Expressions and Equations – Mathematical Phrases and Sentences

- *equation* → L.H.S. = R.H.S. → a complete mathematical "*sentence*"
  e.g. "The sum of two consecutive numbers is 31." → $x+x+1=31$
- *expression* → *not* a complete mathematical "sentence" → more like a *phrase*
  e.g. "Ten more than a number" → $x+10$

Solving the so-called "word problems" that you are given in school is usually just a matter of *translating English sentences into mathematical equations*.

Mathematical Words

Symbol	*English Equivalent*
+	sum, plus, added to, more than, increased by, gain of, total of, combined with
–	difference, minus, subtracted from, less than, fewer than, decreased by, loss of
×	product, times, multiplied by, of, factor of, double (×2), twice (×2), triple (×3)
÷	quotient, divided by, half of (÷2), one-third of (÷3), per, ratio of
=	is, are, was, were, will be, gives, yields

Translating from English into Algebraic Expressions and Equations

Complete the following table.

English	*Algebraic Expression*	*English*	*Algebraic Equation*
Six more than a number	$n+6$	Six more than a number is 5.	$n+6=5$
A number decreased by 7	$x-7$	A number decreased by 7 is –9.	$x-7=-9$
The product of a number and –3	$-3y$	The product of a number and –3 is 4.	$-3y=4$
Half of a number	$\frac{z}{2}$	Half of a number is 16.	$\frac{z}{2}=16$
Triple a number decreased by 5	$3x-5$	Triple a number decreased by 5 is 8.	$3x-5=8$
Double a number plus 5	$2n+5$	Double a number plus 5 gives 13.	$2z+5=13$
One-third of a number minus 2	$\frac{1}{3}x-2$	One-third of a number minus 2 yields 16.	$\frac{1}{3}x-2=16$
One-fourth of a number reduced by 5	$\frac{x}{4}-5$	One-fourth of a number reduced by 5 is -1	$\frac{x}{4}-5=-1$
Sixty-five decreased by a number	$65-a$	Sixty-five decreased by a number gives 7.	$65-a=7$
A number divided by 7	$\frac{x}{7}$	A number divided by 7 is –10.	$\frac{x}{7}=-10$
Quadruple a number subtracted *from* 6	$6-4x$	Quadruple a number subtracted *from* 6 is 2.	$6-4x=2$
The product of 4 and a number subtracted from 2	$2-4t$	The product of 4 and a number subtracted from 2 gives 3.	$2-4t=3$
The quotient of 2, and a number reduced by 4	$\frac{2}{x-4}$	The quotient of 2, and a number reduced by 4 is -9	$\frac{2}{x-4}=-9$
The quotient of 6, and a number subtracted from 3	$\frac{6}{3-x}$	The quotient of 6 and a number subtracted from 3 is 2. (no comma)	$3-\frac{6}{x}=2$
The product of 2, and a number increased by 7	$2(y+7)$	The product of 2, and a number increased by 7 is 13.	$2(y+7)=13$
The difference of triple a number, and a number increased by 3	$3x-(x+3)$	The difference of triple a number, and a number increased by 3 yields 21.	$3x-(x+3)=21$

Summary of Main Ideas

Algebra as a Language

Complete the following statements:

(a) Languages like English are best suited to descriptions of a qualitative nature.

(b) The language of algebra is best suited to descriptions of a quantitative nature.

(c) Math is like a dating service because it's all about relationships.

(d) The language of algebra has many advantages when it comes to describing mathematical relationships. Some of the advantages include 1. Understood by everyone (universal language) 2. mathematical ideas can be expressed very concisely 3. mathematical relationships can be manipulated easily (i.e. put in another form easily)

(e) Expressions and equations can be compared to phrases and sentences respectively.

Give an example of an expression: $5\sqrt{z}-7y$ Give an example of an equation: $2x-7=3$

An expression is like a phrase because it is NOT a complete mathematical statement.

An equation is like a sentence because it IS a complete mathematical statement.

(f) The Pythagorean Theorem is an example of an equation that describes the mathematical relationship among the sides of a right triangle.

(g) Math is much easier to understand when we keep in mind the MEANING of the symbols, operations, expressions and equations. Also, it helps to have good control over one's mental autopilot.

Vocabulary of Algebra

Complete the following table:

Name	Example	Name	Example
Constant	π	Unlike terms	$3x,\ 3y$
Variable	x	Like terms	$3ab^2,\ -10ab^2$
Expression	$-5x^2y-7xy$	Simplify an expression	$3ab^2-10ab^2=-7ab^2$
Term	$-5x^2y$	Evaluate an expression	$3(-3)(-4)^2-10(3)(4)^2=-304$
(Numeric) Coefficient	$-5x^2y$	Polynomial	$-4x^5+2x^3-7x^2+x-1$
Literal Coefficient (Variable Part)	$-5x^2y$	Trinomial	$-4x^5+2x^3-7x^2$
Polynomial	$-3x^2+5x^3-2x+1$	Binomial	$3ab^2-10ab^2$
Monomial	$3x$	Monomial / Term	$-7x^2$
Binomial	$5a+6b$	Expression	$-3x^2y+5abc-\frac{2xy^3z}{ab^2}-5\sqrt{z}$
Trinomial	$5a+6b+7c$	Term	$-5\sqrt{z}$
Evaluate an Expression	$-2(-4)^2+1=-2(16)+1=-31$	(Numeric) Coefficient	$-5\sqrt{z}$
Simplify an Expression	$-5xy-xy=-6xy$	Literal Coefficient (Variable Part)	$-5\sqrt{z}$
Like Terms	$x,\ 2x$	Variable	a
Unlike Terms	$x,\ y$	Constant	-34553476348.467674737

Simplifying Algebraic Expressions

1. Complete the following statements:

(a) "Evaluate an expression" means to perform operations to obtain the final answer.

(b) "Simplify an expression" means to perform operations to put an expression in a simpler form.

(c) When $-3(-3)-10(-3)(5)^2$ is evaluated, the result is $9-10(-3)(25)=9-(-750)=9+750=759$.

(d) The expression $3ab^2-10ab^2$ ***can*** be simplified because the terms are like.

(e) The expression $3ab-10ab^2$ ***cannot*** be simplified because the terms are unlike.

(f) One way to interpret the expression $2p+5p$ is two pizzas plus five pizzas. Using this interpretation, it ***makes sense*** that the simplified form is $7p$ because it would mean seven pizzas.

(g) One way to interpret the expression $2h+5d$ is two hotties plus five dogs. Using this interpretation, it ***does not make sense*** that the simplified form is ~~$7hd$~~ because 2 hotties plus 5 dogs cannot possibly equal 7 "hottie-dogs".

(h) When simplifying expressions, first like terms should be collected. When this is being done, it is very important that the operations move with the terms.

(i) When simplifying expressions containing brackets, the brackets can be removed without making any other changes only if the bracket is preceded by a "plus" sign. This is so because addition can be performed in any order whatsoever without changing the sum. If a bracket is preceded by a "minus" sign, brackets cannot be removed without making other changes. This is so because the result of subtracting (i.e. the difference) ***is*** affected by the order in which it is performed. In this case, it is best to remove brackets by adding the opposite.

2. Simplify each of the following expressions.

a) $(7x-9)+(x-4)$

b) $(3y+8)+(-y-5)$

c) $(8c-6)-(c+7)$

d) $(k+2)-(3k-2)$

e) $(3p^2-8p+1)+(9p^2+4p-1)$

f) $(5xy^2+6x-7y)-(3xy^2-6x+7y)$

g) $(4x-3)+(x+8)-(2x-5)$

h) $(2uv^2-3v)-(v+3u)+(4uv^2-9u)$

Answers:

(a) $8x-13$ (b) $2y+3$

(c) $7c-13$ (d) $-2k+4$

(e) $12p^2-4p$

(f) $2xy^2+12x-14y$

(g) $3x+10$

(h) $6uv^2-4v-12u$

Discover: Exponent Law for Multiplication of Powers

Complete the table below. Make up your own questions for the blank rows.

Product	***Expanded Form***	***Single Power***
$x^4(x^3)$	$=(x\times x\times x\times x)\times(x\times x\times x)$	$=x^7$
$y^5(y^6)$	$=(y\times y\times y\times y\times y)\times(y\times y\times y\times y\times y\times y)$	$=y^{11}$
$m^3(m^2)$	$=(m\times m\times m)\times(m\times m)$	$=m^5$
$a^4(a^5)$	$=(a\times a\times a\times a)\times(a\times a\times a\times a\times a)$	$=a^9$
$t^6(t)$	$=(t\times t\times t\times t\times t\times t)\times(t)$	$=t^7$
$w^7(w^2)$	$=(w\times w\times w\times w\times w\times w\times w)\times(w\times w)$	$=w^9$
$h^2(h^4)$	$=(h\times h)\times(h\times h\times h\times h)$	$=h^6$
$p^8(p^5)$	$=(p\times p\times p\times p\times p\times p\times p\times p)\times(p\times p\times p\times p\times p)$	$=p^{13}$
$x^a(x^b)$	$=(x\times x\times\cdots\times x)\times(x\times x\times\cdots\times x)$	$=x^{a+b}$
$c^2d^3(c^4d^5)$	$=(c\times c\times d\times d\times d)\times(c\times c\times c\times c\times d\times d\times d\times d\times d)$	$=c^6d^8$
$k^3j^4(k^5j^2)$		$=k^8j^6$
$p^4q^2(p^2q^3)$		$=p^6q^5$
$v^5z^3(v^3z^4)$		$=v^8z^7$
$g^2t^5(g^4t^3)$		$=g^6t^8$
$a^4b^2c^5(a^3b^4c^4)$		$=a^7b^6c^9$

4 factors of x plus 3 factors of x is 7 factors of x

$4+3=7$

$5+6=11$

$3+2=5$

$4+5=9$

$6+1=7$

$7+2=9$

$2+4=6$

$8+5=13$

a factors of x $\quad$ b factors of x

$3+5=8$

$4+2=6$

CONTINUE IN THE SAME FASHION

Without using expanded form:

$a^4b^2c^5(a^3b^4c^4)$

$=a^4a^3b^2b^4c^5c^4$

$=a^{4+3}b^{2+4}c^{5+4}$

$=a^7b^6c^9$

multiplication can be done in ANY order

Complete this statement: When multiplying powers with the ***same base***,

KEEP the base (don't change it) and ADD the exponents

Discover: Exponent Law for Division of Powers

Complete the table below. Make up your own questions for the blank rows.

Product	***Expanded Form***	***sSingle Power***
$\frac{x^7}{x^3}$	$=\frac{x\times x\times x\times x\times x\times x\times x}{x\times x\times x}=\left(\frac{x\times x\times x}{x\times x\times x}\right)\left(\frac{x\times x\times x\times x}{1}\right)=1\left(\frac{x^4}{1}\right)$	$=x^4$ 7-3=4
$\frac{y^5}{y^2}$	$=\frac{y\times y\times y\times y\times y}{y\times y}=\left(\frac{y\times y}{y\times y}\right)\left(\frac{y\times y\times y}{1}\right)=1\left(\frac{y^3}{1}\right)$	$=y^3$ 5-2=3
$\frac{a^4}{a^3}$	$=\frac{a\times a\times a\times a}{a\times a\times a}=\left(\frac{a\times a\times a}{a\times a\times a}\right)\left(\frac{a}{1}\right)=1\left(\frac{a}{1}\right)$	$=a^1$ 4-3=1
$\frac{t^6}{t^1}$	$=\frac{t\times t\times t\times t\times t\times t}{t}=\left(\frac{t}{t}\right)\left(\frac{t\times t\times t\times t\times t}{1}\right)=1\left(\frac{t^5}{1}\right)$	$=t^5$ 6-1=5
$\frac{w^7}{w^2}$	Continue in the same manner	$=w^5$
$p^8\div p^5$		$=p^3$
$\frac{x^a}{x^b}$		$=x^{a-b}$
$\frac{c^7d^3}{c^4d^2}$	$=\frac{c\times c\times c\times c\times c\times c\times c\times d\times d\times d}{c\times c\times c\times c\times d\times d}$	$=c^3d^1$
$\frac{k^8j^4}{k^5j^2}$		$=k^3j^2$
$\frac{p^4q^6}{p^2q^3}$		$=p^2q^3$
$\frac{v^5z^8}{v^3z^4}$		$=v^3z^4$
$\frac{g^6t^5}{g^4t^3}$		$=g^2t^2$
$\frac{a^4b^7c^6}{a^3b^4c^4}$		$=a^1b^3c^2$

Complete this statement: When dividing powers with the *same base*,

KEEP the base (don't change it) and SUBTRACT the exponents

Discover: Exponent Law for Power of a Power

Complete the table below. Make up your own questions for the blank rows.

Product	***Expanded Form***	***Single Power***
$(x^4)^2$	$=(x\times x\times x\times x)\times(x\times x\times x\times x)$	$=x^8$
$(y^3)^4$	$=(y\times y\times y)\times(y\times y\times y)\times(y\times y\times y)\times(y\times y\times y)$	$=y^{12}$
$(m^3)^2$	$=(m\times m\times m)(m\times m\times m)$	$=m^6$
$(g^2)^5$	$=(g\times g)\times(g\times g)\times(g\times g)\times(g\times g)\times(g\times g)$	$=g^{10}$
$(t^4)^3$	Continue in the same fashion	$=t^{12}$
$(w^5)^4$		$=w^{20}$
$(k^6)^2$		$=k^{12}$
$(r^3)^3$		$=r^9$
$(x^a)^b$		$=x^{ab}$
	Do these using your shortcut, without the middle step	
$(m^3)^5$	$=m^{3\times5}$	$=m^{15}$
$(q^8)^6$	$=q^{8\times6}$	$=q^{48}$
$(x^{10})^4$	$=x^{10\times4}$	$=x^{40}$
$(n^7)^8$	$=n^{7\times8}$	$=n^{56}$
$(a^3b^4)^5$	$=a^{3\times5}b^{4\times5}$	$=a^{15}b^{20}$
		$=$

Complete this statement: When a power is raised to an exponent,

KEEP the base (don't change it) and MULTIPLY the exponents