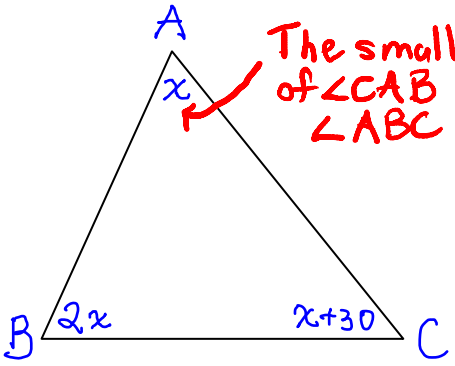
Solutions: TIPS Problems Involving Equations

1. In ΔABC, the measure of $\angle ABC$ is double the measure of $\angle CAB$. The measure of $\angle BCA$ is 30° greater than the measure of the smaller of the other two angles. Find the measure of each angle.



The smaller of $\angle CAB$ and $\angle ABC$

Let x represent the measure of $\angle CAB$.
Then, $2x$ represents the measure of $\angle ABC$
and $x+30$ represents the measure of $\angle BCA$.

(The sum of the interior angles of a triangle) is 180°

$$x + 2x + x + 30 = 180$$
$$\therefore 4x + 30 = 180$$
$$\therefore 4x + 30 - 30 = 180 - 30$$
$$\therefore 4x = 150$$
$$\therefore \frac{4x}{4} = \frac{150}{4}$$
$$\therefore x = 37.5$$

$\angle CAB = 37.5^\circ$
$\angle ABC = 2(37.5^\circ) = 75^\circ$
$\angle BCA = 37.5^\circ + 30 = 67.5^\circ$

2. Naquan is saving nickels and dimes in a jar. The jar contains 10 more nickels than dimes. Altogether, the value of the coins is $16.25. How many nickels and dimes are in the jar?

Coin	Value of One Coin	Number of Coins	Value of Coins
Dime	$0.10	d	$0.10d$
Nickel	$0.05	$d+10$	$0.05(d+10)$
Total	N/A	$d+d+10$	$16.25

(Value of dimes) + (Value of nickels) is 16.25

$$0.10d + 0.05(d+10) = 16.25$$
$$\therefore 0.10d + 0.05d + 0.5 = 16.25$$
$$\therefore 0.15d + 0.5 - 0.5 = 16.25 - 0.5$$
$$\therefore \frac{0.15d}{0.15} = \frac{15.75}{0.15}$$
$$\therefore d = 105$$
$$\therefore d + 10 = 115$$

Nickel = $5^¢$ = $0.05

Dime = $10^¢$ = $0.10

Conclusion
There are 105 dimes and 115 nickels in the jar.

Check
$105(0.10) + 115(0.05) = 10.5 + 5.75$
$= 16.25$ ✓

3. Solution A is 50% hydrochloric acid by volume, while solution B is 75% hydrochloric acid by volume. How many litres of each solution should be used to make 100 litres of a solution which is 60% hydrochloric acid by volume?

"a" litres of solution A
100 litres altogether
∴ 100 − a litres of solution B

Solution (of Problem)

Solution	*% of acid*	*Volume of the Solution (L)*	*Volume of Acid in the Solution (L)*
Solution A	50% = 0.5	a	$0.5a$
Solution B	75% = 0.75	$100 - a$	$0.75(100-a)$
Mixture	60% = 0.6	100	$0.6(100) = 60$

HCl = Hydrochloric Acid

(Volume of HCl in Solution A) + (Volume of HCl in Solution B) = (Total Volume of HCl in the mixture)

$$0.5a + 0.75(100-a) = 60$$

$$\therefore 0.5a + 75 - 0.75a = 60$$

$$\therefore -0.25a + 75 - 75 = 60 - 75$$

$$\therefore \frac{-0.25a}{-0.25} = \frac{-15}{-0.25}$$

$$\therefore a = 60$$

$$\therefore 100 - a = 100 - 60 = 40$$

The mixture should contain 60 L of solution A and 40 L of solution B.