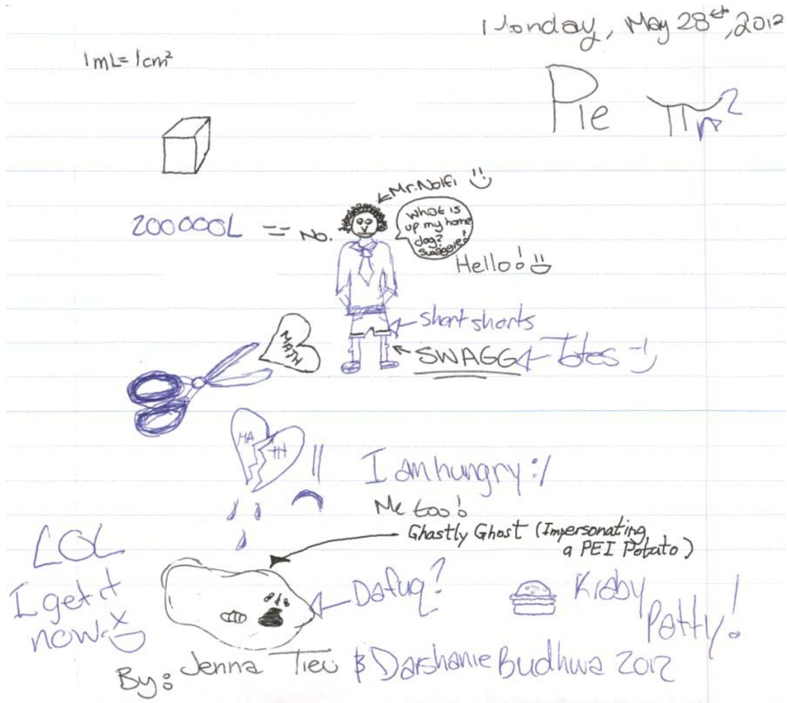
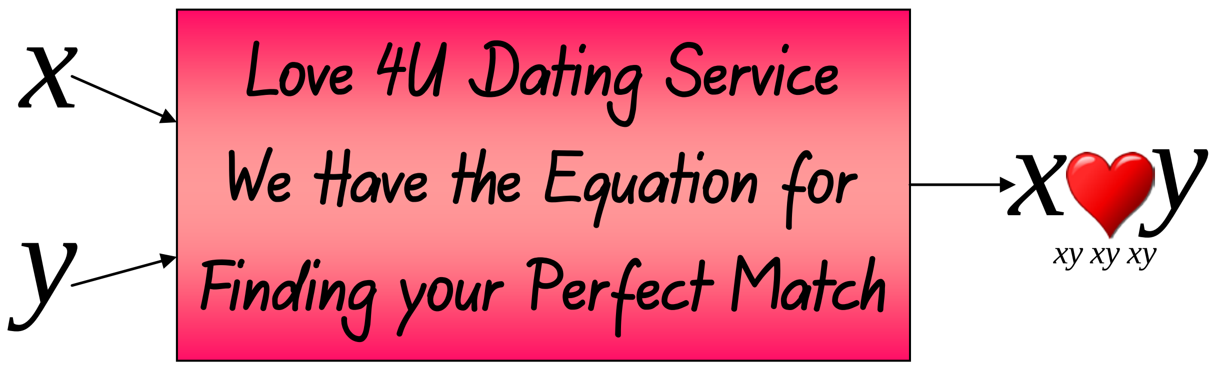
Unit 0 – Measurement and Geometry – Concepts and relationships


Monday, May 28th, 2012
1mL = 1cm²
Pie πr²
Mr. Nolfi
What is up my home dog? Swagger!
200000L == No.
Hello!
Short shorts
MATH
SWAGG Totes
I am hungry :/
Me too!
Ghastly Ghost (Impersonating a PEI Potato)
LOL
I get it now
Dafuq?
Kraby Patty!
By: Jenna Tieu & Darshanie Budhwa 2012

Introduction – Math is like a Dating Service…

Math is Like a Dating Service.

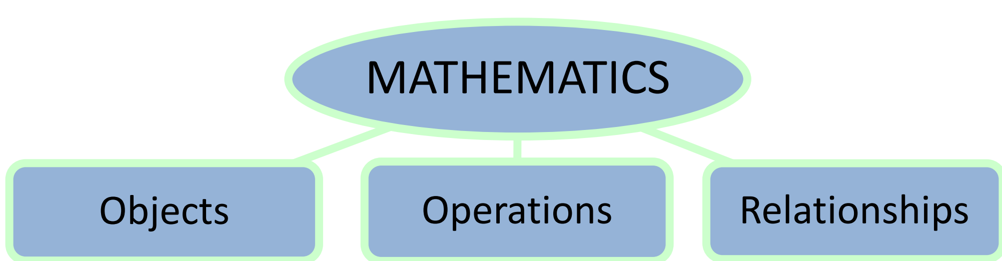
It's all about Relationships!



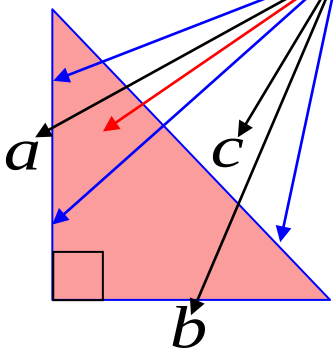
A Framework for understanding Mathematics

As shown below, mathematics can be reduced to ***three basic concepts***:

1. Mathematical Objects (e.g. ***numbers*** are mathematical objects)
2. Mathematical Operations (e.g. $+$, $-$, $\times$, $\div$, $\sqrt{\ }$, etc)
3. Mathematical Relationships (e.g. $c^2 = a^2 + b^2$)



The Pythagorean Theorem – Probably the Most Famous Mathematical Relationship



The Mathematical Objects

- Right Triangle
  Variables (a, b and c)
  Line Segments
  Numbers
- The ***variables*** represent the ***lengths*** of the sides of the right triangle. Therefore, the variables a, b and c represent ***numbers***.

The Mathematical Operations

- **Addition** (+)
- **Squaring** (raise to the exponent 2)

$$c^2 = a^2 + b^2$$

The Mathematical Relationship

- The ***equation*** $c^2 = a^2 + b^2$ expresses how the quantities represented by a, b and c are related to one another by means of mathematical operations.
- This relationship holds for all right triangles, no matter how large or small.

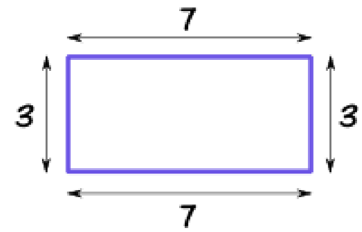
Exercise

Identify the mathematical objects, operations and relationships for the surface area of a cylinder.

Hint: $A = 2\pi r^2 + 2\pi rh$

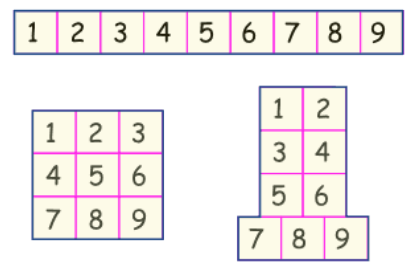
Understanding the Concepts of Perimeter, Area and Volume

Perimeter



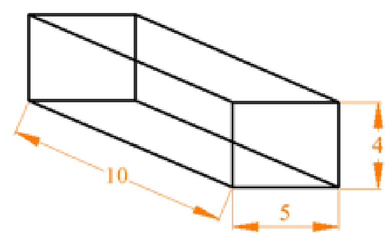
- The ***distance*** around a two-dimensional shape.
- **Example:** the perimeter of this rectangle is 3+7+3+7 = 20
- The perimeter of a circle is called the ***circumference***.
- Perimeter is measured in ***linear units*** such as mm, cm, m, km.

Area



- The "size" or "amount of space" inside the boundary of a two-dimensional surface, including curved surfaces.
- In the case of the surface of a three-dimensional object, the area is usually called ***surface area***.
- **Example:** If each small square at the right has an area of 1 cm^2, the larger shapes all have an area of 9 cm^2.
- Area is measured in ***square units*** such as mm^2, cm^2, m^2, km^2.

Volume



- The "amount of space" contained within the interior of a three-dimensional object. (The ***capacity*** of a three-dimensional object.)
- **Example:** The volume of the "box" at the right is 4×5×10 = 200 m^3. This means, for instance, that 200 m^3 of water could be poured into the box.
- Volume is measured in ***cubic units*** such as mm^3, cm^3, m^3, km^3, mL, L.
  Note: 1 mL = 1 cm^3

Questions

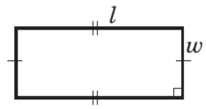
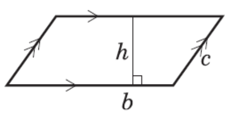
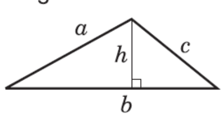
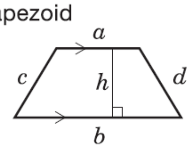
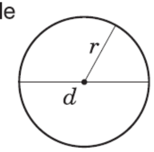
1. You have been hired to renovate an old house. For each of the following jobs, state whether you would measure perimeter, area or volume and explain why.

Job	***Perimeter, Area or Volume?***	***Why?***
Replace the baseboards in a room. 		
Paint the walls.		
Pour a concrete foundation. 		

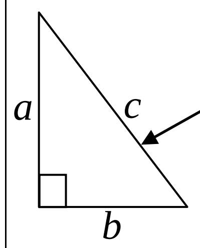
2. Convert 200 m^3 to litres. (**Hint:** Draw a picture of 1 m^3.)

MEASUREMENT RELATIONSHIPS

Perimeter and Area Equations

Geometric Figure	Perimeter	Area
Rectangle 	$P = l + l + w + w$ or $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ or $P = 2(b + c)$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a+b)h}{2}$ or $A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

Pythagorean Theorem



The ***hypotenuse*** is the ***longest side*** of a right triangle. It is always found ***opposite*** the right angle.

In ***any*** right triangle, **the square of the hypotenuse** is equal to **the sum of the squares of the other two sides**. That is,

$$c^2 = a^2 + b^2$$

By using your knowledge of rearranging equations, you can rewrite this equation as follows:

$$b^2 = c^2 - a^2$$

and

$$a^2 = c^2 - b^2$$

The Meaning of π

In ***any*** circle, the ***ratio*** of the ***circumference*** to the ***diameter*** is equal to a ***constant*** value that we call π. That is,

$$C : d = \pi .$$

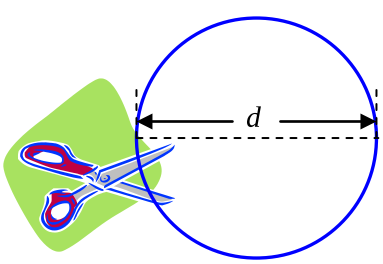
Alternatively, this may be written as

$$\frac{C}{d} = \pi$$

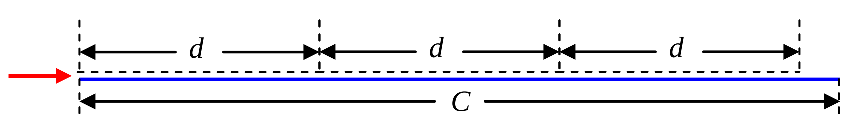
or, by multiplying both sides by d, in the more familiar form

$$C = \pi d .$$

If we recall that $d = 2r$, then we finally arrive at the most common form of this ***relationship***: $C = 2\pi r$.



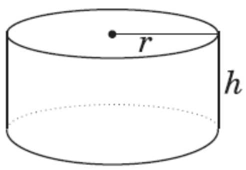
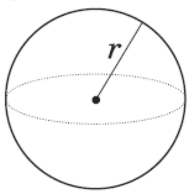
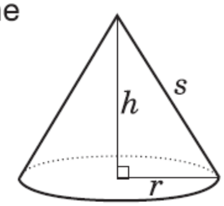
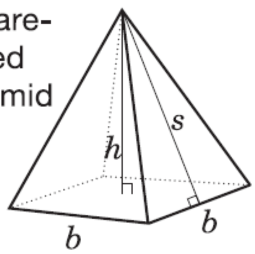
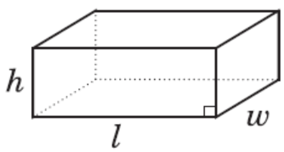
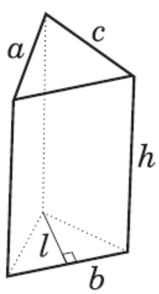
Snip the circle and stretch it out into a straight line.



As you can see, the length of the circumference is slightly more than three diameters. The exact length of C, of course, is π diameters or πd.

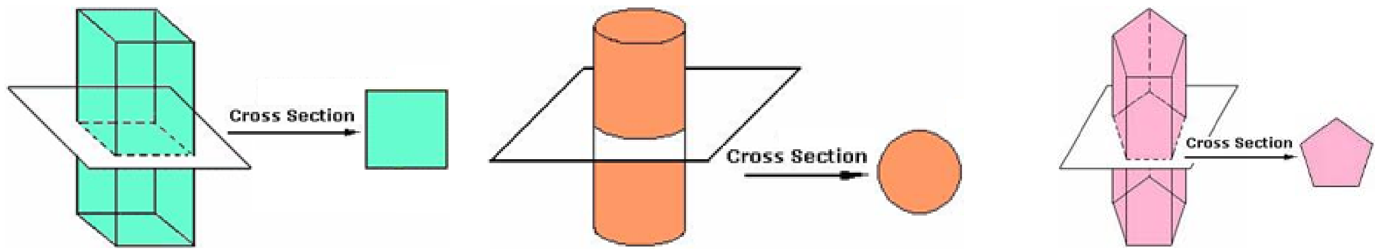
Volume and Surface Area Equations

If you need additional help, Google “area volume solids.”

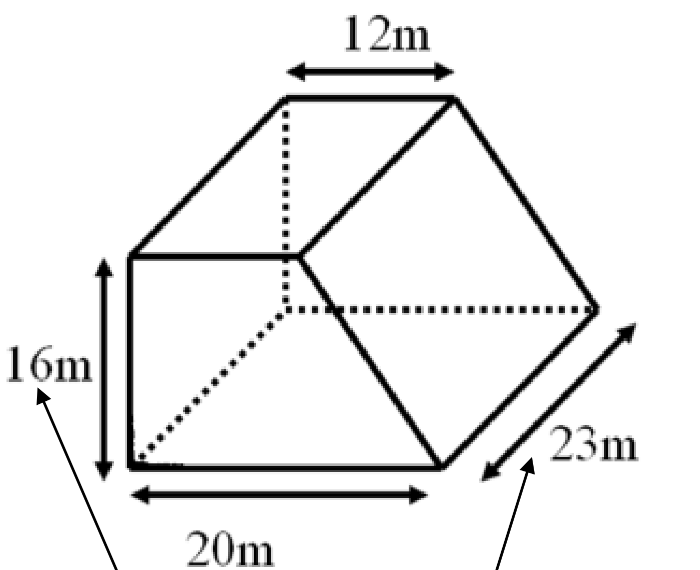
Geometric Figure	Surface Area	Volume
Cylinder 	$A_{\text{base}} = \pi r^2$ $A_{\text{lateral surface}} = 2\pi rh$ $A_{\text{total}} = 2A_{\text{base}} + A_{\text{lateral surface}}$ $= 2\pi r^2 + 2\pi rh$	$\boxed{V = (A_{\text{base}})(\text{height})}$ This is true for all ***prisms*** and ***cylinders***. $V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$ or $V = \frac{4\pi r^3}{3}$ This is true for all ***pyramids*** and ***cones***.
Cone 	$A_{\text{lateral surface}} = \pi rs$ $A_{\text{base}} = \pi r^2$ $A_{\text{total}} = A_{\text{lateral surface}} + A_{\text{base}}$ $= \pi rs + \pi r^2$	$\boxed{V = \dfrac{(A_{\text{base}})(\text{height})}{3}}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$
Square-based pyramid 	$A_{\text{triangle}} = \frac{1}{2}bs$ $A_{\text{base}} = b^2$ $A_{\text{total}} = 4A_{\text{triangle}} + A_{\text{base}}$ $= 2bs + b^2$	$\boxed{V = \dfrac{(A_{\text{base}})(\text{height})}{3}}$ $V = \frac{1}{3}b^2 h$ or $V = \frac{b^2 h}{3}$
Rectangular prism 	$A = 2(wh + lw + lh)$	$\boxed{V = (\text{area of base})(\text{height})}$ This is true for all ***prisms*** and ***cylinders***. $V = lwh$
Triangular prism 	$A_{\text{base}} = \frac{1}{2}bl$ $A_{\text{rectangles}} = ah + bh + ch$ $A_{\text{total}} = A_{\text{rectangles}} + 2A_{\text{base}}$ $= ah + bh + ch + bl$	$\boxed{V = (A_{\text{base}})(\text{height})}$ $V = \frac{1}{2}blh$ or $V = \frac{blh}{2}$

Volumes of Solids with a Uniform Cross-Section

A solid has a ***uniform cross-section*** if any cross-section ***parallel to the base*** is ***congruent*** to the base (i.e. has exactly the same shape and size as the base). Prisms and cylinders have a uniform cross-section. Pyramids and cones do not.



For all solids with a uniform cross-section, $V = (A_{base})(\text{height})$

Problem	***Solution***
Find the volume of the given trapezoidal prism.  	If we stand this solid on end, we can easily see that it is a solid with a uniform cross-section. Any cross-section parallel to the trapezoidal base is a trapezoid that is congruent to the base. Therefore, $V = (A_{base})(\text{height})$ $= \left(\frac{16(12+20)}{2}\right)(23)$ $= \left(\frac{16(32)}{2}\right)(23)$ $= 256(23)$ $= 5888$ The volume of the given trapezoidal prism is 5888 m^3.

Angle Relationships in Polygons

Concepts

Classify Polygons

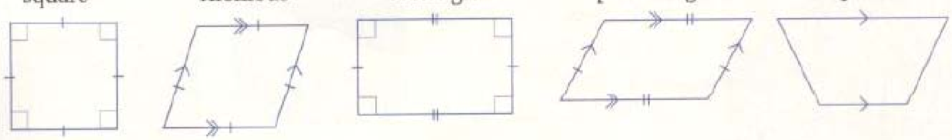
Classify Polygons

A **polygon** is a closed figure formed by three or more line segments.

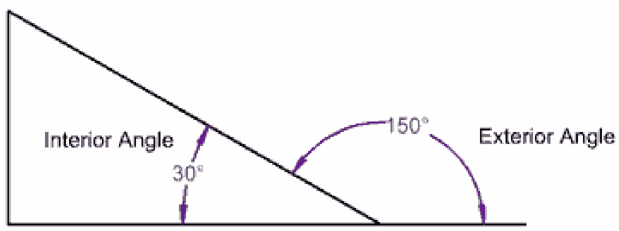
A **regular polygon** has all sides equal and all angles equal.

Some quadrilaterals have special names. A regular quadrilateral is a **square**. An irregular quadrilateral may be a **rectangle**, a **rhombus**, a **parallelogram**, or a **trapezoid**.

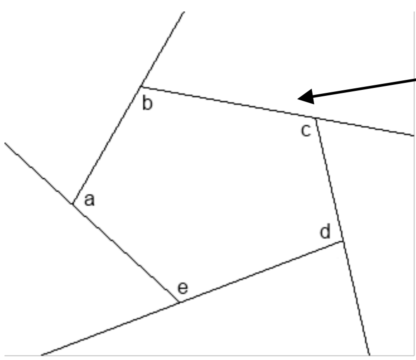
Number of Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon



Interior and Exterior Angles

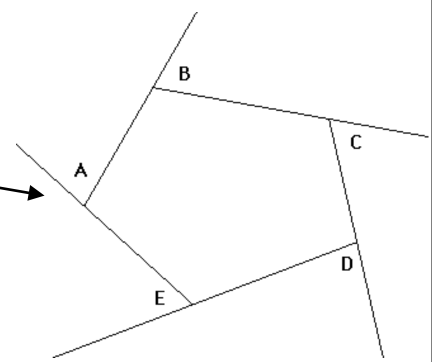


- The diagram at the left shows an ***interior angle*** of a triangle and its corresponding ***exterior angle***.
- Notice that corresponding interior and exterior angles must be ***supplementary*** (i.e. their sum must be 180°).



- The interior angles of a ***pentagon***
- The exterior angles of the same pentagon
- Notice that
  $a+A=180°$
  $b+B=180°$
  $c+C=180°$
  $d+D=180°$
  $e+E=180°$

The corresponding interior and exterior angles are ***supplementary***.



Polygon Definition

A ***polygon*** is a closed plane figure bounded by ***three or more*** line segments.

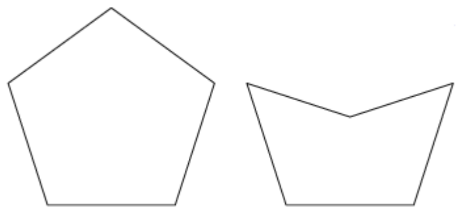
Regular Polygon Definition

A ***regular polygon*** is a polygon in which all sides have the same length (***equilateral***) and all angles have the same measure (***equiangular***).

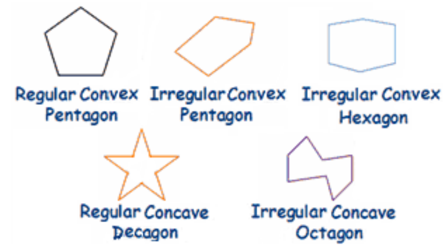
Irregular Polygon Definition

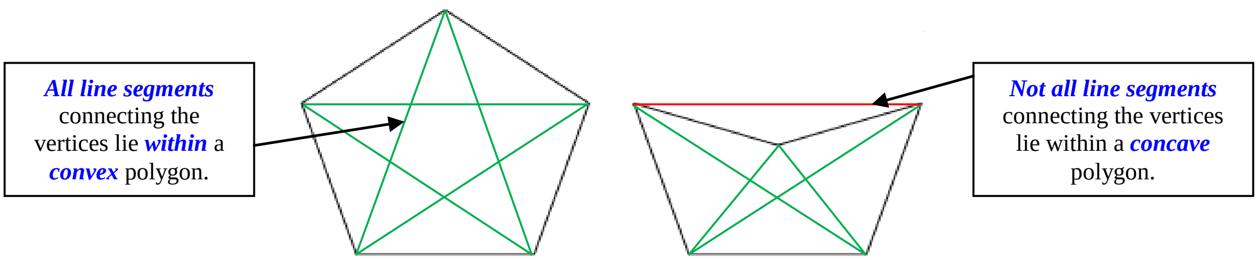
An ***irregular polygon*** is a polygon in which ***not*** all sides have the same length and ***not*** all angles have the same measure.

Convex Polygon Definition	***Concave Polygon Definition***
A ***convex polygon*** is a polygon that contains all line segments connecting any two of its vertices. In a convex polygon, the measure of ***each interior angle*** must be less than 180°.	A ***concave polygon*** is a polygon that ***does not*** contain all line segments connecting any two of its vertices. In a concave polygon, the measure of ***at least one interior angle*** is more than 180°. That is, a concave polygon must contain at least one ***reflex angle***.



convex polygon *concave polygon*

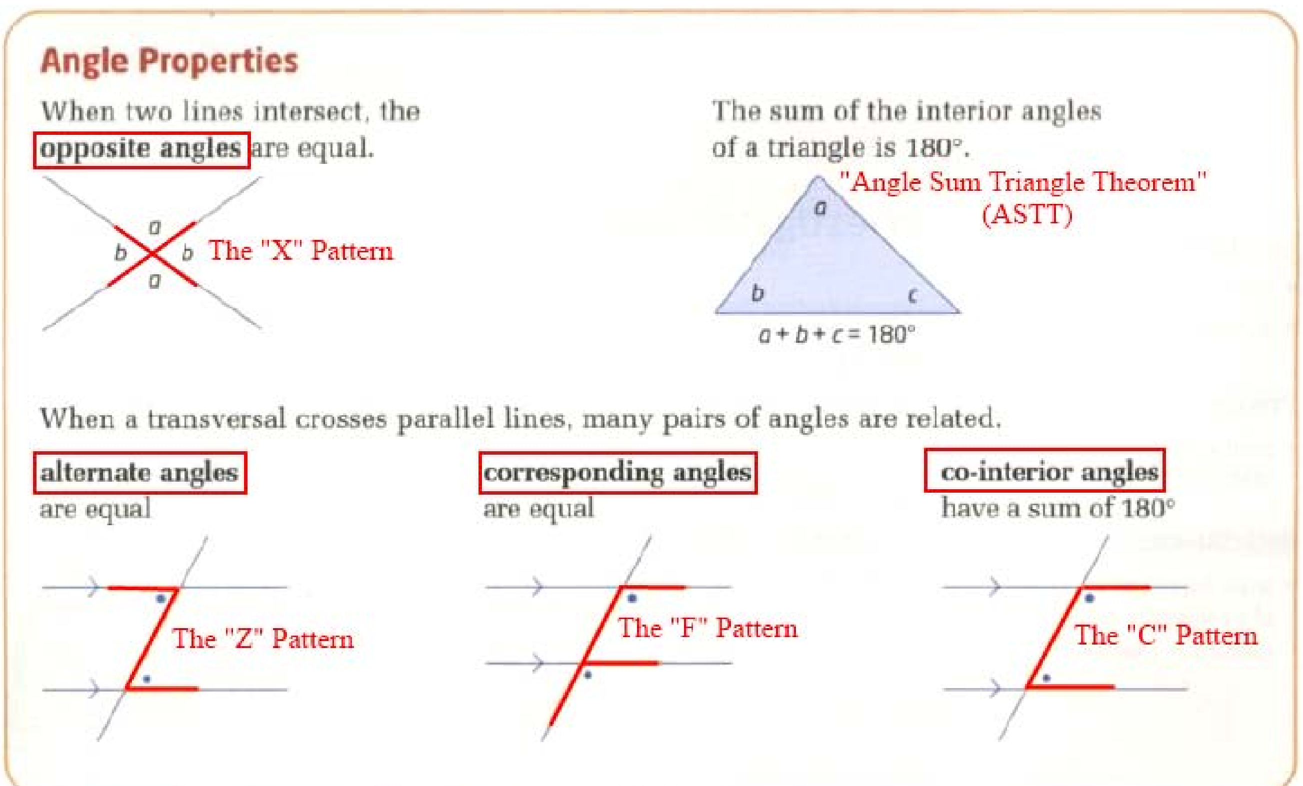




convex polygon *concave polygon*

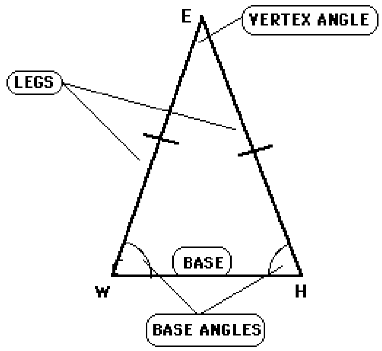
Relationships

Angle Properties – Intersecting Lines, Transversal Passing through a Pair of Parallel Lines, Triangles



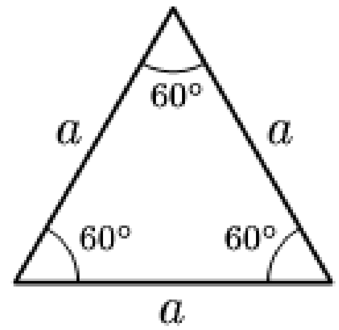
Angles in Isosceles and Equilateral Triangles

- The ***Isosceles Triangle Theorem (ITT)*** asserts that a triangle is isosceles if and only if its ***base angles are equal***.
- This can be proved using ***triangle congruence theorems*** (not covered in this course).



- Using ITT, it can be shown that an ***equilateral triangle*** is also ***equiangular*** (all three angles have the same measure).
- If x represents the measure of each angle, then

$x+x+x=180°$ (ASTT)
$\therefore 3x=180°$
$\therefore x=60°$

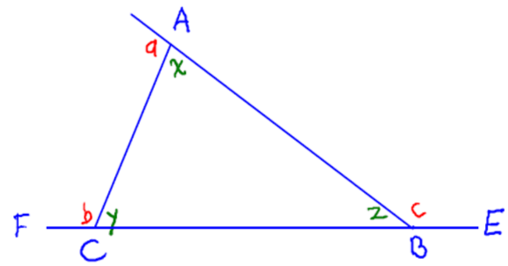


Exterior Angles of a Triangle

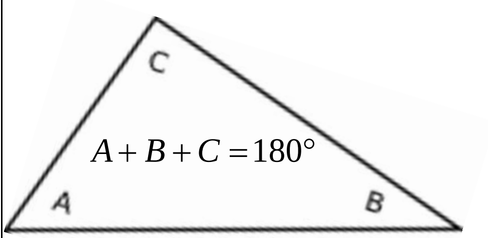
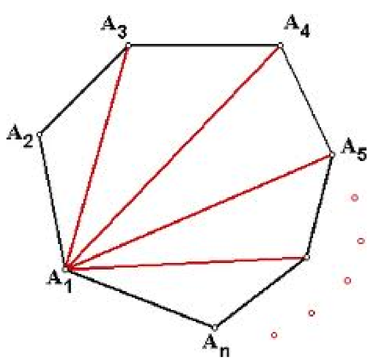
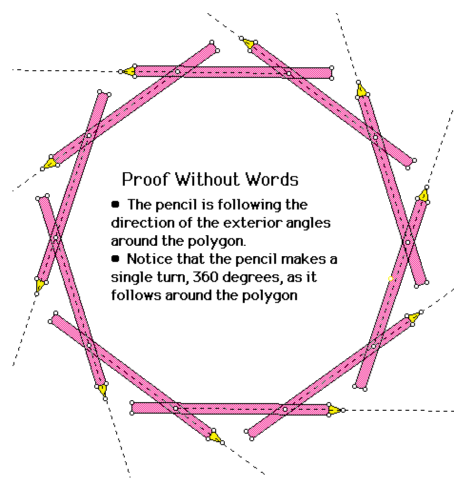
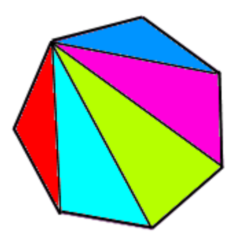
The exterior angle at each vertex of a triangle is equal to the sum of the interior angles at the other two vertices.

Using the diagram at the right, we can rewrite the above statement as follows:

$$a=y+z$$
$$b=x+z$$
$$c=x+y$$



Sum of Interior and Exterior Angles of Polygons

The Relationships	***Understanding Why…***
 The ***sum*** of the ***interior angles*** of any ***triangle*** is 180°.  The ***sum*** of the ***exterior angles*** of ***any convex polygon*** is 360°  The ***sum*** of the ***interior angles*** of ***an n-sided convex polygon*** $=(n-2)\times 180°$ = (number of triangles) × 180°	  Name: Heptagon Number of Sides: 7 Number of Triangles: 5 Sum of Interior Angles: $5(180°)=900°$