# Retrospective Assignment: Units 1 to 5

**Name:** ______________________

**1.** Evaluate.

**(a)** $-4^2+(-4)^2-5[5-7(6)]+5(-2)^5$

$=-16+16-5(5-42)+5(-32)$

$=0-5(-37)+(-160)$

$=185-160$

$=25$

**(b)** $\dfrac{-2\left[14-3(-7)^2\right]-(-16)}{-6^2+3(-7)(-8)-4(3-7)}$

$=\dfrac{-2[14-3(49)]+16}{-36+168-4(-4)}$

$=\dfrac{-2(14-147)+16}{132-(-16)}$

$=\dfrac{-2(-133)+16}{132+16}$

$=\dfrac{266+16}{148}$

$=\dfrac{282}{148}=\dfrac{141}{74}$

**2.** ***Simplify first*** if possible. Then substitute and evaluate.

**(a)** $-5st-6t\left(s-s^2\right)+9s^2t$, $s=-2$, $t=-5$

$=-5st-6st+6s^2t+9s^2t$

$=-11st+15s^2t$ (Unlike terms. Cannot be simplified further)

$=-11(-2)(-5)+15(-2)^2(-5)$

$=-110+15(4)(-5)$

$=-110+(-300)$

$=-110-300$

$=-410$

**(b)** $\boxed{\dfrac{-6t\left(s-s^2\right)^3}{-5st+9s^2t}}$, $s=-2$, $t=-5$ → Cannot be simplified

$=\dfrac{-6(-5)\left[-2-(-2)^2\right]^3}{-5(-2)(-5)+9(-2)^2(-5)}$

$=\dfrac{-30[-2-4]^3}{-50+9(4)(-5)}$

$=\dfrac{-30(-6)^3}{-50+(-180)}$

$=\dfrac{-30(-216)}{-50-180}$

$=\dfrac{6480}{-230}$

$=-\dfrac{648}{23}$

**3.** Simplify fully.

**(a)** $-\left(9ab-7a^2b+1\right)+\left(8ab-11a^2b-19\right)-13a\left(b-ab-2\right)$

$=-9ab+7a^2b-1+8ab-11a^2b-19-13ab+13a^2b+26a$

$=7a^2b-11a^2b+13a^2b-9ab+8ab-13ab+26a+1-19$

$=9a^2b-14ab+26a-18$

**(b)** $\dfrac{\left(-5pq^2\right)^4\left(7p^3q^3\right)}{(-9q)(-14p)^2}$

$=\dfrac{(-5)^4p^4\left(q^2\right)^4\left(7p^3q^3\right)}{(-9q)(-14)^2p^2}$

$=\dfrac{625p^4q^8\left(7p^3q^3\right)}{(-9q)\left(196p^2\right)}$

$=\dfrac{625(7)p^4p^3q^8q^3}{-9(196)p^2q}$

$=\dfrac{4375p^7q^{11}}{-1764p^2q}$

$=\left(\dfrac{\cancel{4375}^{\,625}}{\cancel{-1764}_{\,252}}\right)\left(\dfrac{p^7}{p^2}\right)\left(\dfrac{q^{11}}{q^1}\right)$

$=\dfrac{625}{252}p^5q^{10}$

**4.** Solve each equation.

**(a)** $-\frac{2}{3}(4x-7)-\frac{5}{4}x=\frac{3x+4}{12}-2$

$\therefore \frac{\cancel{12}^{4}}{1}\left[-\frac{2}{\cancel{3}_{1}}(4x-7)\right]-\frac{\cancel{12}^{3}}{1}\left(\frac{5}{\cancel{4}_{1}}x\right)=\frac{\cancel{12}^{1}}{1}\left(\frac{3x+4}{\cancel{12}_{1}}\right)-12(2)$

$\therefore -8(4x-7)-15x=3x+4-24$

$\therefore -32x+56-15x=3x-20$

$\therefore -47x+56=3x-20$

$\therefore -47x+56-3x=3x-20-3x$

$\therefore -50x+56=-20$

$\therefore -50x+56-56=-20-56$

$\therefore -50x=-76$

$\therefore \frac{-50x}{-50}=\frac{\cancel{-76}^{38}}{\cancel{-50}_{25}}$

$\therefore x=\frac{38}{25}$

**(b)** $\frac{4}{5}a+\frac{3a}{10}-5(-3a+7)=-\frac{a+3}{15}+a$

$\therefore \frac{\cancel{30}^{6}}{1}\left(\frac{4}{\cancel{5}_{1}}a\right)+\frac{\cancel{30}^{3}}{1}\left(\frac{3a}{\cancel{10}_{1}}\right)-30(5(-3a+7))=\frac{\cancel{30}^{2}}{1}\left(-\frac{a+3}{\cancel{15}_{1}}\right)+30a$

$\therefore 24a+9a-150(-3a+7)=-2(a+3)+30a$

$\therefore 33a+450a-1050=-2a-6+30a$

$\therefore 488a-1050=28a-6$

$\therefore 488a-1050-28a=28a-6-28a$

$\therefore 460a-1050=-6$

$\therefore 460a-1050+1050=-6+1050$

$\therefore 460a=1044$

$\therefore \frac{460a}{460}=\frac{\cancel{1044}^{261}}{\cancel{460}_{115}}$

$\therefore a=\frac{261}{115}$

**5.** Rearrange each equation to solve for the indicated variable.

**(a)** $V=\frac{1}{3}\pi r^2 h$, solve for $r$.

$\therefore 3V=\frac{\cancel{3}^{1}}{1}\left(\frac{1}{\cancel{3}_{1}}\pi r^2 h\right)$

$\therefore 3V=\pi r^2 h$

$\therefore \frac{3V}{\pi h}=\frac{\pi r^2 h}{\pi h}$

$\therefore \frac{3V}{\pi h}=r^2$

$\therefore \sqrt{\frac{3V}{\pi h}}=\sqrt{r^2}$

$\therefore \sqrt{\frac{3V}{\pi h}}=r$

$\boxed{\therefore r=\sqrt{\frac{3V}{\pi h}}}$

**(b)** $A=\pi r^2+2\pi rh$, solve for $h$.

$\therefore A-\pi r^2=\pi r^2+2\pi rh-\pi r^2$

$\therefore A-\pi r^2=2\pi rh$

$\therefore \frac{A-\pi r^2}{2\pi r}=\frac{2\pi rh}{2\pi r}$

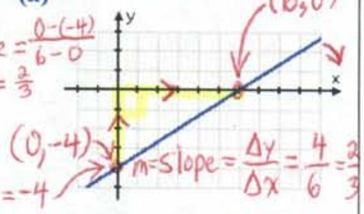
$\therefore \frac{A-\pi r^2}{2\pi r}=h$

$\boxed{\therefore h=\frac{A-\pi r^2}{2\pi r}}$

6. Find an equation of each line. If possible, write the equation in ***standard form*** as well as in ***slope, y-intercept form***.

**(a)**

$m = \text{slope} = \frac{0-(-4)}{6-0} = \frac{2}{3}$




slope, y-intercept form
$\boxed{y = \frac{2}{3}x - 4}$

$\therefore 3y = \frac{\cancel{3}}{1}\left(\frac{2}{\cancel{3}}x\right) - 3(4)$

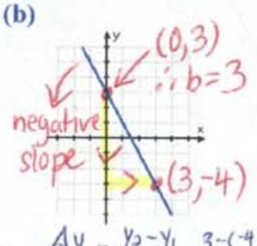
$\therefore 3y = 2x - 12$

$\therefore 3y - 3y = 2x - 12 - 3y$

$\therefore 0 = 2x - 3y - 12$

$\boxed{2x - 3y - 12 = 0}$ standard form

**(b)**




$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-4)}{0-3} = -\frac{7}{3}$

$\boxed{y = -\frac{7}{3}x + 3}$

$\therefore 3y = \frac{\cancel{3}}{1}\left(\frac{-7}{\cancel{3}}x\right) + 3(3)$

$\therefore 3y = -7x + 9$

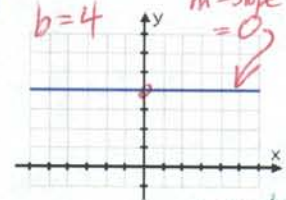
$\therefore 3y + 7x - 9 = -7x + 9 + 7x - 9$

$\boxed{\therefore 7x + 3y - 9 = 0}$
Standard Form

**(c)**

$b = 4$

$m = \text{slope} = 0$



$y = \underbrace{0x}_{\text{equals } 0} + 4$

$\boxed{\therefore y = 4}$ slope, y-intercept form

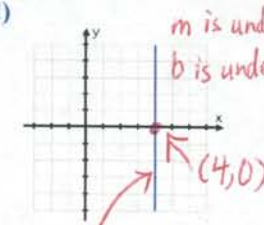
In standard form, this can be written as $0x + y - 4 = 0$

or

$\boxed{y - 4 = 0}$

**(d)**

m is undefined
b is undefined




- every point on this line has x-co-ordinate 4
- there are no restriction on the y-co-ordinate

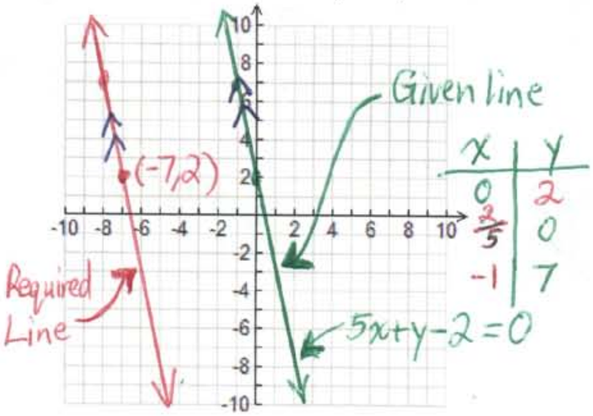
$\boxed{x = 4}$ Standard form

$1x + 0y - 4 = 0$
or $\boxed{x - 4 = 0}$

The equation cannot be written in slope, y-intercept form because m and b are undefined.

7. Find an equation of …

(a) …the line ***parallel*** to the line $5x + y - 2 = 0$ and passing through the point $(-7, 2)$.




| x | y |
|---|---|
| 0 | 2 |
| $\frac{2}{5}$ | 0 |
| -1 | 7 |

$5x + y - 2 = 0$

$\therefore 5x + y - 2 - 5x + 2 = 0 - 5x + 2$

$\therefore y = -5x + 2$

∴ slope of given line is −5

∵ given line is parallel to the required line, slope of required line must by −5

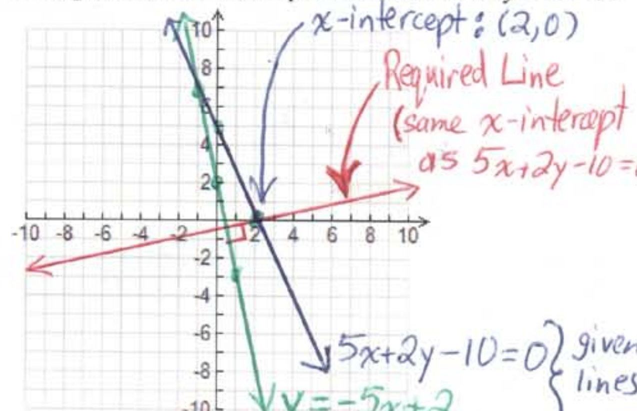
∴ equation is of the form $y = -5x + b$

∵ line passes through $(-7, 2)$, $2 = -5(-7) + b$

$\therefore b = -33$

Equation of required line: $\boxed{y = -5x - 33}$

(b) …the line ***perpendicular*** to the line $y = -5x + 2$ and having the same x-intercept as the line $5x + 2y - 10 = 0$.




Required line:
- perpendicular to $y = -5x + 2$

$\therefore m = +\frac{1}{5}$ (negative reciprocal)

- same x-intercept as $5x + 2y - 10 = 0$

∴ passes through $(2, 0)$

$\therefore y = \frac{1}{5}x + b$

$\therefore 0 = \frac{1}{5}\left(\frac{2}{1}\right) + b$

$\therefore 0 = \frac{2}{5} + b$

$\therefore b = -\frac{2}{5}$

∴ equation of required line is $\boxed{y = \frac{1}{5}x - \frac{2}{5}}$

**8.** For a taxi ride, a Toronto taxi company charges \$5.00 plus \$1.50 per kilometre travelled.

**(a)** Complete the following table of values:

$d$ = distance (km), $C$ = cost (\$)

| $d$ | $C$ | $\Delta C$ (1st differences) |
|---|---|---|
| 0 | \$5.00 | – |
| 10 | \$20.00 | \$15 |
| 20 | \$35.00 | \$15 |
| 30 | \$50.00 | \$15 |
| 40 | \$65.00 | \$15 |
| 50 | \$80.00 | \$15 |

**(b)** Is this relation an example of direct variation or partial variation? Explain.

Partial variation.
When $d=0$, $C=5$
(does not pass through origin)

**(c)** Explain why the relation between $C$ and $d$ must be linear. In addition, state the slope and the $y$-intercept.

The relation is linear because the first differences are constant.

$b=5$

$m=\frac{\Delta C}{\Delta d}=\frac{15}{10}=1.5$

**(d)** Which variable is the dependent variable? Explain.

C is dependent because cost depends on distance travelled.

**(e)** Write an equation, in the form $y=mx+b$, that relates $C$ to $d$.

$C=1.5d+5$

**(f)** Graph the relation.

(See graph below and to the right)

**(g)** Interpret the slope as a rate of change.

$m=1.5$
= cost per kilometre is \$1.50

**(h)** Interpret the $y$-intercept as an initial value.

The initial cost is \$5.
(Cost of entering taxi.)

**(i)** Describe the relation between $C$ and $d$ in words.

The cost is \$5.00 plus \$1.50 per kilometre travelled

**(j)** How much would it cost to take a 100 km taxi ride?

$C=1.5(100)+5$
$=\$155.00$

**(k)** Convert the equation that you obtained in (e) to standard form.

$C=1.5d+5$

$\therefore 0=1.5d-C+5$

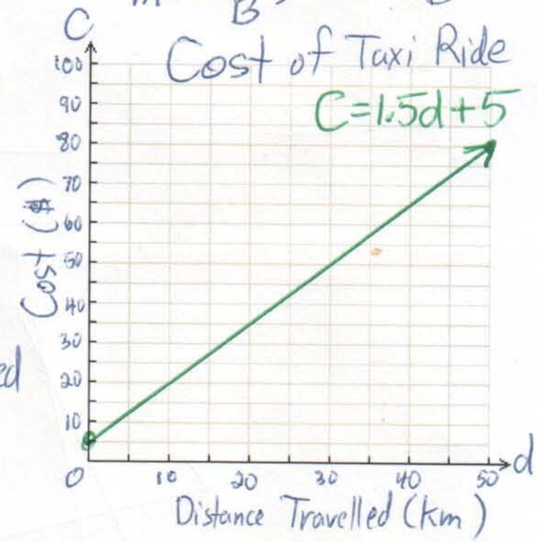
$\therefore 2(0)=2(1.5d)-2C+2(5)$

$\therefore 0=3d-2C+10$

$\therefore 3d-2C+10=0$

**(l)** Is there an easy way to determine the slope and $y$-intercept from the standard form equation of a linear relation? $Ax+By+C=0$

$m=-\frac{A}{B},\ b=-\frac{C}{B}$




**9.** This table shows the numbers of days absent from mathematics class and the math marks for 15 students.

| Number of Days Absent (d) | Math Mark (%) (M) |
|---|---|
| 2 | 82 |
| 0 | 75 |
| 10 | 48 |
| 6 | 62 |
| 1 | 76 |
| 23 | 35 |
| 13 | 42 |
| 2 | 96 |
| 1 | 54 |
| 3 | 73 |
| 7 | 65 |
| 0 | 79 |
| 10 | 60 |
| 16 | 43 |
| 1 | 84 |

Independent Variable

Dependent Variable

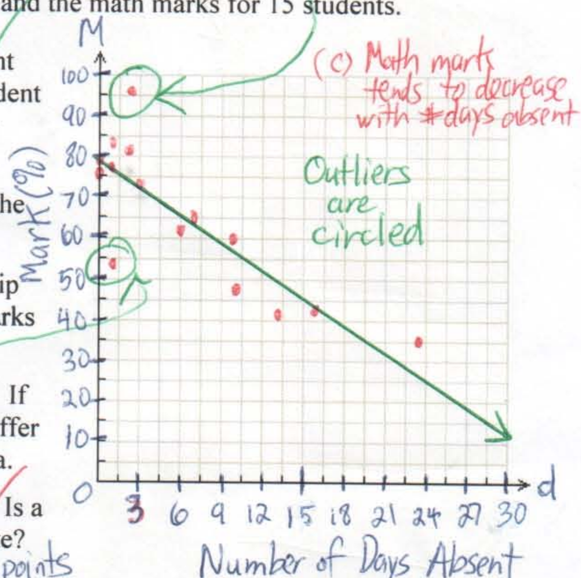
**(a)** Identify the independent variable and the dependent variable. Explain your reasoning.

**(b)** Make a scatter plot of the data.

**(c)** Describe the relationship between a student's marks and attendance.

**(d)** Are there any outliers? If so, explain how they differ from the rest of the data.

**(e)** Draw a line of best fit. Is a linear model appropriate? Explain. Most of the points are clustered around the line of best fit. Therefore, a linear model is appropriate.



$x+x+1+x+2 \quad = -66$

**10.** The sum of three consecutive integers is −66. Find the numbers. Let $x$ represent the smallest integer. Then, the other integers must be $x+1$ and $x+2$.

$\therefore x+x+1+x+2=-66$
$\therefore 3x+3=-66$
$\therefore 3x=-69$
$\therefore x=-23$

The three consecutive integers must be −23, −22 and −21.

$1.5m \quad m$

**11.** Chris has two cats named Toonie and Loonie. Toonie, the older cat, is one and a half times heavier than Loonie. Their combined mass is 18 kg. What is Toonie's mass? Let $m$ represent Loonie's mass.

$\therefore m+1.5m=18$
$\therefore 2.5m=18$
$\frac{2.5m}{2.5}=\frac{18}{2.5}$
$\therefore m=7.2$

Toonie's mass is $1.5(7.2)=10.8$ kg

**12.** Kim's Coffee Shop sells a cup of tea for \$1.05, a cup of coffee for \$1.35, and a cup of hot chocolate for \$2.25. One busy day, 20 more cups of coffee than cups of hot chocolate were sold, and 30 more cups of coffee than tea, for a total of \$202.50 for all three hot drinks. How many cups of each drink were sold?

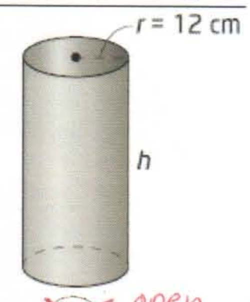
| #cups | | |
|---|---|---|
| tea | coffee | H.C. |
| c | c+30 | c+10 |

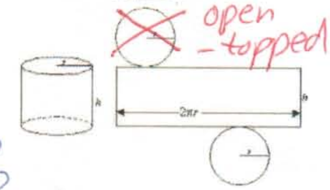
Total Cost of drinks = \$202.50

$\therefore 1.05c+1.35(c+30)+2.25(c+10)=202.50$
$\therefore 1.05c+1.35c+40.5+2.25c+22.5=202.50$
$\therefore 4.65c+63=202.5$
$\therefore 4.65c=202.5-63=139.5$
$\therefore \frac{4.65c}{4.65}=\frac{139.5}{4.65}$
$\therefore c=30$

Thirty cups of tea, sixty cups of coffee and forty cups of hot chocolate were sold.

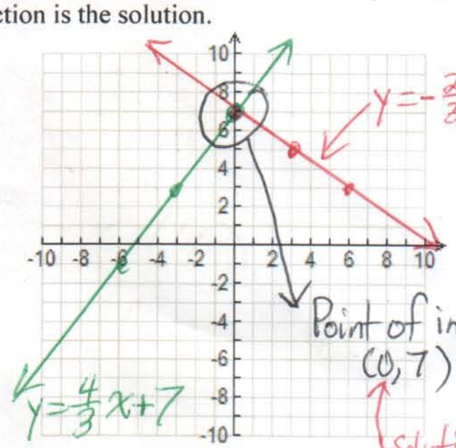
**13.** An *open-topped* cylindrical garbage container has a surface area of 1500 cm$^2$ and a radius of 12 cm. What is its height, to the nearest tenth of a centimetre?
**Note:** The formula for the surface area of a cylinder is $S=2\pi r^2+2\pi rh$. Keep in mind that the garbage container is open-topped! (See the net at the right.)




$\pi r^2+2\pi rh=1500$
$\therefore \pi(12)^2+2\pi(12)h=1500$
$\therefore 144\pi+24\pi h=1500$
$\therefore 24\pi h=1500-144\pi$
$\therefore h=\frac{1500-144\pi}{24\pi} \doteq \frac{1500-144(3.14)}{24(3.14)} \doteq 13.9\text{cm}$

**14.** So far in this course you have only solved equations using algebraic methods. In this question you will solve the equation $-\frac{2}{3}x+7=\frac{4}{3}x+7$ using a graphical method as well as a geometric method.

| ***Graphical Method*** | ***Algebraic Method*** |
|---|---|
| Sketch the graphs of $y=-\frac{2}{3}x+7$ and $y=\frac{4}{3}x+7$ on the same set of axes. Locate the point of intersection of the two lines. One of the co-ordinates of the point of intersection is the solution.  Point of intersection (0, 7) solution $x=0$ | Use an algebraic method to solve $-\frac{2}{3}x+7=\frac{4}{3}x+7$. Does the answer agree with the answer produced by the graphical method? $\therefore \frac{3}{1}\left(\frac{-2}{3}x\right)+3(7)=\frac{3}{1}\left(\frac{4}{3}x\right)+3(7)$ $\therefore -2x+21=4x+21$ $\therefore -2x+21-4x=4x+21-4x$ $\therefore -6x+21=21$ $\therefore -6x+21-21=21-21$ $\therefore -6x=0$ $\therefore \boxed{x=0}$ |



***Brain Teaser***

There are 5 jars of pills containing pills of the same type. Four of the jars contain pills that have a mass of 10 g. One jar, however, contains only contaminated pills, which have a mass of 9 g. Determine which jar has the contaminated pills by making ***exactly one measurement*** with a scale.