Unit 1 – Number Sense and Algebra

THE LANGUAGE OF ALGEBRA

Letters Mixing with Numbers? What on Earth is Going On?

After many years of securely navigating the concrete world of the digits 0 through 9, the dreaded first algebra lesson inevitably arrives. Suddenly, a dizzying array of letters and other symbols swoop upon math students, often leaving them feeling as if a thick, obscuring fog had instantaneously materialized in their mathematical world. There is no need to despair, however. With a consistent focus on ***meaning***, a positive ***attitude*** and a lot of ***effort***, the fog will eventually lift and clarity will be restored.

Definition of Algebra: the part of mathematics in which letters and other symbols are used to represent numbers and other quantities in expressions and equations.

Examples

Algebra is a Language? Quantitative versus Qualitative Description

Quantitative: relating to, measuring, or measured by the ***quantity*** of something rather than its quality.

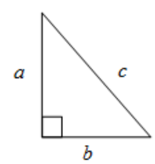
Qualitative: relating to, measuring, or measured by the ***quality*** of something rather than its quantity.

Because it is not widely known that mathematical symbols can be used to express ideas, students are often surprised to learn that mathematics involves language. Admittedly, the language of math fails miserably (at least for the foreseeable future) in describing the beauty of a work of art, the feelings evoked by a haunting passage of music or the joy of being reunited with a loved one. Clearly, natural languages like English are far better suited to descriptions of a ***qualitative*** nature. When considering descriptions of a ***quantitative*** nature, however, math is decidedly the victor. The examples given below point out the advantages of using the language of mathematics in quantitative investigations.

Example 1 – The Pythagorean Theorem

The Pythagorean Theorem describes a relationship that exists among the sides of a right triangle. The table below shows how this theorem can be described using both English and the language of algebra.

English	***Algebra***
In ***any*** right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	In ***any*** right triangle, $c^2 = a^2 + b^2$



As outlined below, a number of difficulties arise when English is used to describe mathematical relationships.

1. The wording tends to be long and cumbersome, which can easily cause confusion. Even a relationship as simple as the Pythagorean Theorem is quite difficult to describe in English.
2. In the absence of a translation into other languages, the description is accessible only to those who have a reading knowledge of English.
3. It is extremely difficult to manipulate relationships expressed in English or any other natural language.

By using an algebraic approach, however, these difficulties can be overcome quite easily.

1. The use of algebraic symbols results in very concise descriptions of relationships.
2. Algebraic descriptions are accessible to all people who have a rudimentary understanding of mathematics. Ethnic background is irrelevant.
3. Relationships can be manipulated very easily as shown in the following example:

$$c^2 = a^2 + b^2$$
$$\therefore b^2 = c^2 - a^2$$
$$\therefore a^2 = c^2 - b^2$$

Example 2 – Using Equations to Solve Problems

There are one hundred and forty coins in a collection of dimes and nickels. If the total value of the coins is \$10.90, how many dimes and how many nickels are there?

Solution

The main difficulty encountered in solving this type of problem is the ***translation*** from English into the language of algebra. Once this is done, the relationship between the number of coins and the total value of the coins is expressed in a form that is easy to manipulate. The following approach is usually not found in textbooks but it clearly shows how the equation that is obtained is nothing more than a ***mathematical restatement of the English description of the problem***.

The total value of the coins **is** \$10.90.

The total value of the coins = \$10.90

The value of all the nickels + the value of all the dimes = \$10.90

0.05×(the number of nickels) + 0.10×(the number of dimes) = \$10.90

Now we are ready to apply the language of algebra. If n represents the number of nickels, then the number of dimes must be equal to $140 - n$ (since there are 140 coins altogether). Finally, it is possible to write the following mathematical representation of the original English description:

$$0.05n + 0.10(140 - n) = \$10.90$$

By solving this equation, the desired result is obtained.

Vocabulary of Algebra

Important Definitions

Constant

Any ***fixed*** value or any expression that evaluates to a fixed value.

e.g. 4, 62, −4573, π

$$\begin{aligned} &-3+5(-3)^2-5(3-4(5)) \\ &=-3+5(9)-5(3-20) \\ &=-3+45-5(-17) \\ &=-3+45-(-85)=127 \end{aligned}$$

Variable

A ***symbol***, usually a ***letter***, which represents an ***unknown*** or ***unspecified*** value. As the name implies, a variable is a quantity that ***can change*** or that may take on ***different values***.

e.g. x, y, a, z, θ, Δ, α, β, ☺, ☹, □

(Algebraic) Expression

Any mathematical calculation combining constants and/or variables using any valid mathematical operations.

e.g. $-3x^2y+5abc-\dfrac{2xy^3z}{ab^2}-5\sqrt{z}$

Term

Parts of an expression separated by ***addition*** and ***subtraction*** symbols. More precisely, a term is any mathematical calculation combining constants and/or variables using any operations ***except for addition and subtraction***.

e.g. In the ***expression***

$-3x^2y+5abc-\dfrac{2xy^3z}{ab^2}-5\sqrt{z}$, the

terms are $-3x^2y$, $5abc$, $-\dfrac{2xy^3z}{ab^2}$

and $-5\sqrt{z}$.

(Numeric) Coefficient

The ***constant*** part of a term.

e.g. In the term $-3x^2y$, the ***numeric coefficient*** (or just coefficient) is -3.

Literal Coefficient

The ***variable*** part of a term.

e.g. In the term $-3x^2y$, the ***literal coefficient*** is x^2y.

Like and Unlike Terms

Two terms with exactly the same literal coefficients are called ***like terms***.

e.g. $-3x^2y$ and $16x^2y$ are ***like terms***.

On the other hand, $-3x^2y$ and $16xy$ are ***unlike terms***.

Evaluate an Expression

To calculate or compute with the objective of finding the "final answer."

e.g. The expression $12+3\sqrt{25}$ ***evaluates*** to 27.

Simplify an Expression

Use the rules of algebra and arithmetic to write an expression in the simplest possible form.

e.g. The expression $2a+5a$ ***simplifies*** to $7a$. (Two apples plus five apples is seven apples.) On the other hand, $2a+5b$ ***cannot be simplified*** because $2a$ and $5b$ are ***unlike terms***. (Two apples plus five bananas ***is not equal*** to "seven apple-bananas.")

Polynomial

A ***polynomial expression*** is an algebraic expression in which each term consists of constants and/or variables combined using only multiplication (including powers).

e.g. 1, −3 $-3x^2y$, $2ab-6a^2bc$,

x^2+2x+1, x^3+3x^2+3x+1

Note that although the prefix "poly" means "many" or "multiple," the term polynomial can be used to describe such an expression with ***any number of terms***.

Monomial

A polynomial with ***exactly one term***.

e.g. Three examples of monomials: 1, -3, $-3x^2y$

Binomial

A polynomial with ***exactly two terms***.

e.g. $2ab-6a^2bc$

Trinomial

A polynomial with ***exactly three terms***.

e.g. x^2+2x+1

Degree of a Polynomial

In any term of a polynomial, the ***degree of the term*** is found by adding the exponents of all the variables.

The ***degree of the polynomial*** is the degree of the ***highest-degree term***.

e.g. $-4x^3y-3x^2y^2+3xy^4+1$

Term	*Degree*
$-4x^3y$	3 + 1 = 4
$-3x^2y^2$	2 + 2 = 4
$3xy^4$	1 + 4 = **5**
1	0

Therefore, the degree of the polynomial is 5.

Equivalent Expressions

Two expressions are ***equivalent*** if they can be simplified to exactly the same expression. For example, $2a+5a$ and $3a+4a$ are equivalent because both expressions simplify to $7a$. In addition, equivalent expressions must agree for all possible values of the variable(s).

Expressions and Equations – Mathematical Phrases and Sentences

- ***equation*** → L.H.S. = R.H.S. → a complete mathematical "***sentence***"
  e.g. "The sum of two consecutive numbers is 31." → $x + x + 1 = 31$
- ***expression*** → ***not*** a complete mathematical "sentence" → more like a ***phrase***
  e.g. "Ten more than a number" → $x + 10$

Solving the so-called "word problems" that you are given in school is usually just a matter of ***translating English sentences into mathematical equations***.

Mathematical Words

Symbol	*English Equivalent*
+	**sum**, plus, added to, more than, increased by, gain of, total of, combined with
–	**difference**, minus, subtracted from, less than, fewer than, decreased by, loss of
×	**product**, times, multiplied by, of, factor of, double (×2), twice (×2), triple (×3)
÷	**quotient**, divided by, half of (÷2), one-third of (÷3), per, ratio of
=	**is**, are, was, were, will be, gives, yields

Translating from English into Algebraic Expressions and Equations

Complete the following table.

English	*Algebraic Expression*	*English*	*Algebraic Equation*
Six more than a number	$n+6$	Six more than a number is 5.	$n+6=5$
A number decreased by 7	$x-7$	A number decreased by 7 is –9.	$x-7=-9$
The product of a number and –3	$-3y$	The product of a number and –3 is 4.	$-3y = 4$
Half of a number	$\frac{z}{2}$	Half of a number is 16.	$\frac{z}{2}=16$
Triple a number decreased by 5	$3x-5$	Triple a number decreased by 5 is 8.	$3x-5=8$
Double a number plus 5		Double a number plus 5 gives 13.	
One-third of a number minus 2		One-third of a number minus 2 yields 16.	
	$\frac{x}{4}-5$		$\frac{x}{4}-5=-1$
Sixty-five decreased by a number		Sixty-five decreased by a number gives 7.	
A number divided by 7		A number divided by 7 is –10.	
Quadruple a number subtracted ***from*** 6		Quadruple a number subtracted ***from*** 6 is 2.	
	$2-4t$		$2-4t=3$
	$\frac{2}{x-4}$		$\frac{2}{x-4}=-9$
The quotient of 6, and a number subtracted from 3		The quotient of 6 and a number subtracted from 3 is 2.	
The product of 2, and a number increased by 7		The product of 2, and a number increased by 7 is 13.	
The difference of triple a number, and a number increased by 3		The difference of triple a number, and a number increased by 3 yields 21.	

Practice: Communicate With Algebra

1. For each term, identify the coefficient (numeric coefficient) and the variable part (literal coefficient).

a) $4x$ **b)** $-5p^4$
c) $3m^2n$ **d)** g^3h^2
e) $-2y^5$ **f)** $-p^4q^5$
g) $\frac{3}{4}ab$ **h)** $0.6r^4s^2$

2. The expression $2x + 5$ is a:
A monomial **B** binomial
C trinomial **D** term

3. The expression $-12m^4n$ is a:
A monomial **B** binomial
C trinomial **D** term

4. The expression $3a^2b^2 + ab^3 + b$ is a:
A monomial **B** binomial
C trinomial **D** term

5. Classify each polynomial by type.

a) $2x + 1$
b) $3p^2 - p + 4$
c) $4b^2d^3$
d) $6 + gh^5$
e) $2 - y^5 - y^2 + 4y$
f) $x^2 - y^2 + 4$
g) $ab - b$
h) $6p^3q^3$

6. What is the degree of each term in question 5?

7. The degree of $5m^2n + mn^3 + 1$ is:
A 1 **B** 2
C 3 **D** 4

8. What is the degree of each polynomial?

a) $6a^2 + 4b^3$ **b)** $5b^4$
c) $3x^2 + x - 1$ **d)** $m^3 - m^2 + 4m$
e) $2p^4q^3$ **f)** $x^2y^2 + 4xy$
g) $a^5b - 7b^3$
h) $-m^4n^3 - m^2n + 4mn^4$

9. Which equation matches this phrase: a number increased by 6 is 8

A $6x = 8$
B $x + 8 = 6$
C $x + 6 = 8$
D $\frac{x}{6} = 8$

10. Write an equation for each phrase.
a) double a number is 14
b) a number decreased by 6 is 5
c) one third of a number is 2
d) triple a number, increased by 1 is 8

11. Maggie earns \$5 per hour when she babysits 1 child. She earns \$8 per hour when she babysits 4 children.

Let x represent the number of hours she babysits 1 child and y represent the number of hours she babysits 4 children.

Which expression represents her total earnings?

A $5x - 8y$ **B** $x + y$
C $5x + 8y$ **D** $x - y$

12. Evaluate each expression for the given values of the variables.

a) $2x - 3$ $x = 4$
b) $3y + 2$ $y = 7$
c) $r^2 - r + 1$ $r = 6$
d) $a^2 - 2b^2$ $a = 3, b = 1$
e) $p^2 + 2p - 3$ $p = 4$
f) $4x^2 - y - 2$ $x = 2, y = 1$

Answers

1.
a) coefficient: 4; variable: x
b) coefficient: –5; variable: p^4
c) coefficient: 3; variable: m^2n
d) coefficient: 1; variable: g^3h^2
e) coefficient: –2; variable: y^5
f) coefficient: –1; variable: p^4q^5
g) coefficient: $\frac{3}{4}$; variable: ab
h) coefficient: 0.6; variable: r^4s^2

2. **B:** binomial

3. **A:** monomial and **D:** term

4. **C:** trinomial

5.
a) binomial **b)** trinomial
c) monomial **d)** binomial
e) four-term polynomial **f)** trinomial
g) binomial **h)** monomial

6. **a)** 1 **b)** 2 **c)** 5 **d)** 6
e) 5 **f)** 2 **g)** 2 **h)** 6

7. **D:** 4

8. **a)** 3 **b)** 4 **c)** 2 **d)** 3
e) 7 **f)** 4 **g)** 6 **h)** 7

9. C

10. **a)** $2x = 14$ **b)** $x - 6 = 5$
c) $\frac{x}{3} = 2$ **d)** $3x + 1 = 8$

11. C

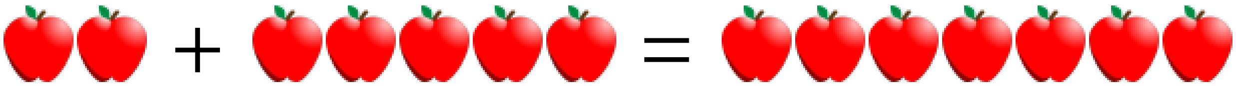
12. **a)** 5 **b)** 23 **c)** 31
d) 7 **e)** 21 **f)** 13

Simplifying Algebraic Expressions Involving Addition and Subtraction

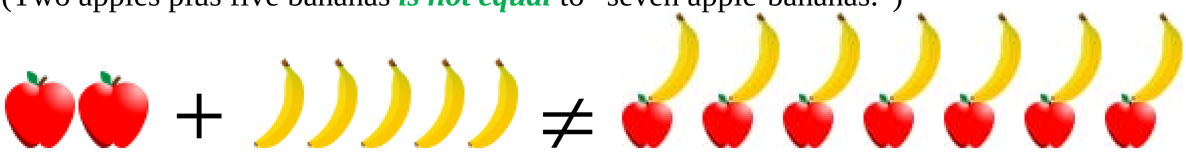
Definition of "Simplify"

Use the ***rules of algebra and arithmetic*** to write an expression in the ***simplest possible form***.

e.g. The expression $2a+5a$ ***simplifies*** to $7a$. (Two apples plus five apples ***is equal to*** seven apples.)



On the other hand, $2a+5b$ ***cannot be simplified*** because $2a$ and $5b$ are ***unlike terms***.
(Two apples plus five bananas ***is not equal*** to "seven apple-bananas.")



Important Points to Remember when Simplifying Polynomials that don't Contain Brackets

- Keep in mind and apply correctly the rules for adding and subtracting integers. The main idea underlying addition and subtraction of integers is that these operations all boil down to either a ***loss*** (move down, move left, etc) or a ***gain*** (move up, move right, etc)

  + (+) = + = ***add*** a ***positive*** value = ***gain***
  +(–) = – = ***add*** a ***negative*** value = ***loss***
  – (–) = + = ***subtract*** a ***negative*** value = ***gain***
  – (+) = – = ***subtract*** a ***positive*** value = ***loss***

- Remember to look for ***like terms***. Do not fall into the trap of attempting to simplify the sum or difference of ***unlike terms***. (2 cows + 2 cows = 4 cows   2 boys + 2 boys = 4 boys   2 cows + 2 boys ≠ 4 cowboys

Simplifying Expressions involving Two or More Terms and NO Brackets

Examples

Simplify each of the following polynomials:

<table>
<tr>
<td>1.
$-5a+3a-6b+4b$
$=-2a-2b$</td>
<td>2.
$-2x^2y-7x^2y+3xy-8xy$
$=-9x^2y-5xy$
Note that x^2y and xy are NOT like terms. The term x^2y means xxy, which is different from xy.</td>
<td>3.
$-5a-6b+3a-4b$
$=-5a+3a-6b-4b$
$=-2a-10b$
Collect Like Terms
The operations move with the terms!</td>
</tr>
<tr>
<td colspan="2">4.
$-5a+6b+3c-4d$
This polynomial cannot be simplified because there are no like terms. (In your notes or on at test, you may write "CBS" as a short form for "cannot be simplified.")</td>
<td>5.
$-15ab^2-6a^2b-13ab^2-4a^2b-10ab$</td>
</tr>
</table>

Practice: Collect Like Terms

1. Which polynomial contains a term like xy^2?

A $4xy - x^2y$ **B** $2x^2 + 3xy^2$

C $-x + y^2 - xy$ **D** $x^2 + y^2 + 4$

2. Are the terms in each pair like or unlike?

a) $5a$ and $-2a$

b) $3x^2$ and x^3

c) $2p^3$ and $-p^3$

d) $4ab$ and $\frac{2}{3}ab$

e) $-3b^4$ and $-4b^3$

f) $6a^2b$ and $3a^2b$

g) $9pq^3$ and $-p^3q$

h) $2x^2y$ and $3x^2y^2$

3. Write one like term and one unlike term for each.

a) $4p$ **b)** $-3a^2$

c) $-k^3$ **d)** $2x$

e) $-4mn^4$ **f)** $2ab$

g) $-pq^3$ **h)** $3b^2d^2$

4. Is it possible to simplify each expression? How do you know?

a) $8a + 3a$ **b)** $5m + 2n$

c) $3p + p$ **d)** $3t - 7t$

e) $4x - 3$ **f)** $-v - 4v + 2v$

g) $6c^2 - c^2 - 3c^2$ **h)** $r^2 + 3r + 7$

5. Simplify each expression.

a) $p + 2p$ **b)** $7g - 4g$

c) $2a - 8a$ **d)** $5x - 2x$

e) $6q + q$ **f)** $4y^2 + 5y^2$

g) $u + 4u - u$ **h)** $7b^3 - 2b^3 - b^3$

6. Collect like terms. Then, simplify.

a) $4b + 3 - 2b + 1$

b) $2p - 7 - p + 4$

c) $1 + 3y + 4 + y$

d) $5 - x - 1 - 2x$

e) $6a - 2b + 3b + 2a$

f) $7r + 2 + 3r - r - 1$

g) $9s - 2s + 5t - 4s$

h) $-g - 3h + 5h + 2g - h$

7. Simplify.

a) $4 + v + 5v - 10$

b) $7a - 2b - a - 3b$

c) $8k + 1 + 3k - 5k + 4 + k$

d) $2x^2 - 4x + 8x^2 + 5x$

e) $12 - 4m^2 - 8 - m^2 + 2m^2$

f) $-6y + 4y + 10 - 2y - 6 - y$

g) $5 + 3h + h - 4 + h + 6 + 2h$

h) $4p^2 + 2q^2 - p^2 + 3p^2 - 7q^2$

8. Simplify.

a) $2a + 6b - 2 + b - 4 + a$

b) $4x + 3xy + y + 5x - 2xy - 3y$

c) $m^4 - m^2 + 1 + 3 - 2m^2 + m^4$

d) $x^2 + 3xy + 2y^2 - x^2 + 2xy - y^2$

9. The length of a rectangle is 2 times the width of the rectangle. Let x represent the width of the rectangle.

a) Write an expression to represent the length of the rectangle.

b) Write a simplified expression for the perimeter of the rectangle.

c) Suppose the width is 6 cm. Find the perimeter of the rectangle.

Answers

1. **B:** $2x^2 + 3xy^2$

2. **a)** like **b)** unlike **c)** like **d)** like **e)** unlike **f)** like **g)** unlike **h)** unlike

3.
a) like: $-7p$; unlike: $4x$
b) like: $2a^2$; unlike: $-3a$
c) like: $5k^3$; unlike: $-k^2$
d) like: $-3x$; unlike: $4p$
e) like: mn^4; unlike: m^4n
f) like: $4ab$; unlike: $4a$
g) like: $2pq^3$; unlike: p^2q^3
h) like: $-b^2d^2$; unlike: $3bd$

4.
a) yes; both terms have the variable a
b) no; the terms have different variables
c) yes; both terms have the variable p
d) yes; both terms have the variable t
e) no; the terms have different variables
f) yes; all terms have the variable v
g) yes; all terms have the variable c^2
h) no; the terms do not all have the same variables

5. **a)** $3p$ **b)** $3g$ **c)** $-6a$ **d)** $3x$ **e)** $7q$ **f)** $9y^2$ **g)** $4u$ **h)** $4b^3$

6.
a) $4b - 2b + 3 + 1$; $2b + 4$
b) $2p - p - 7 + 4$; $p - 3$
c) $1 + 4 + 3y + y$; $5 + 4y$
d) $5 - 1 - x - 2x$; $4 - 3x$
e) $6a + 2a - 2b + 3b$; $8a + b$
f) $7r + 3r - r + 2 - 1$; $9r + 1$
g) $9s - 2s - 4s + 5t$; $3s + 5t$
h) $-g + 2g - 3h + 5h - h$; $g + h$

7. **a)** $6v - 6$ **b)** $6a - 5b$ **c)** $7k + 5$ **d)** $10x^2 + x$ **e)** $4 - 3m^2$ **f)** $-5y + 4$ **g)** $7 + 7h$ **h)** $6p^2 - 5q^2$

8. **a)** $3a + 7b - 6$ **b)** $9x + xy - 2y$ **c)** $2m^4 - 3m^2 + 4$ **d)** $5xy + y^2$

9. **a)** $L = 2x$ **b)** $P = 6x$ **c)** 36cm

Simplifying Expressions involving Two or More Terms AND Brackets

> ***Be Careful!***
>
> Be careful when simplifying expressions that are enclosed in brackets. The brackets can be removed without any changes ***only when a polynomial is being added***. If a polynomial enclosed in brackets is being subtracted, you must bear in mind that the ***operation of subtraction applies to the entire polynomial***, not just the first term.
>
> - Addition ***can*** be performed in any order without changing the result. This means that brackets don't matter! Therefore, to ***add a polynomial*** enclosed in brackets***, simply remove the brackets*** and proceed.
> - Subtraction ***cannot*** be performed in any order without changing the result. This means that brackets ***DO*** matter! To ***subtract a polynomial*** enclosed in brackets, remove the brackets by ***adding the opposite of the polynomial***. This is based on the following property: $x - y = x + (-y)$

Why this Works

- We already know that $x - y = x + (-y)$.
- In other words, ***subtraction*** is the same as "adding the negative of."
- "Adding the negative of" is also the same as "adding the ***opposite*** of."
- Therefore, ***subtraction*** is the same as "adding the ***opposite*** of."

Examples

1.

$$(-5a + 6b) + (3a - 4b)$$
$$= -5a + 6b + 3a - 4b$$
$$= -5a + 3a + 6b - 4b$$
$$= -2a + 2b$$

Since a polynomial is being *added*, the brackets don't matter.

They can be removed without making any changes because addition is unaffected by order.

i.e. $a+(b+c) = (a+b)+c$

2.

$$(-5a + 6b) - (3a - 4b)$$
$$= -5a + 6b + (-3a + 4b)$$
$$= -5a + 6b + (-3a) + 4b$$
$$= -5a + 6b - 3a + 4b$$
$$= -5a - 3a + 6b + 4b$$
$$= -8a + 10b$$

Since a polynomial is being *subtracted*, the brackets matter. Subtraction IS affected by order.

i.e. $a-(b-c) \neq (a-b) - c$

Add the Opposite because subtraction is the same as adding a negative.

3.

$$(-5x + 1) - (2x - 7) - (-3x + 5)$$
$$= -5x + 1 + (-2x + 7) + (3x - 5)$$
$$= -5x + 1 + (-2x) + 7 + 3x - 5$$
$$= -5x + 1 - 2x + 7 + 3x - 5$$
$$= -5x - 2x + 3x + 1 + 7 - 5$$
$$= -4x + 3$$

Two of the brackets are preceded by a negative sign.
In each case, ***add the opposite***:

$$-(2x - 7) \longrightarrow +(-2x + 7)$$

$$-(-3x + 5) \longrightarrow +(3x - 5)$$

The leftmost set of brackets is not preceded by a negative sign. The brackets can be removed without making any changes.

$$(-5x + 1) \longrightarrow -5x + 1$$

Practice: Add and Subtract Polynomials

1. Which expression represents the result of simplifying $(3x - 4) + (2x + 1)$?

A $6x - 4$ **B** $6x + 4$

C $5x + 3$ **D** $5x - 3$

2. Remove brackets and collect like terms. Then, simplify.

a) $(x + 3) + (x + 5)$
b) $(2y - 5) + (y + 9)$
c) $(5m + 1) + (2m + 2)$
d) $(3 - 4d) + (d - 1)$
e) $(3v - 2) + (6 - v)$
f) $(k + 4) + (2 - 3k) + (6k - 1)$
g) $(2p + 4) + (p - 2) + (8 - 3p)$
h) $(3 - r) + (4 + 5r) + (2r - 1)$

3. Write the opposite of each expression.

a) 3 **b)** -5
c) $2p$ **d)** $-x$
e) $m + 4$ **f)** $3b - 1$
g) $x^2 + 2x - 4$ **h)** $-6 - y^2$

4. Which expression represents the result of simplifying $(4x - 1) - (x + 1)$?

A $5x + 2$ **B** $3x - 2$

C $5x$ **D** $3x$

5. Add the opposite. Then, simplify.

a) $(4d - 2) - (d + 1)$
b) $(3x - 4) - (x + 3)$
c) $(2p + 5) - (3p + 2)$
d) $(8 - 4m) - (m - 2)$
e) $(a - 2) - (5 - 3a)$
f) $(z + 7) - (4 - z)$
g) $(p + 1) - (p - 2)$
h) $(5 - 2b) - (6 + 4b)$

6. Simplify.

a) $(6k - 4) + (2k + 4)$
b) $(n + 3) + (3n - 5)$
c) $(2a + 1) - (4a + 2)$
d) $(5 - 3m) + (2m - 1)$
e) $(b - 6) - (2 - 5b) + (b + 4)$
f) $(x + 2) - (1 - x) - (5 + x)$
g) $(g + 12) + (g - 7) - (2 - 3g)$
h) $(1 - b) + (3 + 2b) - (b - 8)$

7. Simplify.

a) $(x^2 + 2x + 1) + (2x^2 + 4)$
b) $(4a + 3b - 6) + (2a - b + 4)$
c) $(2m^2 + m + 12) + (3m^2 + 4m - 6)$
d) $(5n + mn - 3m) + (2m - 5mn + n)$

8. A rectangle has length $4x + 1$ and width $x + 2$.

a) Write a simplified expression for the perimeter of the rectangle.
b) Find the perimeter of the rectangle when $x = 5$.

9. Three artists contributed to a coffee-table book. They each chose to be paid a different way.

Artist	**Fixed Rate ($)**	**Royalty ($ per *n* books sold)**
Ayesha	1000	$2n$
Jorge	–	$5n$
Ioana	4000	–

a) Write an expression for the total earnings for each artist.
b) Write a simplified expression for the total amount paid to Ayesha, Jorge, and Ioana.

Answers

1. **D:** $5x - 3$

2. **a)** $2x + 8$ **b)** $3y + 4$ **c)** $7m + 3$ **d)** $2 - 3d$ **e)** $2v + 4$ **f)** $4k + 5$ **g)** 10 **h)** $6 + 6r$

3. **a)** -3 **b)** 5 **c)** $-2p$ **d)** x **e)** $-m - 4$ **f)** $-3b + 1$ **g)** $-x^2 - 2x + 4$ **h)** $6 + y^2$

4. **B:** $3x - 2$

5. **a)** $3d - 3$ **b)** $2x - 7$ **c)** $-p + 3$ **d)** $10 - 5m$ **e)** $4a - 7$ **f)** $2z + 3$ **g)** 3 **h)** $-1 - 6b$

6. **a)** $8k$ **b)** $4n - 2$ **c)** $-2a - 1$ **d)** $4 - m$ **e)** $7b - 4$ **f)** $x - 4$ **g)** $5g + 3$ **h)** 12

7. **a)** $3x^2 + 2x + 5$ **b)** $6a + 2b - 2$ **c)** $5m^2 + 5m + 6$ **d)** $6n - 4mn - m$

8. **a)** $P = 10x + 6$ **b)** 56

9. **a)** Ayesha: $1000 + 2n$; Jorge: $5n$; Ioana: 4000
b) Total $= 5000 + 7n$

SUMMARY OF MAIN IDEAS

Algebra as a Language

Complete the following statements:

(a) Languages like English are best suited to descriptions of a ____________________________ nature.

(b) The language of algebra is best suited to descriptions of a ____________________________ nature.

(c) Math is like a dating service because it's all about _______________________________________.

(d) The language of algebra has many advantages when it comes to describing mathematical relationships. Some of the advantages include ____

____

(e) Expressions and equations can be compared to phrases and sentences respectively.

Give an example of an expression: ___________________ Give an example of an equation: ___________________

An expression is like a phrase because ____.

An equation is like a sentence because ____.

(f) The Pythagorean Theorem is an example of an _____________ that describes the mathematical _________________ among the sides of a ___________________________.

(g) Math is much easier to understand when we keep in mind the _____________________ of the symbols, operations, expressions and equations. Also, it helps to have good control over one's mental _________________________.

Vocabulary of Algebra

Complete the following table:

Name	*Example*	*Name*	*Example*
Constant			$3x$, $3y$
Variable			$3ab^2$, $-10ab^2$
Expression			$3ab^2 - 10ab^2 = -7ab^2$
Term			$3(-3)(-4)^2 - 10(3)(4)^2 = -304$
(Numeric) Coefficient			$-4x^5 + 2x^3 - 7x^2 + x - 1$
Literal Coefficient (Variable Part)			$-4x^5 + 2x^3 - 7x^2$
Polynomial			$3ab^2 - 10ab^2$
Monomial			$-7x^2$
Binomial			$-3x^2y + 5abc - \frac{2xy^3z}{ab^2} - 5\sqrt{z}$
Trinomial			$-5\sqrt{z}$
Evaluate an Expression			$-5\sqrt{z}$
Simplify an Expression			$-5\sqrt{z}$
Like Terms			a
Unlike Terms			-34553476348.467674737

Simplifying Algebraic Expressions

1. Complete the following statements:

(a) "Evaluate an expression" means ____.

(b) "Simplify an expression" means ____.

(c) When $-3(-3)-10(-3)(5)^2$ is evaluated, the result is ________________________________.

(d) The expression $3ab^2-10ab^2$ ***can*** be simplified because ________________________________.

(e) The expression $3ab-10ab^2$ ***cannot*** be simplified because ______________________________.

(f) One way to interpret the expression $2p+5p$ is two ____________ plus five ____________. Using this interpretation, it ***makes sense*** that the simplified form is $7p$ because ____________________.

(g) One way to interpret the expression $2h+5d$ is two ____________ plus five ____________. Using this interpretation, it ***does not make sense*** that the simplified form is ~~$7hd$~~ because ______________ ____.

(h) When simplifying expressions, first ________________ should be ____________. When this is being done, it is very important that the ____________ move with the ____________.

(i) When simplifying expressions containing brackets, the brackets can be removed without making any other changes only if the bracket is preceded by a ________ sign. This is so because ____________ can be performed in any order whatsoever without changing the sum. If a bracket is preceded by a ________ sign, brackets cannot be removed without making other changes. This is so because the result of ____________ (i.e. the difference) ***is*** affected by the order in which it is performed. In this case, it is best to remove brackets by ____.

2. Simplify each of the following expressions.

a) $(7x-9)+(x-4)$ **b)** $(3y+8)+(-y-5)$

c) $(8c-6)-(c+7)$ **d)** $(k+2)-(3k-2)$

e) $(3p^2-8p+1)+(9p^2+4p-1)$

f) $(5xy^2+6x-7y)-(3xy^2-6x+7y)$

g) $(4x-3)+(x+8)-(2x-5)$

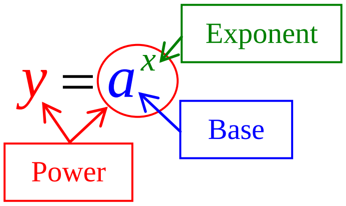
h) $(2uv^2-3v)-(v+3u)+(4uv^2-9u)$

Answers to Question 2

(a) $8x-13$
(b) $2y+3$
(c) $7c-13$
(d) $-2k+4$
(e) $12p^2-4p$
(f) $2xy^2+12x-14y$
(g) $3x+10$
(h) $6uv^2-12u-4v$

Simplifying Algebraic Expressions Involving Multiplication and Division

Powers and Laws of Exponents



Meaning of Powers

Powers are a short form for ***repeated multiplication***.

e.g. $4^6 = (4)(4)(4)(4)(4)(4) = 4096$, $\left(-\frac{2}{3}\right)^5 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = -\frac{32}{243}$

e.g. The mass of the sun is about 2000000000000000000000000000000 kg (2 nonillion kg)
It is much easier to write this as 2×10^{30} kg.

Practice: Work with Exponents

1. What is the base of each power?

a) 5^2 **b)** 2^3
c) $(-3)^4$ **d)** -3^4
e) $\left(\frac{2}{3}\right)^2$ **f)** 2.1^2

2. Write the exponent for each power in question 1.

3. Which expressions are equal to 4 × 4 × 4?

A 3^4 **B** 4^3
C 12 **D** 64

4. Which expression in question 3 is 4 × 4 × 4 written as a power?

5. Which expressions are equal to 2^4?

A 2 × 4 **B** 4 × 4
C 2 × 2 × 2 × 2 **D** 16

6. Which expression in question 5 is 2^4 written in expanded form?

7. Write each expression as a power.

a) 6 × 6 × 6 × 6 × 6 × 6 × 6
b) 9 × 9
c) 0.4 × 0.4 × 0.4
d) (–7) × (–7) × (–7) × (–7) × (–7)
e) (–1.3) × (–1.3) × (–1.3) × (–1.3)
f) $\left(\frac{2}{5}\right)\times\left(\frac{2}{5}\right)\times\left(\frac{2}{5}\right)\times\left(\frac{2}{5}\right)$

8. Write each power in expanded form, then evaluate.

a) 3^4 **b)** 5^3
c) $(-2)^2$ **d)** -3^4
e) $\left(\frac{1}{4}\right)^2$ **f)** 0.4^3

9. Evaluate.

a) 6^3 **b)** 2^7
c) -4^2 **d)** $(-2)^6$
e) 1^{12} **f)** $\left(-\frac{4}{5}\right)^2$

10. Use the correct order of operations to evaluate each expression.

a) $2^4 + 3^2$ **b)** $6^3 - 6$
c) $(2 + 5)^2$ **d)** $(2^2 + 5^2)$
e) $6\left(\frac{1}{3}\right)^2$ **f)** $8^2 \div 2^4$

11. Evaluate each expression for the given values of the variables.

a) $3x^4$ $x = 2$
b) $2x^2 + 5$ $x = 3$
c) $4r^2 - r$ $r = 6$
d) $t^2 - 2t$ $t = 4$
e) $m^2 + m - 4$ $m = 3$
f) $x^2 - y^2$ $x = 7, y = 5$

Answers

1. **a)** 5 **b)** 2 **c)** (–3)
d) 3 **e)** $\frac{2}{3}$ **f)** 2.1

2. **a)** 2 **b)** 3 **c)** 4
d) 4 **e)** 2 **f)** 2

3. B; D

4. 4^3

5. C: D

6. 2 × 2 × 2 × 2

7. **a)** 6^7 **b)** 9^2 **c)** 0.4^3
d) $(-7)^5$ **e)** $(-1.3)^4$ **f)** $\left(\frac{2}{5}\right)^4$

8. **a)** 3 × 3 × 3 × 3; 81 **b)** 5 × 5 × 5; 125
c) (–2) × (–2); 4 **d)** – (3 × 3 × 3 × 3); –81
e) $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$; $\frac{1}{16}$ **f)** 0.4 × 0.4 × 0.4; 0.064

9. **a)** 216 **b)** 128 **c)** –16
d) 64 **e)** 1 **f)** $\frac{16}{25}$

10. **a)** 25 **b)** 210 **c)** 49
d) 29 **e)** $\frac{2}{3}$ **f)** 4

11. **a)** 48 **b)** 23 **c)** 138
d) 8 **e)** 8 **f)** 24

Discover: Exponent Law for Multiplication of Powers

Complete the table below. Make up your own questions for the blank rows.

Product	***Expanded Form***	***Single Power***
$x^4(x^3)$	$=(x\times x\times x\times x)\times(x\times x\times x)$	$=x^7$
$y^5(y^6)$	$=(y\times y\times y\times y\times y)\times(y\times y\times y\times y\times y\times y)$	$=y^{11}$
$m^3(m^2)$	$=$	$=$
$a^4(a^5)$	$=$	$=$
$t^6(t)$	$=$	$=$
$w^7(w^2)$	$=$	$=$
$h^2(h^4)$	$=$	$=$
$p^8(p^5)$	$=$	$=$
	$=$	$=$
	$=$	$=$
$x^a(x^b)$	$=$	$=$
$c^2d^3(c^4d^5)$	$=(c\times c\times d\times d\times d)\times(c\times c\times c\times c\times d\times d\times d\times d\times d)$	$=c^6d^8$
$k^3j^4(k^5j^2)$	$=$	$=$
$p^4q^2(p^2q^3)$	$=$	$=$
$v^5z^3(v^3z^4)$	$=$	$=$
$g^2t^5(g^4t^3)$	$=$	$=$
	$=$	$=$
	$=$	$=$
$a^4b^2c^5(a^3b^4c^4)$	$=$	$=$

Complete this statement: When multiplying powers with the ***same base***,

Discover: Exponent Law for Division of Powers

Complete the table below. Make up your own questions for the blank rows.

Product	***Expanded Form***	***Single Power***
$\frac{x^7}{x^3}$	$=\frac{x\times x\times x\times x\times x\times x\times x}{x\times x\times x}=\left(\frac{x\times x\times x}{x\times x\times x}\right)\left(\frac{x\times x\times x\times x}{1}\right)=1\left(\frac{x^4}{1}\right)$	$=x^4$
$\frac{y^5}{y^2}$	$=\frac{y\times y\times y\times y\times y}{y\times y}=\left(\frac{y\times y}{y\times y}\right)\left(\frac{y\times y\times y}{1}\right)=1\left(\frac{y^3}{1}\right)$	$=y^3$
$\frac{a^4}{a^3}$	=	=
$\frac{t^6}{t}$	=	=
$\frac{w^7}{w^2}$	=	=
$p^8 \div p^5$	=	=
	=	=
$\frac{x^a}{x^b}$	=	
$\frac{c^7d^3}{c^4d^2}$	$=\frac{c\times c\times c\times c\times c\times c\times c\times d\times d\times d}{c\times c\times c\times c\times d\times d}$	$=c^3d$
$\frac{k^8j^4}{k^5j^2}$	=	=
$\frac{p^4q^6}{p^2q^3}$	=	=
$\frac{v^5z^8}{v^3z^4}$	=	=
$\frac{g^6t^5}{g^4t^3}$	=	=
	=	=
$\frac{a^4b^7c^6}{a^3b^4c^4}$	=	=

Complete this statement: When dividing powers with the ***same base***,

Discover: Exponent Law for Power of a Power

Complete the table below. Make up your own questions for the blank rows.

Product	***Expanded Form***	***Single Power***
$\left(x^4\right)^2$	$=(x\times x\times x\times x)\times(x\times x\times x\times x)$	$=x^8$
$\left(y^3\right)^4$	$=(y\times y\times y)\times(y\times y\times y)\times(y\times y\times y)\times(y\times y\times y)$	$=y^{12}$
$\left(m^3\right)^2$	$=$	$=$
$\left(g^2\right)^5$	$=$	$=$
$\left(t^4\right)^3$	$=$	$=$
$\left(w^5\right)^4$	$=$	$=$
$\left(k^6\right)^2$	$=$	$=$
$\left(r^3\right)^3$	$=$	$=$
	$=$	$=$
	$=$	$=$
$\left(x^a\right)^b$	$=$	$=$
	Do these using your shortcut, without the middle step	
$\left(m^3\right)^5$		$=$
$\left(q^8\right)^6$		$=$
$\left(x^{10}\right)^4$		$=$
$\left(n^7\right)^8$		$=$
$\left(a^3b^4\right)^5$		$=$
		$=$

Complete this statement: When a power is raised to an exponent,

How to Read Powers

e.g. Consider the power 2^3. It can be read in a variety of different ways as shown below.

2^3

- "Two to the exponent three"
- "Two cubed"
- "The third power of two"
- "Two to the third"
- "Two raised to the exponent three"

Note that many people will also say, "Two to the ***power*** three." Technically, this is incorrect because the power is 2^3 ***not*** three. However, it is such a common practice to use the word "power" as if it were synonymous with "exponent" that we have no choice but to accept it.

A Common Mistake that you Should Never Make

NEVER confuse powers with multiplication

e.g. 2^3 ***means*** $2\times2\times2=8$ ***NOT*** $2\times3=6$

If you confuse powers with multiplication, then the mass of the sun would be only 600 kg, which is clearly nonsensical!

Mass of sun = 2×10^{30} kg = 2 times10 multiplied by itself 30 times ***NOT*** $2\times10\times30=600$

Simplifying Expressions involving Powers by writing in Expanded Form

In the following examples, the expressions are simplified (written as a single power) by writing powers in expanded form. This is done to help you remember to ***think about the meaning of powers before you write your answers***.

(a) $3^2\left(3^4\right)=3(3)(3)(3)(3)(3)=3^6$

There are six ***factors*** of 3 altogether.

(b) $$\frac{4^5}{4^3}=\frac{4(4)(4)(4)(4)}{4(4)(4)}=\left[\frac{4(4)(4)}{4(4)(4)}\right]\left[\frac{4(4)}{1}\right]=[1][4(4)]=4^2$$

(c) $$\left(5^2\right)^3=\left(5^2\right)\left(5^2\right)\left(5^2\right)=5(5)(5)(5)(5)(5)=5^6$$

(d) $x^2\left(x^4\right)=x(x)(x)(x)(x)(x)=x^6$

(e) $$\frac{a^5}{a^3}=\frac{a(a)(a)(a)(a)}{a(a)(a)}=\left[\frac{a(a)(a)}{a(a)(a)}\right]\left[\frac{a(a)}{1}\right]=[1][a(a)]=a^2$$

(f) $$\left(s^2\right)^3=\left(s^2\right)\left(s^2\right)\left(s^2\right)=s(s)(s)(s)(s)(s)=s^6$$

(g) $3\left(y^3\right)^2$

(h) $\left(3y^3\right)^2$

Understanding the Laws of Exponents

Name of Law	*Law Expressed in Algebraic Form*	*Law Expressed in Verbal Form*	*Example Showing why Law Works*
Product Rule	$a^x a^y = a^{x+y}$	To ***multiply*** two powers with the ***same base***, ***keep the base*** and ***add the exponents***.	$a^2 a^4 = (a)(a)(a)(a)(a)(a) = a^6$ ***Two*** factors of a multiplied by ***four*** factors of a gives ***six*** factors of a.
Quotient Rule	$\frac{a^x}{a^y} = a^{x-y}$	To ***divide*** two powers with the ***same base***, ***keep the base*** and ***subtract the exponents***.	$\frac{a^5}{a^2} = \frac{(a)(a)(a)(a)(a)}{(a)(a)} = a^3$ ***Five*** factors of a divided by ***two*** factors of a leaves ***three*** factors of a. (Two factors of a in the numerator divide out with two factors of a in the denominator.)
Power of a Power Rule	$(a^x)^y = a^{xy}$	To ***raise*** a power to an exponent, ***keep the base*** and ***multiply*** the exponents.	$(a^3)^4 = (a^3)(a^3)(a^3)(a^3) = a^{12}$

Examples

Use the laws of exponents to simplify each of the following expressions.

(a) $x^2(x^4) = x^{2+4} = x^6$

(b) $\frac{y^5}{y^3} = y^{5-3} = y^2$

(c) $x^2(y^4)$ cannot be simplified because the bases are different

(d) $(a^3)^6 = a^{3\times 6} = a^{18}$

(e) $\frac{x^5}{y^3}$ cannot be simplified because the bases are different

(f) $5(x^2)^3 = 5(x^{2\times 3}) = 5x^6$

(g) $(5x^2)^3 = (5x^2)(5x^2)(5x^2)$
$= 5(5)(5)(x^2)(x^2)(x^2)$
$= 125x^{2+2+2}$
$= 125x^6$

Another Law of Exponents

Example (g) above suggests the following shortcut: $(5x^2)^3 = 5^3(x^2)^3 = 125x^{2\times 3} = 125x^6$. In general, this can be expressed as follows:

$$(ab)^x = a^x b^x$$

To ***raise a product to an exponent, raise each factor*** in the product ***to the exponent***.

e.g. $(m^3n^4)^6 = (m^3)^6(n^4)^6 = m^{18}n^{24}$

A Big Example

Use the laws of exponents to simplify $\frac{2ab^2(3a^3b^3)}{(4ab^2)^4}$

$$\frac{2ab^2(3a^3b^7)}{(4ab^2)^4} = \frac{2(3)a^1a^3b^2b^7}{4^4a^4(b^2)^4}$$
$$= \frac{6a^{1+3}b^{2+7}}{256a^4b^{2\times 4}}$$
$$= \frac{6a^4b^9}{256a^4b^8}$$
$$= \left(\frac{6}{256}\right)\left(\frac{a^4}{a^4}\right)\left(\frac{b^9}{b^8}\right)$$
$$= \left(\frac{3}{128}\right)(1)b^{9-8}$$
$$= \frac{3}{128}b$$

Since multiplication can be done in any order, the ***like factors*** can be put together.

Practice: Discover the Exponent Laws

1. Write each expression in expanded form. Then write as a single power.

a) $7^2 \times 7^4$ **b)** $3^5 \times 3^3$
c) 5×5^2 **d)** $3^2 \times 3^4 \times 3^3$
e) $(-2)^2 \times (-2)^3$ **f)** $(-1)^3 \times (-1)^2 \times (-1)$
g) $0.5^3 \times 0.5^2$ **h)** $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^3$

2. Evaluate each expression in question 1.

3. Write each expression in expanded form. Then write as a single power.

a) $8^6 \div 8^4$ **b)** $5^5 \div 5^3$
c) $7^7 \div 7^2$ **d)** $4^8 \div 4^5 \div 4$
e) $(-9)^7 \div (-9)^6$ **f)** $0.1^6 \div 0.1^4$
g) $(-0.3)^4 \div (-0.3)$ **h)** $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^3$

4. Evaluate each expression in question 3.

5. Write each expression in expanded form. Then, write as a single power.

a) $(2^2)^4$ **b)** $(6^2)^2$
c) $(3^3)^2$ **d)** $[(-2)^4]^3$
e) $[(-1)^8]^6$ **f)** $[(-1)^5]^7$
g) $(0.3^2)^2$ **h)** $\left[\left(\frac{2}{5}\right)^2\right]^2$

6. Evaluate each expression in question 5.

7. Use the exponent laws to simplify each expression. Then, evaluate.

a) $4^3 \times 4^4 \div 4^5$ **b)** $8^7 \div 8^7 \times 8$
c) $\dfrac{9^6 \times 9^3}{9^7}$ **d)** $\dfrac{6^5 \times 6^2}{6 \times 6^3}$
e) $(2^4)^2 \times 2^3$ **f)** $\dfrac{(3^2)^4 \times 3^3}{3^8}$
g) $0.2^6 \times 0.2^5 \div (0.2^2)^5$ **h)** $[(-4)^3]^4 \div [(-4)^2]^5$

8. Simplify.

a) $b^5 \times b^3$ **b)** $p^4 \times p$
c) $w^5 \div w^2$ **d)** $x^8 \div x^4$
e) $(m^5)^2$ **f)** $(k^2)^3 \times k^2$
g) $g^5 \times g^5 \div g^7$ **h)** $(a^6)^3 \div (a^5)^2$

9. Simplify.

a) $a^4b^5 \times ab^3$ **b)** $m^2n^4 \times m^3n^3$
c) $p^6q^5 \div (p^3q^2)$ **d)** $6xy^2 \div (2y)$
e) $(gh^4)^3$ **f)** $2k^2m^3 \times (2k^2)^2$
g) $\dfrac{(2g^5h^3)^2}{2gh^6}$ **h)** $\dfrac{6b^2d \times 3b^2d^2}{(3bd)^2}$

Answers

1. **a)** $7 \times 7 \times 7 \times 7 \times 7 \times 7$; 7^6 **b)** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; 3^8
c) $5 \times 5 \times 5$; 5^3 **d)** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; 3^9
e) $(-2) \times (-2) \times (-2) \times (-2) \times (-2)$; $(-2)^5$
f) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$; $(-1)^6$
g) $0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$; 0.5^5
h) $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$; $\left(\frac{1}{2}\right)^4$

2. **a)** 117 649 **b)** 6561 **c)** 125 **d)** 19 683
e) –32 **f)** 1 **g)** 0.031 25 **h)** $\frac{1}{16}$

3. **a)** $\dfrac{8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8}$; 8^2 **b)** $\dfrac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$; 5^2
c) $\dfrac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7}$; 7^5 **d)** $\dfrac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{(4 \times 4 \times 4 \times 4 \times 4) \times 4}$; 4^2
e) $\dfrac{(-9) \times (-9) \times (-9) \times (-9) \times (-9) \times (-9) \times (-9)}{(-9) \times (-9) \times (-9) \times (-9) \times (-9) \times (-9)}$; $(-9)^1$
f) $\dfrac{0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1}{0.1 \times 0.1 \times 0.1 \times 0.1}$; 0.1^2
g) $\dfrac{(-0.3) \times (-0.3) \times (-0.3) \times (-0.3)}{(-0.3)}$; $(-0.3)^3$
h) $\dfrac{\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)}$; $\left(\frac{2}{3}\right)^2$

4. **a)** 64 **b)** 25 **c)** 16 807 **d)** 16
e) –9 **f)** 0.01 **g)** –0.027 **h)** $\frac{4}{9}$

5. **a)** $(2^2) \times (2^2) \times (2^2) \times (2^2)$; 2^8
b) $(6^2) \times (6^2)$; 6^4
c) $(3^3) \times (3^3)$; 3^6
d) $(-2)^4 \times (-2)^4 \times (-2)^4$; $(-2)^{12}$
e) $(-1)^8 \times (-1)^8 \times (-1)^8 \times (-1)^8 \times (-1)^8 \times (-1)^8$; $(-1)^{48}$
f) $(-1)^5 \times (-1)^5 \times (-1)^5 \times (-1)^5 \times (-1)^5 \times (-1)^5 \times (-1)^5$; $(-1)^{35}$
g) $(0.3^2) \times (0.3^2)$; 0.3^4
h) $\left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^2$; $\left(\frac{2}{5}\right)^4$

6. **a)** 256 **b)** 1296 **c)** 729 **d)** 4096
e) 1 **f)** –1 **g)** 0.0081 **h)** $\frac{16}{625}$

7. **a)** 4^2; 16 **b)** 8^1; 8 **c)** 9^2; 81
d) 6^3; 216 **e)** 2^{11}; 2048 **f)** 3^3; 27
g) 0.2^1; 0.2 **h)** $(-4)^2$; 16

8. **a)** b^8 **b)** p^5 **c)** w^3 **d)** x^4
e) m^{10} **f)** k^8 **g)** g^3 **h)** a^8

9. **a)** a^5b^8 **b)** m^5n^7 **c)** p^3q^3 **d)** $3xy$
e) g^3h^{12} **f)** $8k^6m^3$ **g)** $2g^9$ **h)** $2b^2d$

Putting all the Operations Together: The Distributive Property

Review – Operating with Integers

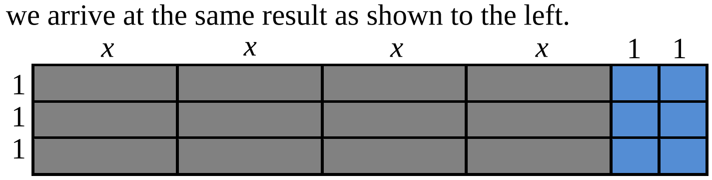
Adding and Subtracting Integers	***Multiplying and Dividing Integers***
• Addition and subtraction of integers always involve ***MOVEMENTS*** ***Add a Positive Value or Subtract a Negative Value*** +(+) ***or*** –(–) → ***GAIN*** (move ***up*** or ***right***) ***Add a Negative Value or Subtract a Positive Value*** +(–) ***or*** –(+) → ***LOSS*** (move ***down*** or ***left***) • Movements on a number line • Moving from one floor to another using an elevator • Loss/gain of yards in football • Loss/gain of money in bank account or stock market • It is ***NOT POSSIBLE*** to predict the sign of the answer to an addition or subtraction only by knowing the signs of the numbers! $2+(-5) = 2-5 = -3$ $5+(-2) = 5-2 = 3$ $8-(+5) = 8-5 = 3$ $-2-(-5) = -2+5 = 3$	• Multiplication is ***repeated addition*** **e.g.** 5(–2) = 5 groups of –2 = (–2)+(–2)+(–2)+(–2)+(–2) = –10 • Division is the ***opposite of*** multiplication **e.g.** –10÷(–2) = How many groups of –2 in –10? = 5 ***Multiply or Divide Two Numbers of Like Sign*** (+)(+) ***or*** (–)(–) → ***POSITIVE RESULT*** ***Multiply or Divide Two Numbers of Unlike Sign*** (+)(–) ***or*** (–)(+) → ***NEGATIVE RESULT*** • The sign of the answer to a multiplication or division is ***DETERMINED ENTIRELY*** by the signs of the numbers! $2(-5) = -10$ $-12 \div 6 = -2$ $-12 \div (-6) = 2$

Understanding how to Multiply a Monomial by a Binomial

Consider the expression shown below. Although BEDMAS would tell us to perform the operations within the brackets first, we cannot do so because 4*x* and 2 are unlike terms. Nevertheless, we can still deal with this expression in a variety of ways.

Cannot be simplified because 4*x* and 2 are ***unlike terms***.

$$3(4x+2)$$

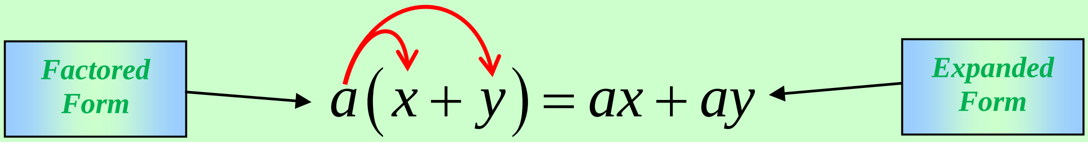
Multiply 3 by $(4x+2)$ ***using the Definition of Multiplication***	***Multiply 3 by*** $(4x+2)$ ***using an Area Model (Algebra Tiles)***
Recall that multiplication is a short form for repeated addition: **e.g.** $3a = a+a+a$ Therefore, $3(4x+2)$ $=(4x+2)+(4x+2)+(4x+2)$ $=4x+2+4x+2+4x+2$ $=4x+4x+4x+2+2+2$ $=12x+6$	We can calculate the area of the following figure in two different ways. By doing so, we arrive at the same result as shown to the left.  Area = width × length $=3(4x+2)$ Area = $x+x+x+x+x+x+x+x+x+x+x+x+1+1+1+1+1+1$ $= 12x + 6$ Therefore, $3(4x+2)=12x+6$

A Shortcut for Multiplying a Monomial by a Polynomial

We performed the multiplication $3(4x+2)$ using two different methods and in both cases, we found that the product was $12x+6$. Unfortunately, while both methods allowed us to "see" clearly what the product should be, a great deal of time was required to arrive at the answer. To resolve this problem, we can use the following law:

The Distributive Property

This property is called the "distributive property" because the monomial *a* is ***distributed*** to each term enclosed in parentheses.



To ***expand*** the product of a monomial by a binomial, ***multiply each term*** of the binomial by the monomial. In other words, ***multiply each term in the brackets by a***.

Examples

1. Expand each of the following:

(a) $-5(3x+y)=-5(3x)-5(y)$

$=-15x-5y$

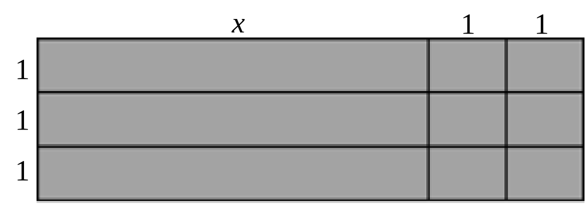
(b) $-7(2a-5b)=-7(2a)+7(5b)$

$=-14a+35b$

(c) $x(-3x^2-y^2)=x(-3x^2)-x(y^2)$

$=-3x^3-xy^2$

2. Use the diagram at the right to show that $3(x+2)=3x+6$.



Solution

Length = $x + 2$
Width = 3

Area = $3(x+2)$

But the area can also be calculated as follows:
Area = $x+x+x+1+1+1+1+1+1=3x+6$

Therefore, $3(x+2)=3x+6$

3. Expand and simplify.

$-3(3x+2y)-5(-4x-6y+1)$

$=-9x-6y+20x+30y-5$

$=-9x+20x-6y+30y-5$

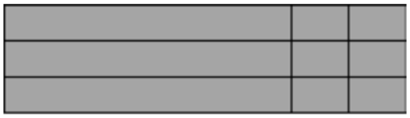
$=11x+24y-5$

Practice: The Distributive Property

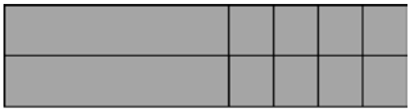
1. Copy and complete the table for each rectangle.

Rectangle	Width	Length	Area	Equation

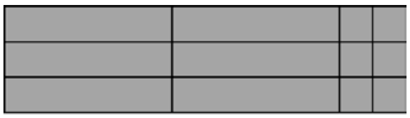
A



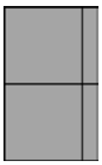
B



C



D



2. Model each expression with algebra tiles. Then, simplify each expression.

a) $5(x + 2)$ **b)** $4(2x + 3)$
c) $x(x + 4)$ **d)** $2x(2x + 5)$

3. Which expression is equal to $6(x – 4)$?

A $6x – 4$ **B** $6x + 4$
C $x – 24$ **D** $6x – 24$

4. Use the distributive property to expand.

a) $3(g + 4)$ **b)** $2(a + 5)$
c) $6(x – 3)$ **d)** $5(b – 1)$
e) $4(3 – r)$ **f)** $–7(q + 3)$
g) $–2(6 – t)$ **h)** $–4(–w – 5)$

5. Expand.

a) $b(b + 1)$ **b)** $m(m + 4)$
c) $x(x – 2)$ **d)** $a(a + 1)$
e) $r(3r + 5)$ **f)** $q(2q + 3)$
g) $k(6 – k)$ **h)** $w(4w – 5)$

6. Expand.

a) $3p(p + 4)$ **b)** $2s(s + 2)$
c) $4x(2x – 1)$ **d)** $6b(3b + 1)$
e) $–r(–5r + 2)$ **f)** $–y(2y – 7)$
g) $5c(8 – 2c)$ **h)** $–3w(2w – 1)$

7. Expand.

a) $(d + 3) \times 2$ **b)** $(k + 1) \times 4$
c) $(w – 2) \times 5$ **d)** $(u – 1) \times (–3)$
e) $(2q + 5) \times 6$ **f)** $(–p + 4) \times (–2)$
g) $(5 – z)(3z)$ **h)** $(6w – 4)(–3w)$

8. Expand.

a) $3(x^2 + x – 4)$ **b)** $2(m^2 – 3m + 5)$
c) $–4(b^2 – 2b – 3)$ **d)** $5c(c^2 – 6c – 1)$
e) $–3h(4 – h^2)$ **f)** $(n^2 + 4n + 3)(–2)$
g) $(5t^2 – 2t)(–t)$ **h)** $(w^2 + 2w – 5)(4w)$

9. Expand and simplify.

a) $2(b + 3) + 5(b + 4)$
b) $3(p – 2) + 6(p + 1)$
c) $–5(m + 5) + 2(m – 7)$
d) $–(d – 4) – 4(d + 2)$

10. Expand and simplify.

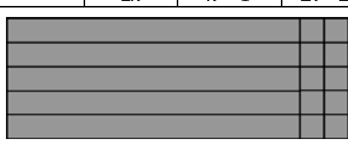
a) $4[b + 3(b + 1)]$
b) $2[3(a + 4) – 4]$
c) $5[4s – (s + 2)]$
d) $3[–2(6 – t) + 5t]$

Answers

1.

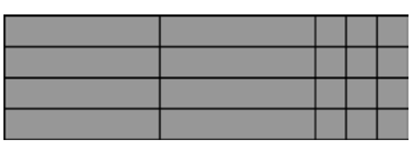
Rectangle	Width	Length	Area	Equation
A	3	$x + 2$	$3x + 6$	$3(x + 2) = 3x + 6$
B	2	$x + 4$	$2x + 8$	$2(x + 4) = 2x + 8$
C	3	$2x + 2$	$6x + 6$	$3(2x + 2) = 6x + 6$
D	$2x$	$x + 1$	$2x^2+2x$	$2x(x + 1) = 2x^2 + 2x$

2. **a)**



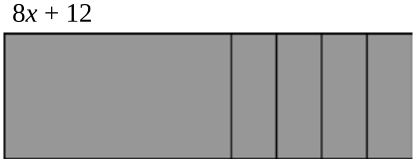
$5x + 10$

b)



$8x + 12$

c)



$x^2 + 4x$

d)

$4x^2 + 10x$

3. **D:** $6x – 24$

4. **a)** $3g + 12$ **b)** $2a + 10$ **c)** $6x – 18$ **d)** $5b – 5$
e) $12 – 4r$ **f)** $–7q – 21$ **g)** $–12 + 2t$ **h)** $4w + 20$

5. **a)** $b^2 + b$ **b)** $m^2 + 4m$ **c)** $x^2 – 2x$ **d)** $a^2 + a$
e) $3r^2 + 5r$ **f)** $2q^2 + 3q$ **g)** $6k – k^2$ **h)** $4w^2 – 5w$

6. **a)** $3p^2 + 12p$ **b)** $2s^2 + 4s$ **c)** $8x^2 – 4x$ **d)** $18b^2 + 6b$
e) $5r^2 – 2r$ **f)** $–2y^2 + 7y$ **g)** $40c–10c^2$ **h)** $–6w^2 + 3w$

7. **a)** $2d + 6$ **b)** $4k + 4$ **c)** $5w – 10$ **d)** $–3u + 3$
e) $12q + 30$ **f)** $2p – 8$ **g)** $15z – 3z^2$ **h)** $–18w^2 + 12w$

8. **a)** $3x^2 + 3x – 12$ **b)** $2m^2 – 6m + 10$
c) $–4b^2 + 8b + 12$ **d)** $5c^3 – 30c^2 – 5c$
e) $–12h + 3h^3$ **f)** $–2n^2 – 8n – 6$
g) $–5t^3 + 2t^2$ **h)** $4w^3 + 8w^2 – 20w$

9. **a)** $7b + 26$ **b)** $9p$ **c)** $–3m – 39$ **d)** $–5d – 4$

10. **a)** $16b + 12$ **b)** $6a + 16$ **c)** $15s – 10$ **d)** $–36 + 21t$

SUMMARY OF SIMPLIFYING ALGEBRAIC EXPRESSIONS

Review of Simplifying Algebraic Expressions

Adding and Subtracting Polynomials with no Brackets

1. Collect like terms. ***Remember that the operation must "travel" with the term!***
2. Use the rules for adding and subtracting integers. (GAINS/LOSSES)

Examples

$$\begin{aligned}&-5a-6b+3a-4b\\=&-5a+3a-6b-4b\\=&-2a-10b\end{aligned}$$

$$\begin{aligned}&-2x^2y+3xy-7x^2y-8xy\\=&-2x^2y-7x^2y+3xy-8xy\\=&-9x^2y-5xy\end{aligned}$$

Adding and Subtracting Polynomials with Brackets

1. If a bracket is preceded by a "+" sign or no sign, the brackets can simply be removed because addition is ***insensitive*** to order.
2. If a bracket is preceded by a "–" sign, brackets cannot be removed because subtraction is sensitive to order. After ***adding the opposite***, the brackets can be removed. This works because − = + (−).
3. Collect like terms.
4. Use the rules for adding and subtracting integers. (GAINS/LOSSES)

Example

$$\begin{aligned}&(-5a+6b)-(3a-4b)\\=&-5a+6b+(-3a+4b)\\=&-5a+6b+(-3a)+4b\\=&-5a+6b-3a+4b\\=&-5a-3a+6b+4b\\=&-8a+10b\end{aligned}$$

Add the opposite of $3a-4b$

Brackets can be removed now because the operation is "+."

The Distributive Property

1. Look for an expression in brackets containing two or more terms (usually the terms are unlike) and a factor outside the brackets.
2. Multiply each term in the brackets by the ***factor*** outside the brackets.

Examples

$$\begin{aligned}x(-3x^2-y^2)&=x(-3x^2)-x(y^2)\\&=-3x^3-xy^2\end{aligned}$$

$$\begin{aligned}&(-5a+6b)-(3a-4b)\\=&-5a+6b-1(3a-4b)\\=&-5a+6b-3a+4b\\=&-5a-3a+6b+4b\\=&-8a+10b\end{aligned}$$

The distributive property can be used as an ***alternative*** to adding the opposite.

Multiplying and Dividing Monomials

1. Make sure there is ***only one term***. If there are two or more terms, make sure that you work on each term separately.
2. Put ***like factors*** together. You are allowed to do this because multiplication can be performed in any order.
3. Use the ***laws of exponents***.

Example

$$\frac{2ab^2(3a^3b^7)}{(4ab^2)^4}=\frac{2(3)a^1a^3b^2b^7}{4^4a^4(b^2)^4}=\frac{6a^{1+3}b^{2+7}}{256a^4b^{2\times4}}=\frac{6a^4b^9}{256a^4b^8}$$

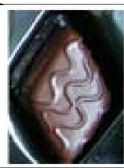
$$=\left(\frac{6}{256}\right)\left(\frac{a^4}{a^4}\right)\left(\frac{b^9}{b^8}\right)=\left(\frac{3}{128}\right)(1)b^{9-8}=\frac{3}{128}b$$

A more Complicated Example Involving the Distributive Property

$$\begin{aligned}&-3a^2b(-6abc+9a^3b^4-7bc)\\=&\,3a^2b(6abc)-3a^2b(9a^3b^4)+3a^2b(7bc)\\=&\,18a^3b^2c-27a^5b^5+21a^2b^2c\end{aligned}$$

Understanding the Distributive Property from a Different Point of View

Consider the box of chocolates shown at the right. As shown in the table given below, a variable is used to represent the type of chocolate.

Variable	*What it Represents*	*Total Number of this type in a Box*
x	One 	$8x$
y	One 	y
z	One 	$2z$
u	One 	$2u$
v	One 	$2v$
w	One 	$5w$

1. Write an algebraic expression that expresses the total number of chocolates in one box.

$$8x + y + 2z + 2u + 2v + 5w$$

2. There are 1000 boxes of these chocolates in a warehouse. Write an algebraic expression that represents the total number of chocolates in the warehouse.

$$1000(8x + y + 2z + 2u + 2v + 5w)$$

3. Now use the distributive property to expand your expression. Does the answer make sense?

$$1000(8x + y + 2z + 2u + 2v + 5w)$$
$$= 8000x + 1000y + 2000z + 2000u + 2000v + 5000w$$

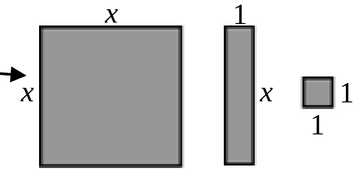
The answer makes sense because in 1000 boxes of chocolates, there should be 8000 of type *x*, 1000 of type *y*, 2000 of type *z*, 2000 of type *u*, 2000 of type *v* and 5000 of type *w*.

UNIT 1 REVIEW

General Review

1. Use ***algebra tiles*** ~~to model~~ each algebraic expression.

a) $4x + 2$ **b)** $2x^2$
c) $x^2 + 2x$ **d)** $2x^2 + x + 4$



2. One face of a cube has area 36 cm^2.
a) What is the side length of the cube?
b) Find the volume of the cube.

3. Evaluate.
a) 5^3 **b)** 2^8
c) -3^4 **d)** $(-2)^4$
e) $(-1)^{10}$ **f)** $\left(\frac{2}{3}\right)^3$

4. Evaluate. Use the correct order of operations.
a) $3^4 + 4^2$ **b)** $7^2 - 7$
c) $9^2 \div 3^2$ **d)** $5 \times \left(\frac{2}{5}\right)^3$
e) $(3^2 + 4^2)$ **f)** $(3 + 4)^2$

5. A scientist studying a type of bacteria notices that the population doubles every 30 minutes. The initial population is 500.
a) Copy and complete the table.

Time (min)	Population
0	500
30	1000
60	
90	
120	

b) Construct a graph of population versus time. Connect the points with a smooth curve.

6. Write as a single power. Then, evaluate.
a) $8^5 \times 8^4 \div 8^7$
b) $6^7 \div 6^5 \div 6$
c) $(3^3)^4 \div 3^9$
d) $\dfrac{(5^3)^4 \times 5^2}{5^{10}}$
e) $2^7 \times 2^5 \div (2^2)^4$
f) $[(-6)^3]^3 \div [(-6)^2]^4$

7. Simplify.
a) b^6b^3 **b)** $g^2g^8 \div g^7$
c) $(a^5)^3 \div (a^4)^2$ **d)** $m^5n \times m^2n^4$
e) $\dfrac{p^7q^4}{p^3q^4}$ **f)** $\dfrac{8b^3d\left(4bd^2\right)}{2\left(2bd\right)^2}$

8. Identify the coefficient and the variable for each term.
a) $7m$ **b)** $-3x^5$
c) $\frac{3}{7}m^2n$ **d)** gh

9. Classify each expression as a monomial, binomial, trinomial, or polynomial.
a) $a^2 - 2a + 1$
b) $2 - 3x^4 - 5x^2 + 4x$
c) $6m^2n^5$
d) $h^3 + 6$
e) $12x$
f) $4x^2 - 3y^2 + 8$

10. State the degree of each term.
a) $-8b^4$
b) $-x^4y^3$
c) $\frac{3}{4}mn^2$
d) $6r^6s$

11. What is the degree of each polynomial?
a) $5a^4 + b^3$
b) $7b^6$
c) $2x^2 + 3x - 1$
d) $8m^4 - m^2 + 2m$

12. Classify each pair of terms as like or unlike.
a) $4a^2$ and $4a$
b) $6x^3$ and $-x^3$
c) $12p^4$ and $-p^4$
d) $4a^2b^3$ and $6a^3b^2$

13. Simplify each expression.

a) $2b + 7g - 5b - 8g$

b) $3x + y^2 + 5y^2 - 7x$

c) $6q + u + 4u + q + u + 4u - u$

d) $10 - m^2 - 7 - m^2 + 4m^2$

e) $-3v + 2v + 6 - 3v - 9 - v$

f) $7 + h + h - 5 + 6h + 2 + 3h$

14. Simplify.

a) $(6k - 4) + (2k + 4)$

b) $(2a + 1) - (4a + 2)$

c) $(b - 6) - (2 - 5b) + (b + 4)$

d) $(g + 12) + (g - 7) - (2 - 3g)$

e) $(x^2 + 2x + 1) + (2x^2 + 4)$

f) $(2m^2 + m + 12) - (3m^2 + 4m - 6)$

15. The length of the Cheungs' backyard is double its width.

a) Write an expression for the perimeter of their back yard.

b) The width of their back yard is 9 m. What is its perimeter?

16. Expand.

a) $5(x + 3)$ **b)** $4(b + 2)$

c) $w(2w + 1)$ **d)** $q(q + 4)$

e) $3c(6 - 4c)$ **f)** $-p(2p - 1)$

g) $-5(a^2 - 4a - 2)$ **h)** $2d(d^2 - 3d - 1)$

17. Expand and simplify.

a) $3(x + 3) + 2(x + 1)$

b) $-4(m + 2) + 3(m - 7)$

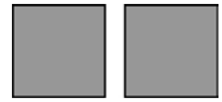
c) $-(d - 3) - 5(d + 2)$

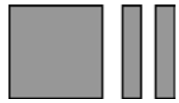
d) $5[b + 2(b + 1)]$

e) $-2[3(a + 3) - 4]$

f) $4[-2(4 - t) + 3t]$

Answers

1. **a)**  **b)** 

c)  **d)** 

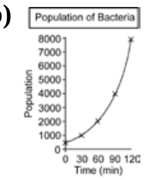
2. **a)** 6 cm **b)** 216 cm^3

3. **a)** 125 **b)** 256 **c)** –81 **d)** 16 **e)** 1 **f)** $\frac{8}{27}$

4. **a)** 97 **b)** 42 **c)** 9 **d)** $\frac{8}{25}$ **e)** 25 **f)** 49

5. **a)**

Time (min)	Population
0	500
30	1000
60	2000
90	4000
120	8000

b) 

6. **a)** 8^2; 64 **b)** 6^1; 6 **c)** 3^3; 27 **d)** 5^4; 625 **e)** 2^4; 16 **f)** $(-6)^1$; –6

7. **a)** b^9 **b)** g^3 **c)** a^7 **d)** m^7n^5 **e)** p^4 **f)** $4b^2d$

8. **a)** coefficient: 7; variable m
b) coefficient: –3; variable x^5

9. **a)** trinomial **b)** polynomial
c) monomial **d)** binomial
e) monomial **f)** trinomial

10. **a)** 4 **b)** 7 **c)** 3 **d)** 7

11. **a)** 4 **b)** 6 **c)** 2 **d)** 4

12. **a)** unlike **b)** like **c)** like **d)** unlike

13. **a)** $-3b - g$ **b)** $-4x + 6y^2$
c) $7q + 9u$ **d)** $3 + 2m^2$
e) $-5v - 3$ **f)** $4 + 11h$

14. **a)** $8k$ **b)** $-2a - 1$
c) $7b - 4$ **d)** $5g + 3$
e) $3x^2 + 2x + 5$ **f)** $-m^2 - 3m + 18$

15. **a)** $P = 6x$ **b)** 54 m

16. **a)** $5x + 15$ **b)** $4b + 8$
c) $2w^2 + w$ **d)** $q^2 + 4q$
e) $18c - 12c^2$ **f)** $-2p^2 + p$
g) $-5a^2 + 20a + 10$ **h)** $2d^3 - 6d^2 - 2d$

17. **a)** $5x + 11$ **b)** $-m - 29$
c) $-6d - 7$ **d)** $15b + 10$
e) $-6a - 10$ **f)** $-32 + 20t$

Problem Solving Review

1.

Meredith has a summer job at a fitness club. She earns a $5 bonus for each student membership and a $7 bonus for each adult membership she sells.

a) Write a polynomial expression that describes Meredith's total bonus.

b) Identify the variable and the coefficient of each term and explain what they mean.

c) How much will Meredith's bonus be if she sells 12 student memberships and 10 adult memberships?

2.

An arena charges $25 for gold seats, $18 for red seats, and $15 for blue seats.

a) Write an expression that describes the total earnings from seat sales.

b) Identify the variable and the coefficient of each term and explain what they mean.

c) How much will the arena earn if it sells 100 gold seats, 200 blue seats, and 250 red seats?

3.

In a soccer league, teams receive 3 points for a win, 2 points for a loss, and 1 point for a tie.

a) Write an algebraic expression to represent a team's total points.

b) What variables did you choose? Identify what each variable represents.

c) The Falcons' record for the season was 5 wins, 2 losses, and 3 ties. Use your expression to find the Falcons' total points.

d) The 10-game season ended with the Falcons tied for second place with the same number of points as the Eagles. The Eagles had a different record than the Falcons. How is this possible?

4.

On a multiple-choice test, you earn 2 points for each correct answer and lose 1 point for each incorrect answer.

a) Write an expression for a student's total score.

b) Maria answered 15 questions correctly and 3 incorrectly. Find Maria's total score.

5.

Alberto is training for a triathlon, where athletes swim, cycle, and run. During his training program, he has found that he can swim at 1.2 km/h, cycle at 25 km/h, and run at 10 km/h. To estimate his time for an upcoming race, Alberto rearranges the formula distance = speed × time to find that: time = $\frac{\text{distance}}{\text{speed}}$.

a) Choose a variable to represent the distance travelled for each part of the race. For example, choose s for the swim.

b) Copy and complete the table. The first row is done for you.

Part of the Race	Speed (km/h)	Distance (km)	Time (h)
swim	1.2	s	$\frac{s}{1.2}$
cycle			
run			

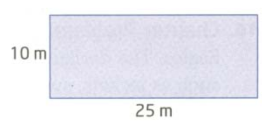
c) Write a trinomial to model Alberto's total time.

d) A triathlon is advertised in Kingston. Participants have to swim 1.5 km, cycle 40 km, and run 10 km. Using your expression from part c), calculate how long it will take Alberto to finish the race.

e) Is your answer a reasonable estimate of Alberto's triathlon time? Explain.

6.

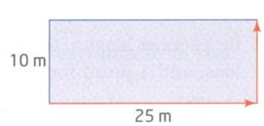
Ashleigh can walk 2 m/s and swim 1 m/s. What is the quickest way for Ashleigh to get from one corner of her pool to the opposite corner?



a) Predict whether it is faster for Ashleigh to walk or swim.

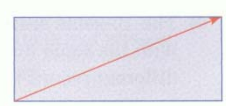
b) Ashleigh can walk at a speed of 2 m/s. The time, in seconds, for Ashleigh to walk is $\frac{w}{2}$, where w is the distance, in metres, she walks. Use this relationship to find the travel time if Ashleigh walks around the pool.

Path 1: Walk the entire distance.



c) Write a similar expression to represent the time taken for Ashleigh to swim a distance s. Her swimming speed is 1 m/s. Use this relationship to find the travel time if Ashleigh swims straight across.

Path 2: Swim the entire distance.

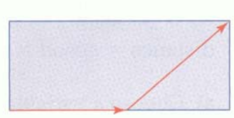


d) Which route is faster, and by how much?

7. Refer to question 6

a) Do you think it will be faster for Ashleigh to walk half the length and then swim? Explain your reasoning.

Path 3: Walk half the length, then swim.



b) Find the travel time for this path. Compare this with your answers to question 6.

c) Do you think this is the fastest possible path? Find the fastest path and the minimum time required to cross the pool, corner to opposite corner. Describe how you solved this.

8.

10. E. coli is a type of bacteria that lives in our intestines and is necessary for digestion. It doubles in population every 20 min. The initial population is 10.

a) Copy and complete the table. (The population of listeria doubles every 60 minutes

Time (min)	Population of Listeria	Population of E. Coli
0	800	
20		
40		
60		
80		
100		
120		

b) When will the population of E. coli overtake the population of Listeria?

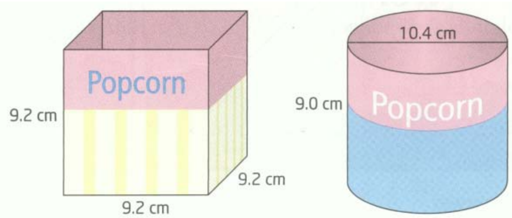
c) What population will the two cultures have when they are equal?

9.

The durations (lengths of time) of musical notes are related by powers of $\frac{1}{2}$, beginning with a whole note. Copy and complete the table.

Note	Symbol	Duration (in beats)	Power Form
whole		1	
half		$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
quarter		$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
eighth			
sixteenth			
thirty-second			

10.



Mega-Box **Jumbo Drum**

Which container holds more popcorn? How much more? Assume that each container is filled just to the top. Round your answer to the nearest cubic centimetre.

11.

Refer to question 9. Look at the pattern in the last column. Extend this pattern backward to write the power form for a whole note. Does this answer make sense? Use a calculator to evaluate this power. Describe what you observe.

12.

Uranium-233 is another isotope that is used in nuclear power generation. 1 kg of U-233 can provide about the same amount of electrical power as 3 000 000 kg of coal. This number can be written in **scientific notation** as 3×10^6.

a) Another isotope of uranium, U-238, has a half-life of 4 500 000 000 years. Write this number in scientific notation.

b) What is the half-life of U-238, in seconds? Write your answer in scientific notation.

c) The number 6.022×10^{23} is a very important number in chemistry. It is called "one mole." One mole is the amount of a substance that contains as many atoms, molecules, ions, or other elementary units as the number of atoms in 12 g of carbon-12. Carbon-12 is the basic building block of living things. Write one mole in standard notation.

d) Describe any advantages you see to using scientific notation.

13.

Math Contest Determine the last digit of the number 3^{1234} when written in expanded form. Justify your answer.

14.

Math Contest If $3^x = 729$, the value of x is

A 3 **B** 5 **C** 6 **D** 7 **E** 8

15.

Math Contest Numbers are called perfect powers if they can be written in the form x^y for positive integer values of x and y. Find all perfect powers less than 1000.

16.

Math Contest x^x is always greater than y^y as long as $x > y$. For what whole-number values of x and y is $x^y > y^x$? Justify your answer.

17.

A computer repair technician charges \$50 per visit plus \$30/h for house calls.

a) Write an algebraic expression that describes the service charge for one household visit.

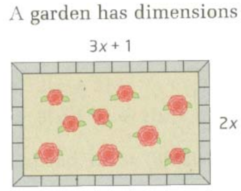
b) Use your expression to find the total service charge for a 2.5-h repair job.

c) Suppose all charges are doubled for evenings, weekends, and holidays. Write a simplified expression for these service charges.

d) Use your simplified expression from part c) to calculate the cost for a 2.5-h repair job on a holiday. Does this answer make sense? Explain.

18.

A garden has dimensions as shown.



a) Find a simplified expression for the perimeter.

b) Find a simplified expression for the area.

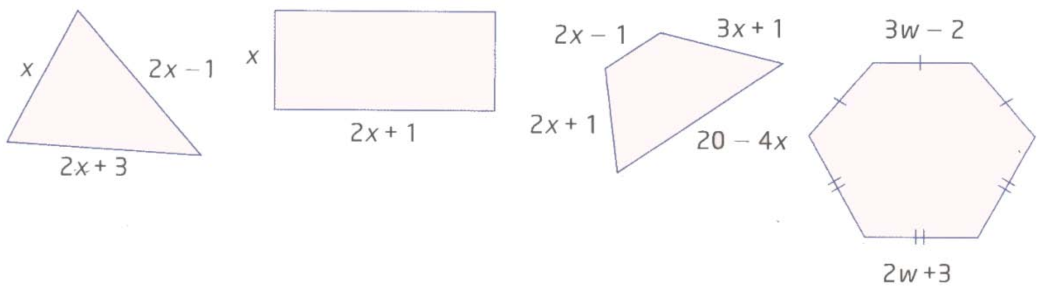
c) Repeat parts a) and b) if both the length and width are tripled.

d) Has this tripled the perimeter? Justify your answer.

e) Has this tripled the area? Justify your answer.

19.

a) Find a simplified expression for the perimeter of each figure. Use algebra tiles if you wish.



b) A rectangle has length $2x - 1$ and width $8 - 2x$. What is unusual about the perimeter?

c) For what value of x is the rectangle in part b) also a square?

Answers

1. (a) $5s + 7a$

(b) s = # student memberships sold
a = # adult memberships sold
5 = cost in $ per student membership
7 = cost in $ per adult membership

(c) $130.00

2. (a) $25g + 18r + 15b$

(b) g = # gold tickets sold, r = # red tickets sold
b = # blue tickets sold
25 = cost in $ per gold seat ticket
18 = cost in $ per red seat ticket
15 = cost in $ per blue seat ticket

(c) $10,000.00

3. (a) $3w + 2l + t$

(b) w = # wins, l = # losses, t = # ties
3 = points per win, 2 = points per loss, 1 = points per tie

(c) 22

(d) There is more than one way to get 22 points. For example, 2 wins and 8 losses also results in 22 points in 10 games. (You should note that this point system is quite silly because a loss is more valuable than a tie. A team that is losing has no incentive to work hard to achieve a tie.)

4. (a) $2c - i$ (c = # correct answers, i = # incorrect answers)

(b) 27

5. (a) s = distance travelled swimming (km)
c = distance travelled cycling (km)
r = distance travelled running (km)

(b)

Part of the Race	Speed (km/h)	Distance (km)	Time (h)
swim	1.2	s	$\frac{s}{1.2}$
cycle	25	c	$c/25$
run	10	r	$r/10$

(c) $\frac{s}{1.2} + \frac{c}{25} + \frac{r}{10}$ (d) $\frac{1.5}{1.2} + \frac{40}{25} + \frac{10}{10} = 3.85$ h = 3 h, 51 min

(e) The given speeds are reasonable because they are significantly lower than world record speeds. Therefore, the final answer seems reasonable.

6. (a) Ashleigh should take less time walking because her walking speed is twice her swimming speed but her walking distance is less than twice her swimming distance.

(b) 17.5 s

(c) 26.9 s

(d) walking is faster by 9.4 s

7. (a) It's possible that this route is faster because it is much shorter than path 1. Also, the swimming distance for this route is much shorter than for path 2.

(b) 22.3 s (which means this path is not faster)

(c) Path 1 is the fastest of the three routes. Calculus can be used to show that path 1 is the fastest route possible.

8. (a)

Time (min)	Population of Listeria	Population of E. Coli
0	800	10
20		20
40		40
60	1600	80
80		160
100		320
120	3200	640

(b) About 3 h, 10 min (190 minutes)

(c) About 7000

9.

Note	Symbol	Duration (in beats)	Power Form
whole	𝅝	1	
half	𝅗𝅥	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
quarter	♩	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
eighth	♪	$\frac{1}{8}$	$\left(\frac{1}{2}\right)^3$
sixteenth	𝅘𝅥𝅯	$\frac{1}{16}$	$\left(\frac{1}{2}\right)^4$
thirty-second	𝅘𝅥𝅰	$\frac{1}{32}$	$\left(\frac{1}{2}\right)^5$

10. Volume of Mega-Box = $(9.2)^3 \doteq 778.7 \text{ cm}^3$

Volume of Jumbo Drum = $\pi(5.2)^2(9) \doteq 764.6 \text{ cm}^3$

Mega-Box holds more popcorn (about 14.1 cm^3 more)

11. $\left(\frac{1}{2}\right)^0 = 1$. This makes sense according to the pattern that exists for the rest of the notes.

12. (a) 4.5×10^9

(b) $4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60 = \mathbf{1.41912 \times 10^{17}}$

(c) 602,200,000,000,000,000,000,000
(six hundred and two sextillion, two hundred quintillion)

(d) Scientific notation is very useful for expressing long numbers containing long strings of zeroes. Such numbers, when written in scientific notation, are much easier to read and understand.

13. $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$

$3^5 = 243$, $3^6 = 729$, $3^7 = 2187$, $3^8 = 6561$

$3^9 = 19683$, $3^{10} = 59049$, $3^{11} = 177147$, $3^{12} = 531441$

This pattern continues indefinitely.

Note that when the exponent is divisible by 4, the last digit is 1.

Therefore, 3^{1232} must end in 1 because 1232 is divisible by 4.

To continue the pattern, 3^{1233} must end in 3 and

3^{1234} must end in 9.

14. C ($x = 6$)

15. $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$

$2^7 = 128$, $2^8 = 256$, $2^9 = 512$

$3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, $3^6 = 729$

$4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$

$5^1 = 5$, $5^2 = 25$, $5^3 = 125$, $5^4 = 625$

$6^1 = 6$, $6^2 = 36$, $6^3 = 216$

$7^1 = 7$, $7^2 = 49$, $7^3 = 243$

$8^1 = 8$, $8^2 = 64$, $8^3 = 512$

$9^1 = 9$, $9^2 = 81$, $9^3 = 729$

$10^1 = 10$, $10^2 = 100$

16. $2^1 > 1^2$, $3^1 > 1^3$, $4^1 > 1^4$, $5^1 > 1^5$, …, $x^1 > 1^x$ for $x > 1$.

$3^2 > 2^3$, $3^4 > 4^3$, $3^5 > 5^3$, $3^6 > 6^3$, $3^7 > 7^3$, …

There are infinitely many other examples. A general answer to this question can be obtained using logarithms. You will learn about logarithms in grade 12.

17. (a) $30t + 50$ (t = number of hours)

(b) \$125.00

(c) $2(30t + 50) = 60t + 100$

(d) \$250.00 (This answer makes sense because it is double the answer for (b)).

18. (a) $P = 2(2x + 3x + 1) = 2(5x + 1) = 10x + 2$

(b) $A = 2x(3x + 1) = 6x^2 + 2x$

(c) $P = 2(6x + 9x + 3) = 2(15x + 3) = 30x + 6$

$A = 6x(9x + 3) = 54x^2 + 18x$

(d) Tripling the length and width ***also*** tripled the perimeter because $3(10x + 2) = 30x + 6$.

(e) Tripling the length and width ***did not*** triple the area. The area is actually ***nine times greater*** because $9(6x^2 + 2x) = 54x^2 + 18x$.

19. (a) $P = 5x + 2$ $\quad$ $P = 6x + 2$ $\quad$ $P = 3x + 21$ $\quad$ $P = 3(2w + 3) + 3(3w - 2) = 6w + 9 + 9w - 6 = 15w + 3$

(b) $P = 2(2x - 1 + 8 - 2x) = 2(7) = 14$ This is unusual because the perimeter is constant. No matter what the value of x is, the perimeter always turns out to be 14.

(c) The rectangle is a square if $2x - 1 = 8 - 2x$. This is true if $x = 2.25$.