

SOLUTIONS - PREPARATION FOR CUMULATIVE TEST #1: UNITS 1 TO 3

Name: _____

1. Evaluate.

$$\begin{aligned}
 \text{(a)} \quad & -4^2 + (-4)^2 - 5[5 - 7(6)] + 5(-2)^5 \\
 & = -16 + 16 - 5(5 - 42) + 5(-32) \\
 & = 0 - 5(-37) + (-160) \\
 & = 185 - 160 \\
 & = 25
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{-2[14 - 3(-7)^2] - (-16)}{-6^2 + 3(-7)(-8) - 4(3 - 7)} \\
 & = \frac{-2[14 - 3(49)] + 16}{-36 + 168 - 4(-4)} \\
 & = \frac{-2(14 - 147) + 16}{132 - (-16)} \\
 & = \frac{-2(-133) + 16}{132 + 16} \\
 & = \frac{266 + 16}{148} \\
 & = \frac{282}{148} = \frac{141}{74}
 \end{aligned}$$

2. Simplify first if possible. Then substitute and evaluate.

$$\begin{aligned}
 \text{(a)} \quad & -5st - 6t(s - s^2) + 9s^2t, s = -2, t = -5 \\
 & = -5st - 6st + 6s^2t + 9s^2t \\
 & = -11st + 15s^2t \quad (\text{Unlike terms. Cannot be simplified further.}) \\
 & = -11(-2)(-5) + 15(-2)^2(-5) \\
 & = -110 + 15(4)(-5) \\
 & = -110 + (-300) \\
 & = -110 - 300 \\
 & = -410
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{-6t(s - s^2)^3}{-5st + 9s^2t}, s = -2, t = -5 \rightarrow \text{Cannot be simplified} \\
 & = \frac{-6(-5)[-2 - (-2)^2]^3}{-5(-2)(-5) + 9(-2)^2(-5)} \\
 & = \frac{30[-2 - 4]^3}{-50 + 9(4)(-5)} = \frac{-6480}{-230} \\
 & = \frac{30(-6)^3}{-50 + (-180)} = \frac{648}{23} \\
 & = \frac{30(-216)}{-50 - 180}
 \end{aligned}$$

3. Simplify fully.

$$\begin{aligned}
 \text{(a)} \quad & -(9ab - 7a^2b + 1) + (8ab - 11a^2b - 19) - 13a(b - ab - 2) \\
 & = -9ab + 7a^2b - 1 + 8ab - 11a^2b - 19 - 13ab + 13a^2b + 26a \\
 & = 7a^2b - 11a^2b + 13a^2b - 9ab + 8ab - 13ab + 26a \\
 & \quad + 1 - 19 \\
 & = 9a^2b - 14ab + 26a - 18
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{(-5pq^2)^4(7p^3q^3)}{(-9q)(-14p)^2} \\
 & = \frac{(-5)^4 p^4 q^8 (7p^3 q^3)}{(-9q)(-14)^2 p^2} \\
 & = \frac{625 p^4 q^8 (7p^3 q^3)}{(-9q)(196 p^2)} \\
 & = \frac{625(7) p^4 p^3 q^8 q^3}{-9(196) p^2 q} \\
 & = \frac{4375 p^7 q^{11}}{-1764 p^2 q} \\
 & = \frac{625}{252} p^5 q^{10}
 \end{aligned}$$

4. Solve each equation.

(a) $-\frac{2}{3}(4x-7) - \frac{5}{4}x = \frac{3x+4}{12} - 2$

$$\therefore \frac{12}{1} \left[-\frac{2}{3}(4x-7) \right] - \frac{12}{1} \left(\frac{5}{4}x \right) = \frac{12}{1} \left(\frac{3x+4}{12} \right) - 12(2)$$

$$\therefore -8(4x-7) - 15x = 3x+4 - 24$$

$$\therefore -32x + 56 - 15x = 3x - 20$$

$$\therefore -47x + 56 = 3x - 20$$

$$\therefore -47x + 56 - 3x = 3x - 20 - 3x$$

$$\therefore -50x + 56 = -20$$

$$\therefore -50x + 56 - 56 = -20 - 56$$

$$\therefore -50x = -76$$

$$\therefore \frac{-50x}{-50} = \frac{-76}{-50}$$

$$\therefore x = \frac{38}{25}$$

(b) $\frac{4}{5}a + \frac{3a}{10} - 5(-3a+7) = -\frac{a+3}{15} + a$

$$\therefore \frac{30}{1} \left(\frac{4}{5}a \right) + \frac{30}{1} \left(\frac{3a}{10} \right) - 30(5(-3a+7)) = \frac{30}{1} \left(-\frac{a+3}{15} \right) + 30a$$

$$\therefore 24a + 9a - 150(-3a+7) = -2(a+3) + 30a$$

$$\therefore 33a + 450a - 1050 = -2a - 6 + 30a$$

$$\therefore 483a - 1050 = 28a - 6$$

$$\therefore 483a - 1050 - 28a = 28a - 6 - 28a$$

$$\therefore 455a - 1050 = -6$$

$$\therefore 455a - 1050 + 1050 = -6 + 1050$$

$$\therefore 455a = 1044$$

$$\therefore \frac{455a}{455} = \frac{1044}{455}$$

$$\therefore a = \frac{1044}{455}$$

5. Rearrange each equation to solve for the indicated variable.

(a) $V = \frac{1}{3}\pi r^2 h$, solve for r .

$$\therefore 3V = \pi r^2 h$$

$$\therefore 3V = \pi r^2 h \quad \therefore \sqrt{\frac{3V}{\pi h}} = r$$

$$\therefore \frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\therefore \frac{3V}{\pi h} = r^2$$

$$\therefore \sqrt{\frac{3V}{\pi h}} = \sqrt{r^2}$$

$$\therefore r = \sqrt{\frac{3V}{\pi h}}$$

(b) $A = \pi r^2 + 2\pi r h$, solve for h .

$$\therefore A - \pi r^2 = \pi r^2 + 2\pi r h - \pi r^2$$

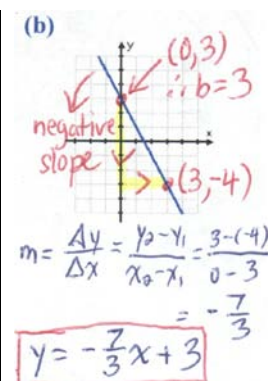
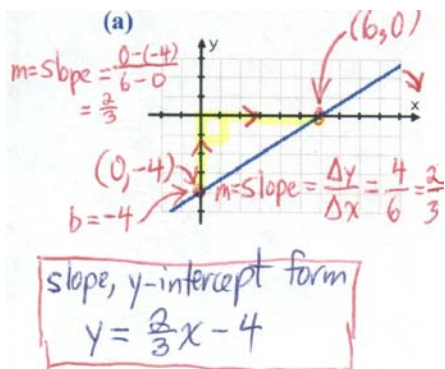
$$\therefore A - \pi r^2 = 2\pi r h$$

$$\therefore \frac{A - \pi r^2}{2\pi r} = \frac{2\pi r h}{2\pi r}$$

$$\therefore \frac{A - \pi r^2}{2\pi r} = h$$

$$\therefore h = \frac{A - \pi r^2}{2\pi r}$$

6. Find an equation of each line in **slope, y-intercept form** (i.e. in the form $y = mx + b$).



7. For a taxi ride, a Toronto taxi company charges \$5.00 plus \$1.50 per kilometre travelled.

- (a) Complete the following table:

d = distance (km), C = cost (\$)

d	C	ΔC (1 st differences)
0	\$5.00	—
10	\$20.00	\$15
20	\$35.00	\$15
30	\$50.00	\$15
40	\$65.00	\$15
50	\$80.00	\$15

- (b) Is this relation an example of direct variation or partial variation? Explain.

Partial variation.
 When $d=0$, $C=5$
 (does not pass through origin)

- (c) Explain why the relation between C and d must be linear. In addition, state the **slope** and the **y-intercept**.

The relation is linear because the first differences are constant.
 $b = 5$
 $m = \frac{\Delta C}{\Delta d} = \frac{15}{10} = 1.5$

- (d) Which variable is the dependent variable? Explain.

C is dependent because cost depends on distance travelled.

- (e) Write an equation, in the form $y = mx + b$, that relates C to d .

$$C = 1.5d + 5$$

- (f) Graph the relation.

(See graph below and to the right)

- (g) Interpret the slope as a rate of change.

$m = 1.5$
 = cost per kilometre is \$1.50

- (h) Interpret the y-intercept as an initial value.

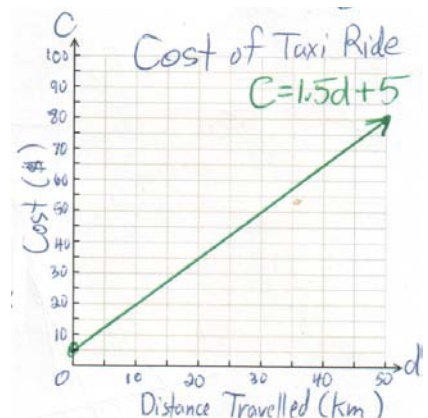
The initial cost is \$5.
 (Cost of entering taxi.)

- (i) Describe the relation between C and d in words.

The cost is \$5.00 plus \$1.50 per kilometre travelled

- (j) How much would it cost to take a 100 km taxi ride?

$$C = 1.5(100) + 5 = \$155.00$$



8.

$x + x + 1 + x + 2 = -66$

The sum of three consecutive integers is -66 . Find the numbers. Let x represent the smallest integer. Then, the other integers must be $x+1$ and $x+2$.

$$\begin{aligned} \therefore x + x + 1 + x + 2 &= -66 \\ \therefore 3x + 3 &= -66 \\ \therefore 3x &= -69 \\ \therefore x &= -23 \end{aligned}$$

The three consecutive integers must be $-23, -22$ and -21 .

9.

Chris has two cats named Toonie and Loonie. Toonie, the older cat, is one and a half times heavier than Loonie. Their combined mass is 18 kg. What is Toonie's mass? Let m represent Loonie's mass.

$$\begin{aligned} \therefore m + 1.5m &= 18 \\ \therefore 2.5m &= 18 \\ \therefore m &= 7.2 \end{aligned}$$

Toonie's mass is $1.5(7.2) = 10.8$ kg

10. Kim's Coffee Shop sells a cup of tea for \$1.05, a cup of coffee for \$1.35, and a cup of hot chocolate for \$2.25. One busy day, 20 more cups of coffee than cups of hot chocolate were sold, and 30 more cups of coffee than tea, for a total of \$202.50 for all three hot drinks. How many cups of each drink were sold? (Use an equation to solve this problem.)

#cups		
tea	coffee	H.C.
C	C+30	C+10

Total Cost of drinks = \$202.50

$$1.05C + 1.35(C+30) + 2.25(C+10) = 202.50$$

$$1.05C + 1.35C + 40.5 + 2.25C + 22.5 = 202.50$$

$$4.65C + 63 = 202.5$$

$$4.65C = 202.5 - 63 = 139.5$$

$$\frac{4.65C}{4.65} = \frac{139.5}{4.65}$$

$$C = 30$$

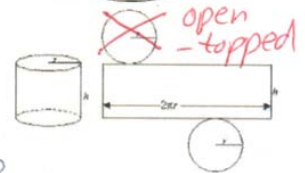
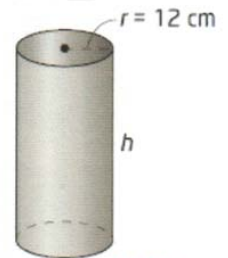
Thirty cups of tea, sixty cups of coffee and forty cups of hot chocolate were sold.

11.

An open-topped cylindrical garbage container has a surface area of 1500 cm^2 and a radius of 12 cm. What is its height, to the nearest tenth of a centimetre?

Note: The formula for the surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$. Keep in mind

that the garbage container is open-topped! (See the net at the right.)



$$2\pi r^2 + 2\pi rh = 1500$$

$$\pi(12)^2 + 2\pi(12)h = 1500$$

$$144\pi + 24\pi h = 1500$$

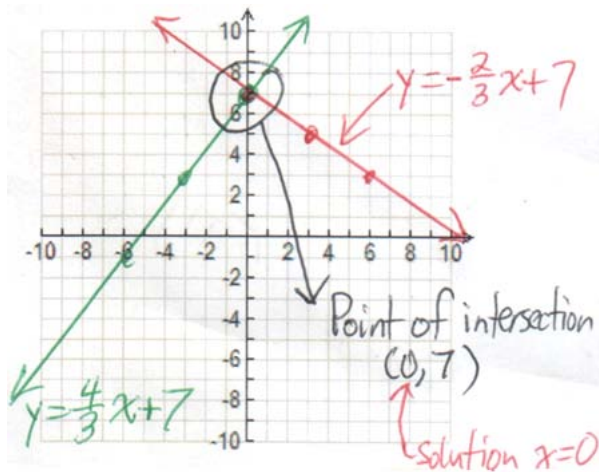
$$24\pi h = 1500 - 144\pi$$

$$h = \frac{1500 - 144\pi}{24\pi} = \frac{1500 - 144(3.14)}{24(3.14)} = 13.9 \text{ cm}$$

12. So far in this course you have only solved equations using algebraic methods. In this question you will solve the equation $-\frac{2}{3}x + 7 = \frac{4}{3}x + 7$ using a graphical method as well as a geometric method.

Graphical Method

Sketch the graphs of $y = -\frac{2}{3}x + 7$ and $y = \frac{4}{3}x + 7$ on the same set of axes. Locate the point of intersection of the two lines. One of the co-ordinates of the point of intersection is the solution.



Algebraic Method

Use an algebraic method to solve $-\frac{2}{3}x + 7 = \frac{4}{3}x + 7$.

Does the answer agree with the answer produced by the graphical method?

$$\frac{2}{3}(-\frac{2}{3}x) + 3(7) = \frac{2}{3}(\frac{4}{3}x) + 3(7)$$

$$-2x + 21 = 4x + 21$$

$$-2x + 21 - 4x = 4x + 21 - 4x$$

$$-6x + 21 = 21$$

$$-6x + 21 - 21 = 21 - 21$$

$$-6x = 0$$

$$x = 0$$

Brain Teaser

There are 5 jars of pills containing pills of the same type. Four of the jars contain pills that have a mass of 10 g each. One jar, however, contains only defective pills, each of which has a mass of 9 g. Determine which jar has the defective pills by making exactly one measurement with a scale.