# RETROSPECTIVE ASSIGNMENT 1: NUMBER SENSE AND ALGEBRA

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**Name:** Mr. Solutions

1. Complete the following statements: /4

Two algebraic expressions are said to be ***equivalent*** if they can be simplified to exactly the same expression. Equivalent expressions must agree for all possible values of the variable(s).

The following is an example of two equivalent expressions: $2d+5d$, $3d+4d$ (both simplify to $7d$)

2. The expressions $2x$ and $x^2$ are ***not equivalent***. Show this in the following ways. /16

(a) $2x$ ***means*** double a number while $x^2$ ***means*** a number times itself.

For example, if $x=-7$,

$2x =$ $2(-7)$ $=$ $-14$

and $x^2 =$ $(-7)^2$ $=$ $49$.

(b) $2x$ ***means*** $2$ groups of $x$ while $x^2$ ***means*** $x$ groups of $x$.

For example, if $x=4$,

$2x=2(4)$, which means $2$ groups of $4$ and

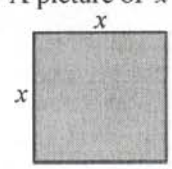
$x^2=4^2=(4)(4)$, which means $4$ groups of $4$.

(c) Complete the table. Then draw conclusions by completing the statement to the right of the table.

| $x$ | $2x$ | $x^2$ |
|---|---|---|
| $-5$ | $-10$ | $25$ |
| $-4$ | $-8$ | $16$ |
| $-3$ | $-6$ | $9$ |
| $-2$ | $-4$ | $4$ |
| $-1$ | $-2$ | $1$ |
| $0$ | $0$ | $0$ |
| $1$ | $2$ | $1$ |
| $2$ | $4$ | $4$ |
| $3$ | $6$ | $9$ |
| $4$ | $8$ | $16$ |
| $5$ | $10$ | $25$ |

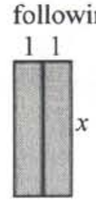
From the table, we can see that $2x$ and $x^2$ agree ***only when*** $x=$ $0$ and when $x=$ $2$. For all other values of $x$, $2x$ and $x^2$ do not agree. Therefore, $2x$ and $x^2$ ***cannot be*** equivalent.

(d) A picture of $x^2$ could look like the following:




This picture can represent $x^2$ because the area of the square is $x^2$.

On the other hand, a picture of $2x$ could look like the following:




This picture can represent $2x$ because the area of the rectangle is $2x$.

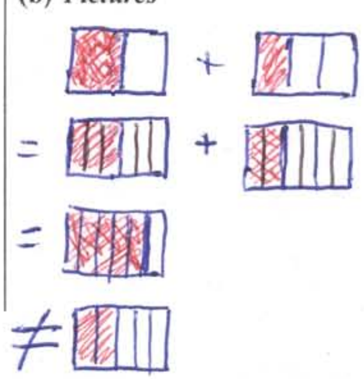
From these pictures we must conclude that $2x$ and $x^2$ are ***not equivalent*** because the areas are not the same. (The areas only agree if $x=2$.)

3. Using both a ***logical argument*** and ***pictures***, explain why $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$. Then complete the statement at the right. /6

(a) ***Logical Argument***

The value of $\frac{1}{2}+\frac{1}{3}$ must be greater than $\frac{1}{2}$. However, $\frac{2}{5}$ is less than $\frac{1}{2}$. Therefore, $\frac{1}{2}+\frac{1}{3}$ cannot possibly equal $\frac{2}{5}$.

(b) ***Pictures***




Whenever I ***add*** or ***subtract fractions***, I must always remember to express each fraction using a common denominator because each whole must be divided into the same number of equal parts.

4: First complete the statements found below. Then *evaluate* the expression shown at the right. Show all steps!

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Whenever I ***evaluate expressions***, I must always remember:

1. ***Adding*** and ***subtracting*** involves losses and gains.
2. ***Multiplying*** and ***dividing*** involves counting negative signs.
3. I ***should not*** use the distributive property because the expressions in brackets CAN be simplified.
4. I ***should*** use BEDMAS so that I'll know how to apply the operations in the correct order.
5. I ***should*** separate the expression into terms.

$$\frac{-2\left[4^2-3(-7)^2\right]-\left(3^2-2^4\right)}{-6^2+(-6)^2+3(-7)(-8)-4(3-7)}$$

$$=\frac{-2[16-3(49)]-(9-16)}{-36+36+168-4(-4)}$$

$$=\frac{-2[16-147]-(-7)}{168+16}$$

$$=\frac{-2(-131)+7}{184}$$

$$=\frac{262+7}{184}$$

$$=\frac{269}{184}$$

5. First complete the statements found below. Then *substitute* the given values into the expression shown at the right and *evaluate*. Show all steps!

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Whenever I ***substitute values into expressions***, I must:

1. Replace the ***variables*** with empty brackets, taking care to ensure that operations are not changed and exponents remain the same.
2. Then the given values should be inserted into the empty brackets, taking care to ensure that the correct values are used.
3. Finally, the resulting expression should be evaluated using BEDMAS and keeping in mind all the points made in question 4.

$$\frac{-a^2+3ab^3-6ab^2}{(a-b)(a+b)},\ a=4,\ b=-\frac{1}{2}$$

$$=\frac{-(4)^2+3(4)\left(-\frac{1}{2}\right)^3-6(4)\left(-\frac{1}{2}\right)^2}{\left(4-\left(-\frac{1}{2}\right)\right)\left(4+\left(-\frac{1}{2}\right)\right)}$$

$$=\frac{-16+\frac{12}{1}\left(\frac{-1}{8}\right)-\frac{24}{1}\left(\frac{1}{4}\right)}{\left(\frac{8}{2}+\frac{1}{2}\right)\left(\frac{8}{2}-\frac{1}{2}\right)}$$

$$=\frac{-16+\left(\frac{-3}{2}\right)-6}{\frac{9}{2}\left(\frac{7}{2}\right)}$$

$$=\frac{\frac{-32}{2}-\frac{3}{2}-\frac{12}{2}}{\left(\frac{63}{4}\right)}$$

$$=-\frac{47}{2}\div\frac{63}{4}$$

$$=\frac{-47}{\cancel{2}_1}\times\frac{\cancel{4}^2}{63}=\frac{-94}{63}$$

6. First complete the statements found below. Then *simplify* the expression shown at the right. Show all steps!

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$$\frac{(ab^2)^3(-2a^2b)^4}{(-4a)^3} - 2a^5b^3(3a^3b^7-6ab) - ab(a^5b^3+7ab)$$

Whenever I ***simplify expressions***, I must remember:

1. When ***adding*** and ***subtracting expressions***, I must collect like terms. I should add the opposite ***only when*** subtracting a polynomial in brackets.
2. When ***multiplying*** and ***dividing*** expressions, I must put like factors together and use the laws of exponents. In addition, if I multiply a monomial by a polynomial with two or more terms, I should use the distributive property.
3. I ***must never confuse*** addition and subtraction with multiplication and division. For example,
   - $x^2+x^2 = 2x^2$ but $x^2(x^2) = x^4$

$$=\frac{a^3(b^2)^3(-2)^4(a^2)^4b^4}{(-4)^3a^3} - 6a^8b^{10} + 12a^6b^4 - a^6b^4 - 7a^2b^2$$

$$=\frac{a^3b^6(16)a^8b^4}{-64a^3} + 12a^6b^4 - a^6b^4 - 6a^8b^{10} - 7a^2b^2$$

$$=\frac{16a^{11}b^{10}}{-64a^3} + 11a^6b^4 - 6a^8b^{10} - 7a^2b^2$$

$$=\left(\frac{16}{-64}\right)\left(\frac{a^{11}}{a^3}\right)\left(\frac{b^{10}}{1}\right) + 11a^6b^4 - 6a^8b^{10} - 7a^2b^2$$

$$=-\frac{1}{4}a^8b^{10} + 11a^6b^4 - 6a^8b^{10} - 7a^2b^2$$

$$=-\frac{1}{4}a^8b^{10} - 6a^8b^{10} + 11a^6b^4 - 7a^2b^2$$

$$=-\frac{1}{4}a^8b^{10} - \frac{24}{4}a^8b^{10} + 11a^6b^4 - 7a^2b^2$$

$$=-\frac{25}{4}a^8b^{10} + 11a^6b^4 - 7a^2b^2$$

7. ***Write expressions*** for the area and perimeter of the following shape.

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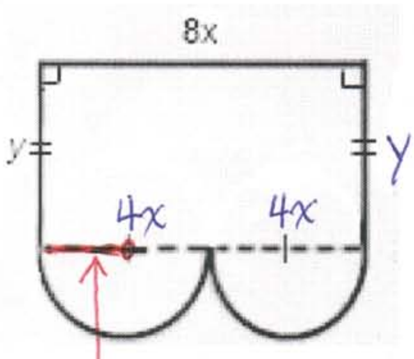


$r = 2x$

Total area

$= \underbrace{\text{area of rectangle} + \text{area of 2 semi-circles}}$ area of one circle

$= 8x(y) + \pi r^2$

$= 8xy + \pi(2x)^2$

$= 8xy + 4\pi x^2$

circumference of 2 semi-circles put together

Perimeter $= 8x + y + y + 2\pi r$

$= 8x + 2y + 2\pi(2x)$

$= 8x + 2y + 4\pi x$

This can be simplified further because $8x$ and $4\pi x$ are like terms

$= 8x + 4\pi x + 2y$

$= (8+4\pi)x + 2y$