number

(b) q^2: A number squared

(c) $n-14$: A number decreased by 14

(d) $3z$: The product of 3 and a number

(e) s^3: A number cubed

(f) $14-s$: 14 decreased by a number

15. *Solve* each problem by writing and solving an *equation*.

(a) The sum of three consecutive integers is 6144. Find the integers.

Let x represent the smallest of the three integers. Then $x+1$ and $x+2$ must be the next two consecutive integers.

sum of integers is 6144

$x+x+1+x+2 = 6144$

$\therefore 3x+3 = 6144$

$\therefore 3x+3-3 = 6144-3$

$\therefore 3x = 6141$

$\therefore \frac{3x}{3} = \frac{6141}{3}$

$\therefore x = 2047$

$x+1 = 2048$

$x+2 = 2049$

(b) Bhav has a pocket-full of nickels and pennies to pay for lunch. If the total number of coins is 103 and their total value is \$3.99, how many of each coin does he have?

Let p represent the number of pennies. Then the number of nickels must be $103-p$.

total value of coins is \$3.99

$0.01p + 0.05(103-p) = 3.99$

$\therefore 0.01p + 5.15 - 0.05p = 3.99$

$\therefore -0.04p + 5.15 = 3.99$

$\therefore -0.04p + 5.15 - 5.15 = 3.99 - 5.15$

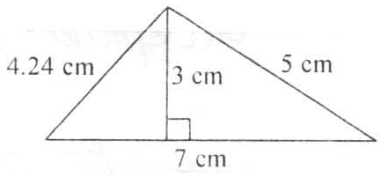
$\therefore -0.04p = -1.16$

$\therefore \frac{-0.04p}{-0.04} = \frac{-1.16}{-0.04}$

$\therefore p = 29$

$n = 103-29 = 74$

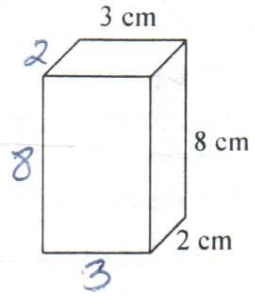
16. Find the *perimeter* and *area* of the triangle shown at the right.



$P = 4.24 + 7 + 5 = 16.24 \text{ cm}$

$A = \frac{bh}{2} = \frac{7(3)}{2} = \frac{21}{2} = 10.5 \text{ cm}^2$

17. Find the ***surface area*** and ***volume*** of the given rectangular prism.



Surface Area: $A = 2(3)(8) + 2(2)(8) + 2(3)(2)$

$= 48 + 32 + 12$

$= 92\ cm^2$

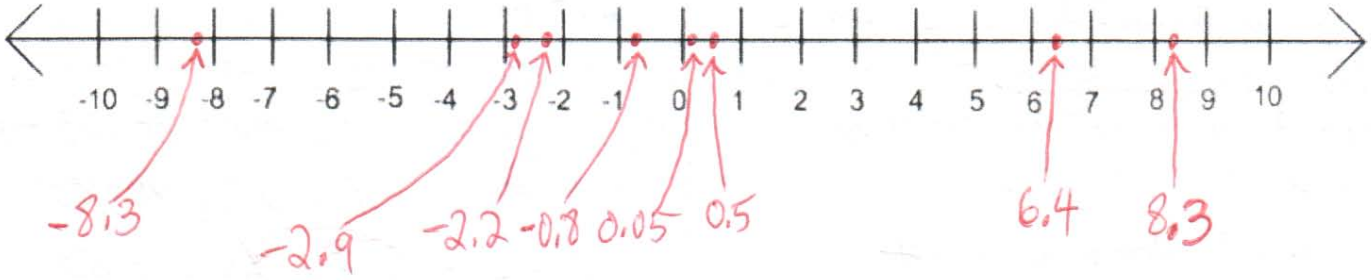
Volume: $V = lwh$

$= 3(2)(8)$

$= 48\ cm^3$

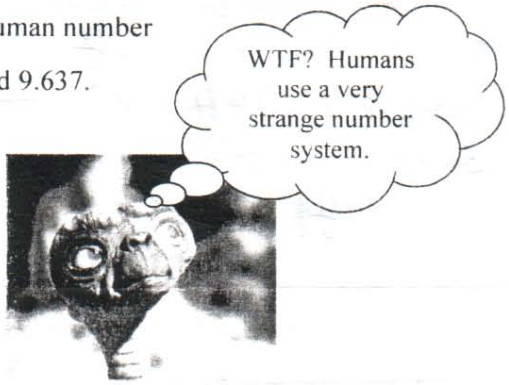
18. Use the given number line to arrange the following numbers in order from smallest to largest.

8.3, –2.9, 0.05, 6.4, –0.8, –2.2 –8.3, 0.5



19. An extraterrestrial being is seeking your help in learning how to use the human number system. He/she/it asks you to explain the meaning of the numbers $2\frac{4}{7}$ and 9.637.

Draw diagrams to help the extraterrestrial understand our number system.



Various answers are acceptable,

To be discussed in class.