

More on Identities

Equations that are Identities	$2 + 2 = 4$	$x + x = 2x$	$3x^2 - 5x - 7x + 2 - 3 = 3x^2 - 12x - 1$	$a + 2b + 3c - a - 2b - 3c = 0$
Equations that are NOT Identities	$x + 1 = 4$	$3x - 7 = -14$	$x^2 + 3x + 2 = 0$	$x^2 + 2 = 0$

- An **identity** is an equation in which the expression on the L.H.S. is **equivalent** ("identical") to the expression on the R.H.S. In such equations, the L.H.S. equals the R.H.S. for **all possible value(s)** of the unknown(s).
- The **expressions** in equations that are **not identities**, on the other hand, are **not equivalent**. The L.H.S. equals the R.H.S. only for a specific value or specific values of the unknown. The values for which the L.H.S. equals the R.H.S. are called **solutions** of the equation. In addition, such values are said to **satisfy** the equation.

Exercise

1. State whether the given value(s) of the unknown(s) **satisfy(ies)** the given equation. Show your work!

(a) $-3x - 5 = -4$, $x = -4$	$L.S. = -3(-4) - 5 = 12 - 5$	$R.S. = -4$	$L.S. \neq R.S.$	NO!
(b) $-3x - 5 = -4$, $x = -3$	$L.S. = -3(-3) - 5 = 9 - 5 = 4$	$R.S. = -4$	$L.S. \neq R.S.$	NO!
(c) $x^2 + 5x + 6 = 0$, $x = -2$	$L.S. = (-2)^2 + 5(-2) + 6 = 4 - 10 + 6 = 0$	$R.S. = 0$	$L.S. = R.S.$	YES!
(d) $x^2 + 5x + 6 = 0$, $x = -3$	$L.S. = (-3)^2 + 5(-3) + 6 = 9 - 15 + 6 = 0$	$R.S. = 0$	$L.S. = R.S.$	YES!
(e) $d = -7t + 10$, $t = 0$, $d = 10$	$L.S. = 10$	$R.S. = -7(0) + 10 = 0 + 10 = 10$	$L.S. = R.S.$	YES!

2. Classify each of the following equations as **identities** (I) or **equations that need to be solved** (S).

- (a) $x + 3 = 4$ I/S (S) (b) $(2x^3)(3x)^4 = 162x^7$ I/S (I) (c) $3a + 4a = 7a$ I/S (I) (d) $2x - 7 = 4$ I/S (S) (e) $4 - y = 2$ I/S (S) (f) $3g - 4g = -g$ I/S (I)

3. Classify each of the following equations as **equations to be solved** (S), **equations that describe a relationship** (R) or **identities** (I). State reasons for each choice.

(a) $x - 5 = -4$ (S) R / I Reasons: Only 1 value of x satisfies the equation ($x=1$)	(b) $x - 5x = -4x$ S / R (I) Reasons: All values of x satisfy the equation	(c) $-3xy(-5xy^3) = 15x^2y^4$ S / R (I) Reasons: All values of x and y satisfy the equation
(d) $c^2 = a^2 + b^2$ S (R) I Reasons: The Pythagorean Theorem Relationship of lengths of sides of a right triangle	(e) $V = \frac{4}{3}\pi r^3$ S (R) I Reasons: Volume of a sphere how volume is related to radius.	(f) $a^2 + 3a = -2$ (S) R / I Reasons: Only has 2 solutions
(g) $x^3 + 27 = 0$ (S) R / I Reasons: Only has one solution	(h) $\frac{1}{2}(-3a-7) - \frac{3}{4}(2a) = -a+7$ (S) R / I Reasons: Only has one solution	(i) $3xy(1-5xy^3) = 3xy - 15x^2y^4$ S / R (I) Reasons: Satisfied by all possible values of x and y

4. Use **trial and error** to find solutions for each of the following equations:

<p>(a) $x - 5 = -4$</p> <p>$x = 1$ because $1 - 5 = -4$</p>	<p>(b) $-5x - 7 = -47$</p> <p>$x = 8$ because $-5(8) - 7$ $= -40 - 7$ $= -47$</p>	<p>(c) $-5(x - 7) + 3 = -4x - 15$</p> <p>This one is difficult to solve by trial and error It turns out that $x = 53$ (who would have guessed this?)</p>
<p>(d) $a^2 + 3a = -2$</p> <p>This has 2 solutions $a = -1$ and $a = -2$ Again, it would be time consuming to find these solutions by trial and error</p>	<p>(e) $x^3 + 27 = 0$</p> <p>$x = -3$ because $(-3)^3 + 27$ $= (-3)(-3)(-3) + 27$ $= -27 + 27$ $= 0$</p>	<p>(f) $\frac{1}{2}(-3a - 7) - \frac{3}{4}(2a) = -a + 7$</p> <p>Good luck with this one! It turns out that $a = -\frac{21}{4}$ ←</p>

5. Explain why **trial and error** is generally **not** a useful strategy when it comes to solving equations.

YIKES!!

Trial and error is an effective problem solving technique only when a given problem has a small number of possible solutions.

e.g. Find two positive even integers, each of which is less than 10 and which have a sum of 14.

2, 4, 6, 8 → Positive even integers less than 10.

By choosing pairs of these numbers, one quickly comes to the conclusion that the numbers must be 6 and 8.

Trial and Error: $2+4=6$, $2+6=8$, $2+8=10$ ✗
 $4+6=10$, $4+8=12$ ✗
 $6+8=14$ ✓

Since equations generally have an infinite number of possible solutions, trial and error is effective **ONLY** for very simple equations.

TECHNIQUES FOR SOLVING EQUATIONS

The Golden Rules of Solving Equations

1. Whatever operation is performed to one side of an equation **must also be** performed to the other side!
2. The goal of solving an equation is to **isolate** the unknown (get it "by itself"). This is accomplished by **undoing** the operations performed to the unknown in the order **opposite** of **BEDMAS**.

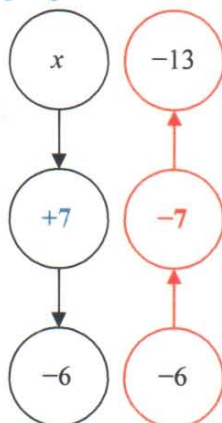
Examples of One-Step and Two-Step Equations

(a)

$$\begin{aligned} x + 7 &= -6 \\ \therefore x + 7 - 7 &= -6 - 7 \\ \therefore x &= -13 \end{aligned}$$

In Words

Add 7 to a number and the result is -6. The number must be -13.

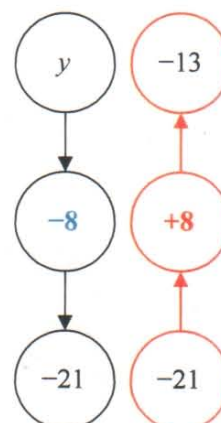


(b)

$$\begin{aligned} y - 8 &= -21 \\ \therefore y - 8 + 8 &= -21 + 8 \\ \therefore y &= -13 \end{aligned}$$

In Words

Subtract 8 from a number and the result is -21. The number must be -13.

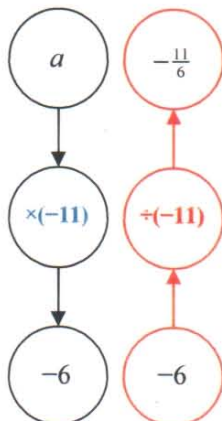


(c)

$$\begin{aligned} -11a &= -6 \\ \therefore \frac{-11a}{-11} &= \frac{-6}{-11} \\ \therefore a &= \frac{6}{11} \end{aligned}$$

In Words

Multiply a number by -11 and the result is -6. The number must be $\frac{6}{11}$.

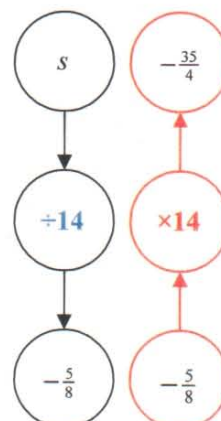


(d)

$$\begin{aligned} \frac{s}{14} &= -\frac{5}{8} \\ \therefore \frac{14}{1} \left(\frac{s}{14} \right) &= \frac{14}{1} \left(-\frac{5}{8} \right) \\ \therefore s &= -\frac{70}{8} = -\frac{35}{4} \end{aligned}$$

In Words

Divide a number by -14 and the result is $-\frac{5}{8}$. The number must be $-\frac{35}{4}$.

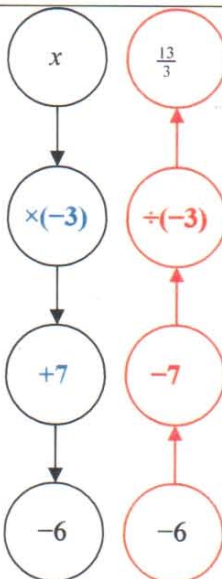


(e)

$$\begin{aligned} -3x + 7 &= -6 \\ \therefore -3x + 7 - 7 &= -6 - 7 \\ \therefore -3x &= -13 \\ \therefore \frac{-3x}{-3} &= \frac{-13}{-3} \\ \therefore x &= \frac{13}{3} \end{aligned}$$

In Words

Multiply a number by 3 then add 7 and the result is -6. The number must be $\frac{13}{3}$.

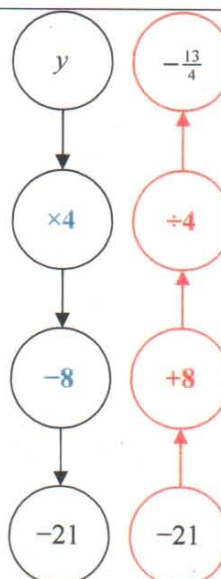


(f)

$$\begin{aligned} 4y - 8 &= -21 \\ \therefore 4y - 8 + 8 &= -21 + 8 \\ \therefore 4y &= -13 \\ \therefore \frac{4y}{4} &= \frac{-13}{4} \\ \therefore y &= \frac{-13}{4} \end{aligned}$$

In Words

Multiply a number by 4 then subtract 8 and the result is -21. The number must be $-\frac{13}{4}$.

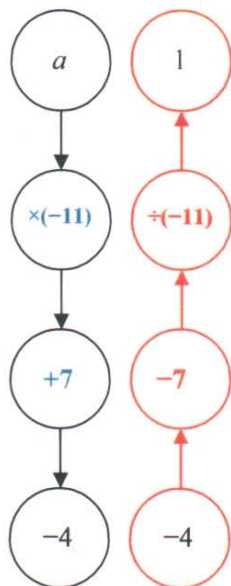


(g)

$$\begin{aligned}
 -11a + 7 &= -4 \\
 \therefore -11a + 7 - 7 &= -4 - 7 \\
 \therefore -11a &= -11 \\
 \therefore \frac{-11a}{-11} &= \frac{-11}{-11} \\
 \therefore a &= 1
 \end{aligned}$$

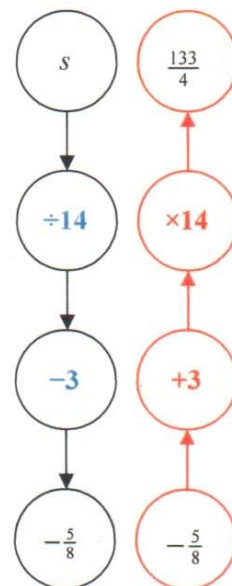
In Words

Multiply a number by -11 then add 7 and the result is -4. The number must be 1.



(h)

$$\begin{aligned}
 \frac{s}{14} - 3 &= -\frac{5}{8} \\
 \therefore \frac{s}{14} - 3 + 3 &= -\frac{5}{8} + 3 \\
 \therefore \frac{s}{14} &= -\frac{5}{8} + \frac{3}{1} \\
 \therefore \frac{s}{14} &= -\frac{5}{8} + \frac{24}{8} \\
 \therefore \frac{s}{14} &= \frac{19}{8} \\
 \therefore \frac{14}{1} \left(\frac{s}{14} \right) &= \frac{14}{1} \left(\frac{19}{8} \right) \\
 \therefore s &= \frac{266}{8} = \frac{133}{4}
 \end{aligned}$$

**Checking your Solutions**

Once you have obtained a *tentative* solution, it is a very good idea to check whether it *satisfies* both sides of the equation. Here are some examples.

(g) **Tentative Solution:** $a = 1$

L.H.S.	R.H.S.
$-11a + 7$	-4
$= -11(1) + 7$	
$= -11 + 7$	
$= -4$	

Since L.H.S. = R.H.S., $a = 1$ is the correct solution.

(h) **Tentative Solution:** $s = \frac{133}{4}$

L.H.S.	R.H.S.
$\frac{s}{14} - 3$	$-\frac{5}{8}$
$= \frac{\left(\frac{133}{4}\right)}{14} - 3$	
$= \frac{133}{4} \div \frac{14}{1} - 3$	
$= \frac{133}{4} \times \frac{1}{14} - \frac{3}{1}$	
$= \frac{133}{56} - \frac{168}{56}$	
$= \frac{-35}{56}$	
$= -\frac{35 \div 7}{56 \div 7}$	
$= -\frac{5}{8}$	

Since L.H.S. = R.H.S., $s = \frac{133}{4}$ is the correct solution.

Examples of Solving More Complicated Equations

(a) $-3(2x-7)+5x=4$
 $\therefore -6x+21+5x=4$
 $\therefore -6x+5x+21=4$
 $\therefore -x+21=4$
 $\therefore -x+21-21=4-21$
 $\therefore -x=-17$
 $\therefore x=17$

(b) $-3(2x-7)+5x=-4x+5$
 $\therefore -6x+21+5x=-4x+5$
 $\therefore -6x+5x+21=-4x+5$
 $\therefore -x+21=-4x+5$
 $\therefore -x+21+4x=-4x+5+4x$
 $\therefore 3x+21=5$
 $\therefore 3x+21-21=5-21$
 $\therefore 3x=-16$
 $\therefore \frac{3x}{3}=\frac{-16}{3}$
 $\therefore x=-\frac{16}{3}$

(c) $-5(-4y-7)+5(-y-3)=4(2y-7)+3$
 $\therefore 20y+35-5y-15=8y-28+3$
 $\therefore 15y+20=8y-25$
 $\therefore 15y+20-8y=8y-25-8y$
 $\therefore 7y+20=-25$
 $\therefore 7y+20-20=-25-20$
 $\therefore 7y=-45$
 $\therefore \frac{7y}{7}=\frac{-45}{7}$
 $\therefore y=-\frac{45}{7}$

(d) $-15(z-4)-(-15z-4)=4-3z$
 $\therefore -15z+60+(15z+4)=4-3z$
 $\therefore -15z+60+15z+4=4-3z$
 $\therefore 64=4-3z$
 $\therefore 64+3z=4-3z+3z$
 $\therefore 64+3z=4$
 $\therefore 64+3z-64=4-64$
 $\therefore 3z=-60$
 $\therefore \frac{3z}{3}=\frac{-60}{3}$
 $\therefore z=-20$

Exercises: Check the Solutions to (a) and (b) Above

(a) Tentative Solution: $x=17$

L.H.S.	R.H.S.
$-3(2x-7)+5x$	4
$=-3(2(17)-7)+5(17)$	
$=-3(34-7)+85$	
$=-3(27)+85$	
$=-81+85$	
$=4$	

Since L.H.S. = R.H.S., $x=17$ is the solution.

(b) Tentative Solution: $x=-\frac{16}{3}$

L.H.S.	R.H.S.
$-3(2x-7)+5x$	$-4x+5$
$=-3(\frac{2}{1}(-\frac{16}{3})-\frac{7}{1})+\frac{5}{1}(-\frac{16}{3})$	$=-\frac{4}{1}(-\frac{16}{3})+5$
$=-3(-\frac{32}{3}-\frac{21}{3})-\frac{80}{3}$	$=\frac{64}{3}+\frac{15}{3}$
$=-\frac{3}{1}(-\frac{53}{3})-\frac{80}{3}$	$=\frac{79}{3}$
$=\frac{159}{3}-\frac{80}{3}$	
$=\frac{79}{3}$	

Since L.H.S. = R.H.S., $x=-\frac{16}{3}$ is the solution.

Summary

1. If possible, **simplify** both sides of the equation. **Remember!** Like Terms, Distributive Property, Add the Opposite.
2. If the **variable** (i.e. the unknown) **appears on both sides** of the equation, eliminate it from one side by performing the **opposite** operation to **both sides** of the equation.
3. If you have done everything correctly, by this stage you should have an equation with **at most two** operations to undo. **Undo** the operations in the order **opposite** of **BEDMAS**. Remember to perform the same operations to both sides!

Try this One!

Solve the following equation. Then check your solution.

$$-13(-2z-3) - 1(15z+4) = -3 - 1(4-3z) - 7(3z-2)$$

$$\therefore 26z + 39 - 15z - 4 = -3 - 4 + 3z - 21z + 14$$

$$\therefore 26z - 15z + 39 - 4 = 3z - 21z - 3 - 4 + 14$$

$$\therefore 11z + 35 = -18z + 7$$

$$\therefore 11z + 35 + 18z = -18z + 7 + 18z$$

$$\therefore 29z + 35 = 7$$

$$\therefore 29z + 35 - 35 = 7 - 35$$

$$\therefore 29z = -28$$

$$\therefore \frac{29z}{29} = \frac{-28}{29}$$

$$\therefore z = \frac{-28}{29}$$

L.H.S.

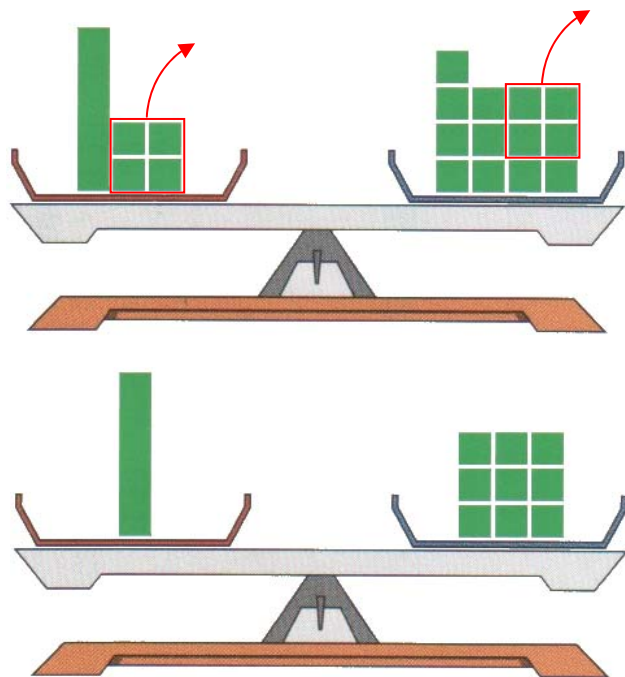
R.H.S.

$$\begin{aligned} & -13\left[\frac{-2(-28)}{29} - \frac{3}{1}\right] - \left[\frac{15(-28)}{29} + \frac{4}{1}\right] \\ &= -13\left(\frac{56}{29} - \frac{87}{29}\right) - \left(\frac{-420}{29} + \frac{116}{29}\right) \\ &= -13\left(\frac{-31}{29}\right) - \left(\frac{-304}{29}\right) \\ &= \frac{403}{29} + \frac{304}{29} \\ &= \frac{707}{29} \end{aligned}$$

$$\begin{aligned} & -3 - \left[\frac{4 - \frac{3(-28)}{29}}{1}\right] - 7\left[\frac{3(-28)}{29} - \frac{2}{1}\right] \\ &= -3 - \left(\frac{116}{29} + \frac{84}{29}\right) - 7\left(\frac{-84}{29} - \frac{58}{29}\right) \\ &= -\frac{87}{29} - \frac{200}{29} - 7\left(\frac{-142}{29}\right) \\ &= -\frac{287}{29} - \left(\frac{-994}{29}\right) \\ &= -\frac{287}{29} + \frac{994}{29} \\ &= \frac{707}{29} \end{aligned}$$

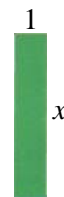
Why it Makes Sense to Perform the same Operation to Both Sides

If the same **operation is performed to both sides** of an equation, then **equality still holds**. This can be compared to a balance scale as shown in the diagrams below. Balance is maintained as long as the weight on each side is the same. If weight is removed from or added to one side, then exactly the same weight must be removed from or added to the other side. Otherwise, one side will be heavier than the other and the balance will be lost.



This picture represents the equation $x + 4 = 13$. If a total of 4 units is removed from each side, the balance is maintained and the unknown is isolated.

Now that the unknown has been isolated, it's clear that the solution is $x = 9$.



This rectangle represents x because its area is equal to x .



This square represents 1 because its area is equal to 1.

Homework

pp. 200 – 202, #5, 6, 7, 8, 9, 13, 14, 15

$$\frac{1}{4}(2y-7) + \frac{y-5}{6} = -3 - (5y-8)$$

← LCD is 12

- ① Multiply both sides (each term) by LCD to eliminate fractions
- ② Simplify each side
- ③ Get rid of variable on one side (if variable appears on both sides)
- ④ Undo the operations in order opposite of BEDMAS

$$\textcircled{1} \therefore \frac{12}{1} \left(\frac{1}{4} \right) (2y-7) + \frac{12}{1} \left(\frac{y-5}{6} \right) = 12(-3) - 12(5y-8)$$

$$\therefore \frac{12}{4}(2y-7) + \frac{12}{6} \left(\frac{y-5}{1} \right) = -36 - 60y + 96$$

$$\textcircled{2} \therefore 3(2y-7) + 2(y-5) = 60 - 60y$$

$$\therefore 6y - 21 + 2y - 10 = 60 - 60y$$

$$\therefore 8y - 31 = 60 - 60y$$

$$\textcircled{3} \therefore 8y - 31 + 60y = 60 - 60y + 60y$$

$$\therefore 68y - 31 = 60$$

$$\therefore 68y - 31 + 31 = 60 + 31$$

$$\therefore 68y = 91$$

$$\therefore \frac{68y}{68} = \frac{91}{68}$$

$$\therefore y = \frac{91}{68}$$

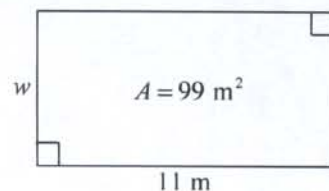
MANIPULATING (REARRANGING) EQUATIONS

Example 1

A rectangle has an area of 99 m^2 and a length of 11 m . What is its width?

Solution

1. **Given:** $A = 99$, $l = 11$ **Required to Find (RTF):** $w = ?$



2. Use the techniques that you have learned to **solve** for w **in terms of** A and l .

3. Substitute the given information into the rearranged equation:

$$\begin{aligned}
 A &= lw \\
 \therefore \frac{A}{l} &= \frac{lw}{l} \\
 \therefore \frac{A}{l} &= w \\
 \therefore w &= \frac{A}{l}
 \end{aligned}$$

• Solve for w (i.e. "isolate" it, get it "by itself") in terms of A and l .
 • Since w is multiplied by l , we can isolate w by performing the **opposite** operation (i.e. by dividing both sides by l).

$$\begin{aligned}
 w &= \frac{A}{l} \\
 &= \frac{99}{11} \\
 &= 9
 \end{aligned}$$

The width of the rectangle is 9 m .

Example 2

The United States still uses the Fahrenheit temperature scale for weather reports and many other everyday purposes. The **relationship** between the Celsius and Fahrenheit temperature scales is given by the following equation:

$$F = \frac{9}{5}C + 32$$

This type of equation describes a **relationship** between the **unknowns**.

Where F represents the temperature in degrees Fahrenheit and C represents the temperature in degrees Celsius.

(a) Use the given equation to convert -40°C to $^\circ\text{F}$. Is there anything strange about your result? Explain.

$$\begin{aligned}
 C &= -40, F = ? \\
 F &= \frac{9}{5}C + 32 \\
 &= \frac{9}{5}(-40) + 32 \\
 &= \frac{-360}{5} + 32 \\
 &= -72 + 32 \\
 &= -40 \\
 \therefore -40^\circ\text{C} &= -40^\circ\text{F} \\
 \text{The two scales "cross"} & \text{ at } -40^\circ
 \end{aligned}$$

(b) **Solve** for C in terms of F . (Rearrange the equation to get C "by itself.")

$$\begin{aligned}
 F &= \frac{9}{5}C + 32 \\
 \therefore F - 32 &= \frac{9}{5}C + 32 - 32 \\
 \therefore F - 32 &= \frac{9}{5}C \\
 \therefore 5(F - 32) &= 5\left(\frac{9}{5}C\right) \\
 \therefore 5F - 160 &= 9C \\
 \therefore \frac{5F - 160}{9} &= \frac{9C}{9} \\
 \therefore \frac{5F - 160}{9} &= C \\
 \therefore C &= \frac{5F - 160}{9}
 \end{aligned}$$

This one can also be done by multiplying both sides by 5 in the first step!

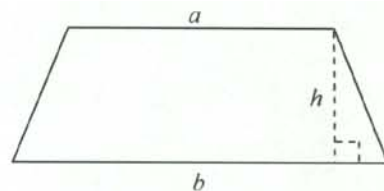
(c) While travelling in the U.S., you read a weather report in an American newspaper. According to the report, the forecast high temperature for the day is 70°F . How would you dress for a high of 70°F ?

$$\begin{aligned}
 F &= 70, C = ? \\
 \therefore C &= \frac{5(70) - 160}{9} \\
 &= \frac{350 - 160}{9} \\
 &= \frac{190}{9} \\
 &\approx 21.1
 \end{aligned}$$

$\therefore 70^\circ\text{F} \approx 21.1^\circ\text{C}$
 \therefore light clothing should be sufficient

Example 3

The area of a trapezoid is found by using the equation $A = \frac{h(a+b)}{2}$.



(a) Solve for h in terms of a , b and A .

$$A = \frac{h(a+b)}{2}$$

$$\therefore 2A = \frac{2}{1} \left(\frac{h(a+b)}{2} \right)$$

$$\therefore 2A = h(a+b)$$

$$\therefore \frac{2A}{a+b} = \frac{h(a+b)}{a+b}$$

$$\therefore \frac{2A}{a+b} = h$$

$$\therefore h = \frac{2A}{a+b}$$

(b) Solve for a in terms of h , b and A .

$$A = \frac{h(a+b)}{2}$$

$$\therefore 2A = \frac{2}{1} \left(\frac{h(a+b)}{2} \right)$$

$$\therefore 2A = h(a+b)$$

$$\therefore 2A = ha + hb$$

$$\therefore 2A - hb = ha + hb - hb$$

$$\therefore 2A - hb = ha$$

$$\therefore \frac{2A - hb}{b} = \frac{ha}{h}$$

$$\therefore a = \frac{2A - hb}{h}$$

(c) Solve for b in terms of a , h and A .

$$A = \frac{h(a+b)}{2}$$

$$\therefore 2A = \frac{2}{1} \left(\frac{h(a+b)}{2} \right)$$

$$\therefore 2A = h(a+b)$$

$$\therefore 2A = ha + hb$$

$$\therefore 2A - ha = ha + hb - ha$$

$$\therefore \frac{2A - ha}{h} = \frac{hb}{h}$$

$$\therefore \frac{2A - ha}{h} = b$$

$$\therefore b = \frac{2A - ha}{h}$$

Example 4

Einstein's famous equation, $E = mc^2$, describes the relationship between **energy** (E) and **mass** (m). In this equation, c represents the **speed of light** (approximately 300 000 km/s), a very important constant of nature.



(a) Solve for m in terms of E and c .

$$E = mc^2$$

$$\therefore \frac{E}{c^2} = \frac{mc^2}{c^2}$$

$$\therefore \frac{E}{c^2} = m$$

$$\therefore m = \frac{E}{c^2}$$

(b) Solve for c in terms of E and m .

$$E = mc^2$$

$$\therefore \frac{E}{m} = \frac{mc^2}{m}$$

$$\therefore \frac{E}{m} = c^2$$

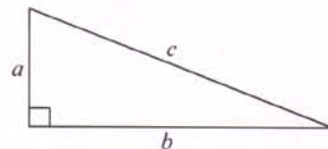
$$\therefore \sqrt{\frac{E}{m}} = \sqrt{c^2}$$

$$\therefore \sqrt{\frac{E}{m}} = c$$

$$\therefore c = \sqrt{\frac{E}{m}}$$

Example 5

The Pythagorean Theorem (also known as Pythagoras' Theorem) relates the lengths of the sides of a right triangle. According to this theorem, if c represents the length of the *hypotenuse* (the longest side of a right triangle) and a and b represent the lengths of the other two sides, then



$$c^2 = a^2 + b^2$$

(a) Solve for a^2 .

$$\begin{aligned} c^2 &= a^2 + b^2 \\ \therefore c^2 - b^2 &= a^2 + b^2 - b^2 \\ \therefore c^2 - b^2 &= a^2 \\ \therefore a^2 &= c^2 - b^2 \end{aligned}$$

(b) Solve for b^2 .

$$\begin{aligned} c^2 &= a^2 + b^2 \\ \therefore c^2 - a^2 &= a^2 + b^2 - a^2 \\ \therefore c^2 - a^2 &= b^2 \\ \therefore b^2 &= c^2 - a^2 \end{aligned}$$

(c) Solve for c .

$$\begin{aligned} c^2 &= a^2 + b^2 \\ \therefore \sqrt{c^2} &= \sqrt{a^2 + b^2} \\ \therefore c &= \sqrt{a^2 + b^2} \end{aligned}$$

Einsteinian Challenge!

Albert Einstein discovered that the universe can behave in strange and unexpected ways. For example, he discovered that the mass of an object is *not constant*! According to Einstein's Special Theory of Relativity, the mass of an object *depends* on the velocity at which it is travelling! As counterintuitive as this startling result might seem, it has been confirmed by every experiment ever performed.

The *relationship* between the *mass* and *velocity* of an object is described by the equation given below. This equation is derived from revolutionary results that Einstein published in 1905. These results, along with their consequences, later came to be known as the *Special Theory of Relativity*. (The two groundbreaking papers published in 1905 that formed the foundation of Special Relativity are entitled *On the Electrodynamics of Moving Bodies* and *Does the Inertia of a Body Depend on its Energy-Content?*)

The Equation	The Meaning of the Symbols	Example of Use
$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$	<ul style="list-style-type: none"> $v \rightarrow$ The <i>velocity</i> of the object This is a <i>variable</i> quantity. $c \rightarrow$ The <i>speed of light</i> This is a <i>constant</i> quantity. $m_0 \rightarrow$ The <i>rest mass</i> (mass when $v = 0$) This is a <i>constant</i> quantity. $m \rightarrow$ The <i>mass</i> (mass when $v > 0$) This is a <i>variable</i> quantity. 	<p>Calculate the mass of a 100.0 kg object moving at three-quarters the speed of light.</p> <p>Solution</p> <p>$m_0 = 100 \text{ kg}$, $c \doteq 299792 \text{ km/s}$, $m = ?$</p> <p>$v = \frac{3}{4}c \doteq \frac{3}{4}(299792) = 224844 \text{ km/s}$</p> <p>$\therefore m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \doteq \frac{100.0}{\sqrt{1 - \frac{224844^2}{299792^2}}} \doteq 151.2$</p>

The Challenge

Solve for v in the equation given above (i.e. Einstein's equation that relates mass to velocity).

\$5.00 bonus!!

Summary

1. An equation that contains *two or more variables* and that *is not an identity* is often called a *formula*. Such equations describe how the values of two or more variables are *related* to one another.
2. Such equations can be *rearranged* or *manipulated* by performing the *same operation to both sides*.
3. The *purpose* of *rearranging* is to *solve* for one variable *in terms of* all the others.

Homework

Write down your homework assignment in this space.