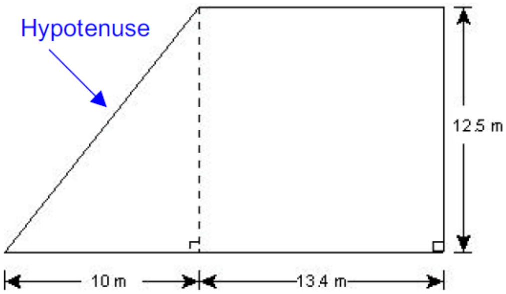
Unit 6 – Measurement and Geometry – Practice

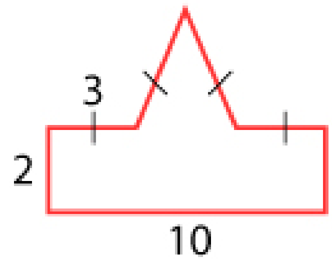
Perimeter and Area Problems

1. Calculate the ***perimeter*** and ***area*** of each of the following shapes.

(a)



(b)



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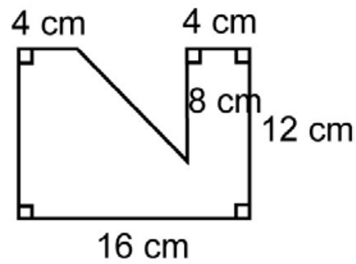

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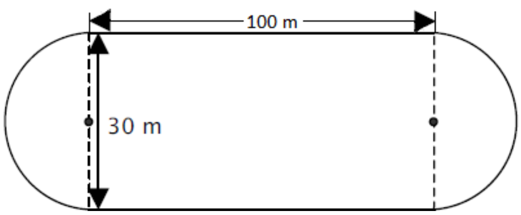

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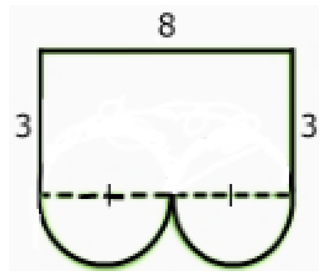

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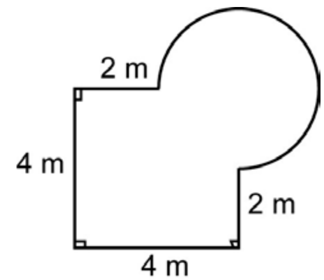
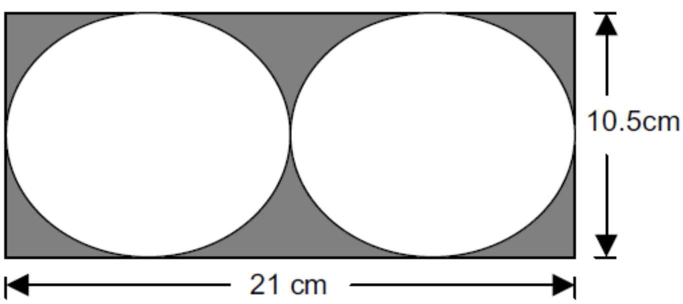


2. Calculate the area of the ***shaded region***.

(a)



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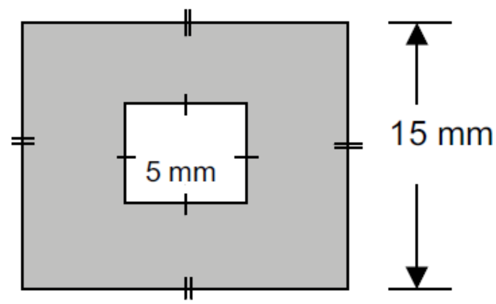

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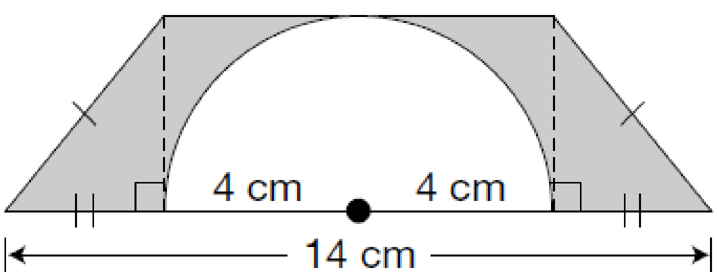

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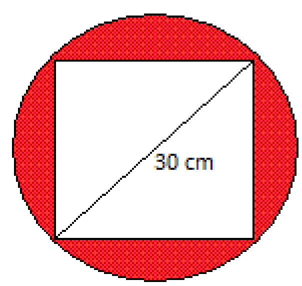
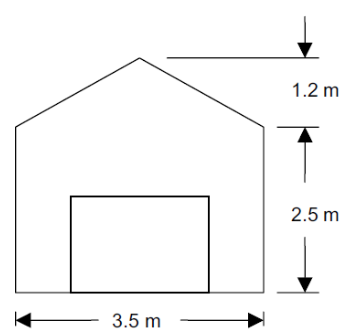


3. The front of a garage, excluding the door, needs to be painted.

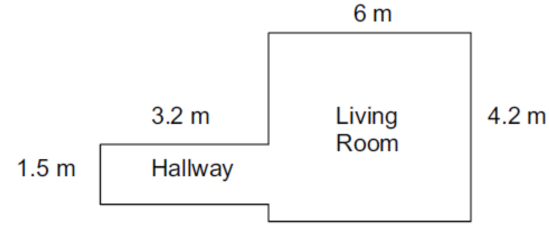
(a) Calculate the area of the region that needs to be painted assuming that the door is 1.5m high and 2 m wide. (See the diagram at the right for all other dimensions.)

(b) If one can of paint covers an area of 2.5 m^2, how many cans will need to be purchased?

(c) If one can of paint sells for $19.89, how much will it cost to buy the paint? (Include 13% HST.)



4. Bill wishes to replace the carpet in his living room and hallway with laminate flooring. A floor plan is shown at the right.



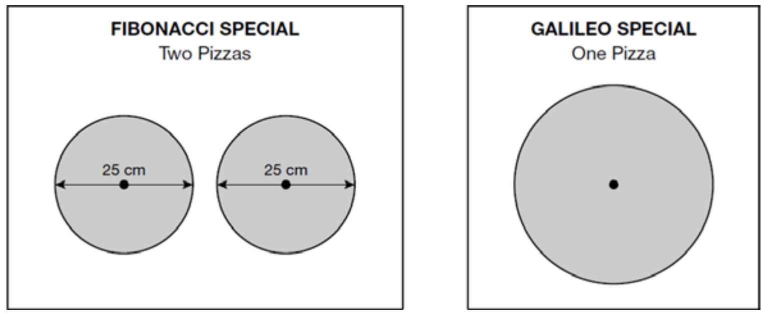
(a) Find the total area of floor to be covered.

(b) Laminate flooring comes in boxes that contain 2.15m^2 of material. How many boxes will Bill require?

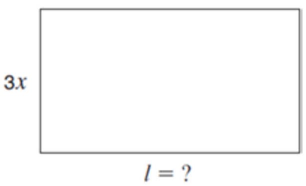
(c) One box costs \$43.25. How much will the flooring cost? (Include 13% HST.)

(d) When laying laminate flooring, it is estimated there will be 5% waste. How much waste can Bill expect?

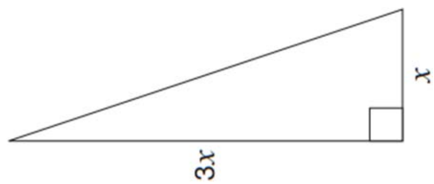
5. As shown in the diagram, a pizza shop has a weekend special. What should be the diameter of the Galileo Special pizza so that both specials contain the same amount of pizza? (You may assume that the pizzas all have the same thickness.)



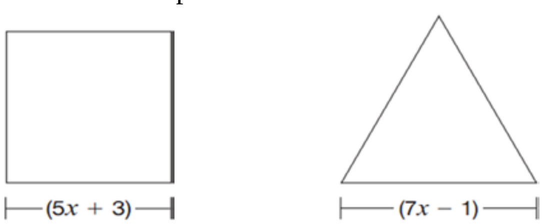
6. The area of the following rectangle is $6xy^2$ square units and its width is $3x$ units. What is its length?



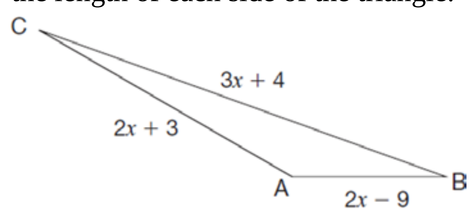
7. The are of the following triangular garden is 96 m^2. Determine the value of x.



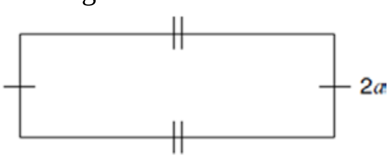
8. The following square and equilateral triangle both have the same perimeter. Find the value of x.



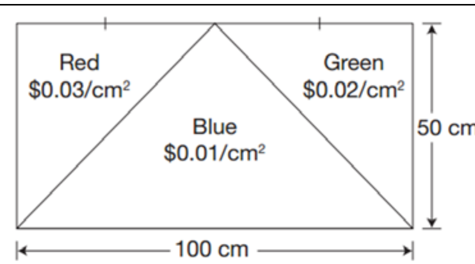
9. The perimeter of the following triangle is 75 m. Determine the length of each side of the triangle.



10. The following rectangular field has a width of $2a$ metres and a perimeter of $10a - 6$ metres. What is the length of the field?



11. A flag that is to be made of three different materials. What is the total cost of the flag?



Answers

1. **(a)** $P \doteq 65.3$ m, A=230 m^2 **(b)** P=26, $A \doteq 24.5$ **(c)** $P \doteq 67.3$ cm, A=160 cm^2 **(d)** $P \doteq 294.2$ m, $A \doteq 3706.5$ m^2 **(e)** $P \doteq 26.56$, $A \doteq 36.56$ **(f)** $P \doteq 21.42$ m, $A \doteq 25.42$ m^2	**2.** **(a)** $A \doteq 47.3$ m^2 **(b)** A=200 mm^2 **(c)** $A \doteq 18.9$ cm^2 **(d)** $A \doteq 257$ cm^2 **3.** **(a)** A=7.85 m^2 **(b)** 4 cans **(c)** \$89.90
4. **(a)** A=30 m^2 **(b)** At least 14 boxes, probably 15 **(c)** \$684.22 **(d)** About 1.5 m^2 of waste ($\therefore$ 15 boxes are needed) **5.** About 35.4 cm **6.** $l = 2y^2$ units **7.** $x = 8$	**8.** $x = 15$ **9.** $x = 11$ lengths of sides: 13, 25, 37 **10.** $l = 3a - 3$ units **11.** Red: \$37.50 Green: \$25.00 Blue: \$25.00 **Total: \$87.50**

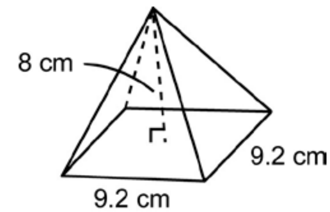
VOLUME AND SURFACE AREA PROBLEMS

1. A cone has radius of 8 cm and slant height of 10 cm. What is its surface area to the nearest tenth of a cm^2?

A 670.2 cm^2 **B** 804.2 cm^2 **C** 452.4 cm^2 **D** 640 cm^2

2. What is the volume of this pyramid, to the nearest tenth of a cubic centimetre?

A 677.1 cm^3 **B** 231.8 cm^3 **C** 338.6 cm^3 **D** 225.7 cm^3

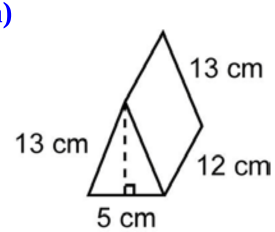


3. A sphere has radius 7 cm. What is its volume to the nearest tenth of a cubic centimetre?

A 1436.8 cm^3 **B** 615.8 cm^3 **C** 4310.3 cm^3 **D** 205.3 cm^3

4. Find the ***surface area*** and ***volume*** of each object. Round your answers to one decimal place.

(a)



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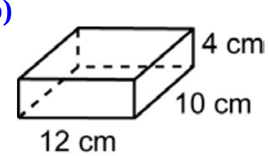

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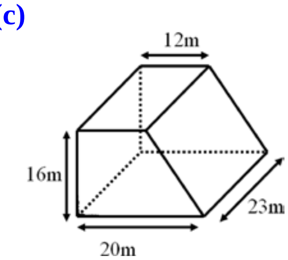

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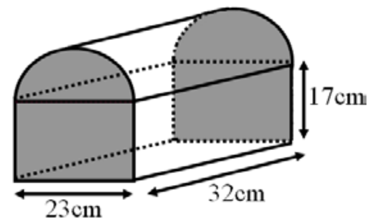
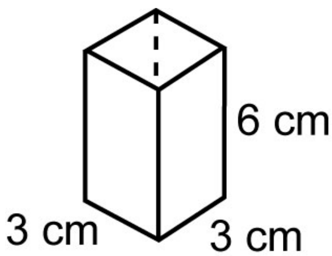


5. The shape at the right is a

____________________ .

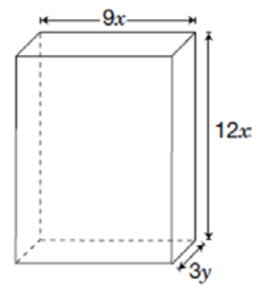
What is the maximum volume of a cone that would fit in this shape?



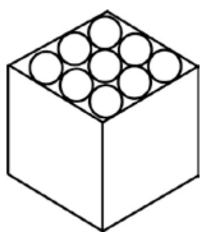
6. The shape at the right is a

____________________ .

What is the volume of this shape?



7. Carmine is packing 27 superballs in 3 square layers. Each ball has diameter 4 cm.

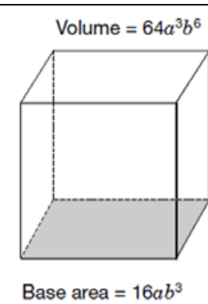


(a) What is the minimum volume of the box?

(b) What is the surface area of the box?

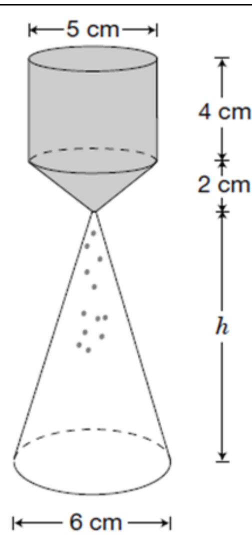
(c) How much empty space is in the box?

8. Expressions are given for the volume and base area of a rectangular prism. What is the height of the prism?



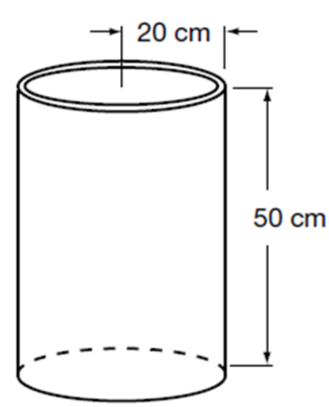
9. As shown below, sand is being poured from one container to another. The sand flows from the shaded part to the unshaded cone. The shaded part starts full of sand. By the time the shaded part is empty, the unshaded cone is filled to the top.

What is the height of the unshaded cone?

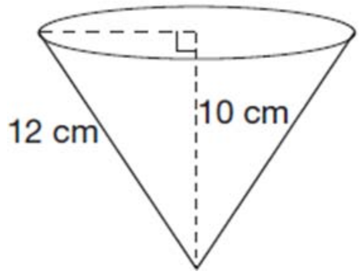


10. Brad has a cylindrical container that is open at the top. He wants to paint the outer surfaces of the container, including the bottom.

Calculate the area of that surface that needs to be painted.

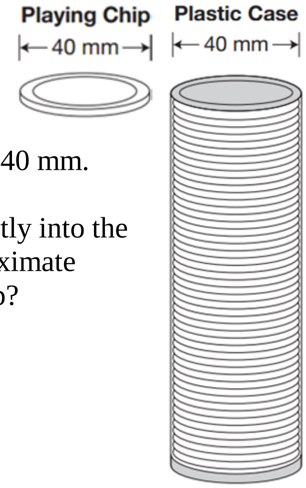


11. Calculate the surface area of the cone shown below.

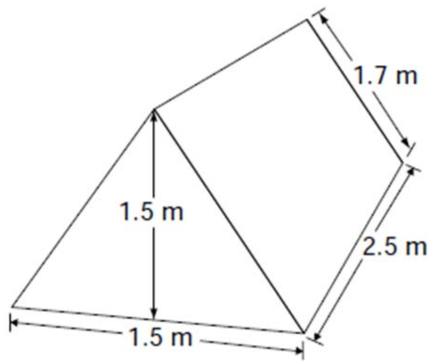


12. The playing chips of a board game are stored in cylindrical plastic cases. The plastic cases have a volume of 25120 mm^3 and a diameter of 40 mm.

If 50 playing chips can fit tightly into the plastic case, what is the approximate height (thickness) of each chip?

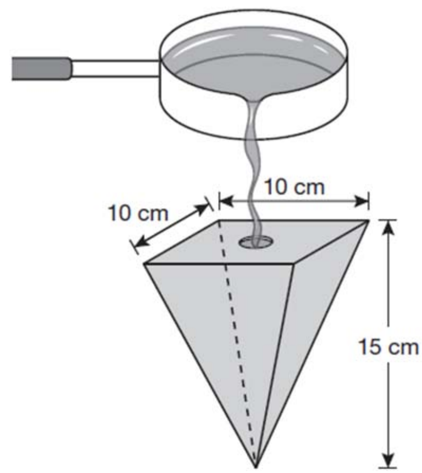


13. Including the ends and the floor, calculate the surface area of the tent shown below.



14. The mould shown at the right is used to make a candle in the shape of a square-based pyramid.

What is the volume of the mould?



Answers

1. C **2.** D **3.** A

4. **(a)** $A \doteq 435.8$ cm^2, $V \doteq 382.7$ cm^3 **(b)** A=416 cm^2, V=480 cm^3 **(c)** $A \doteq 2027.7$ m^2, V=5888 m^3 **(d)** $A \doteq 2850$ cm^2, $V \doteq 19156$ cm^3

5. square prism, 14.1 cm^3

6. rectangular prism, $324x^2y$ cubic units

7. **(a)** 1728 cm^3 **(b)** 864 cm^2 **(c)** 823.2 cm^3

8. $h = 4a^2b^3$ units

9. Vol. of shaded part = vol. of cylinder + vol. of small cone = $\pi(2.5)^2(4) + \dfrac{\pi(2.5)^2(2)}{3} \doteq 91.6 \doteq$ volume of unshaded cone

$\therefore$ height of unshaded cone = $h = \dfrac{3V}{\pi r^2} \doteq \dfrac{3(91.6)}{\pi(3)^2} \doteq 9.7$ cm

10. Area to be painted = area of bottom + area of lateral surface = $\pi(20)^2 + 2\pi(20)(50) \doteq 7540$ cm^2

11. First use the Pythagorean Theorem to calculate the radius of the cone ($r \doteq 6.6$). Then $A \doteq \pi(6.6)(12) + \pi(6.6)^2 \doteq 386$ cm^2

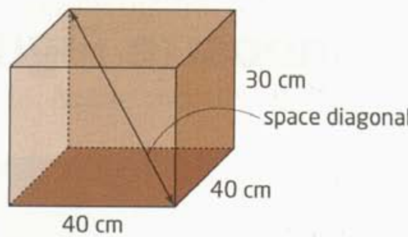
12. Each chip is about 0.4 mm thick.

13. The tent has a surface area of 14.5m^2

14. The mould has a volume of 500 cm^3

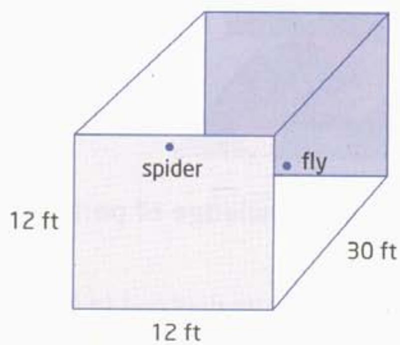
Extend

10. A cardboard box measures 40 cm by 40 cm by 30 cm. Calculate the length of the space diagonal, to the nearest centimetre.



11. The Spider and the Fly Problem is a classic puzzle that originally appeared in an English newspaper in 1903. It was posed by H.E. Dudeney. In a rectangular room with dimensions 30 ft by 12 ft by 12 ft, a spider is located in the middle of one 12 ft by 12 ft wall, 1 ft away from the ceiling. A fly is in the middle of the *opposite* wall 1 ft away from the floor. If the fly does not move, what is the shortest distance that the spider can crawl along the walls, ceiling, and floor to capture the fly?

Hint: Using a net of the room will help you get the answer, which is less than 42 ft!

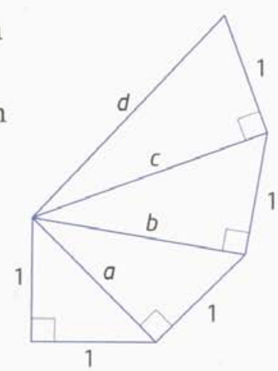


12. A spiral is formed with right triangles, as shown in the diagram.

a) Calculate the length of the hypotenuse of each triangle, leaving your answers in square root form. Describe the pattern that results.

b) Calculate the area of the spiral shown.

c) Describe how the expression for the area would change if the pattern continued.



13. Math Contest

a) The set of whole numbers (5, 12, 13) is called a *Pythagorean triple*. Explain why this name is appropriate.

b) The smallest Pythagorean triple is (3, 4, 5). Investigate whether multiples of a Pythagorean triple make Pythagorean triples.

c) Substitute values for m and n to investigate whether triples of the form $(m^2 - n^2, 2mn, m^2 + n^2)$ are Pythagorean triples.

d) What are the restrictions on the values of m and n in part c)?

Answers

10. 64 cm

11. 40 ft

12. a) $\sqrt{2}$; $\sqrt{3}$; $\sqrt{4}$; $\sqrt{5}$

b) $\frac{\sqrt{1}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{4}}{2}$

c) As you add right triangles to the spiral pattern, the area will increase by $\frac{\sqrt{\text{Number of Triangles}}}{2}$.

13. a) This name is appropriate because this set of three whole numbers satisfies the Pythagorean theorem.

b) Yes.

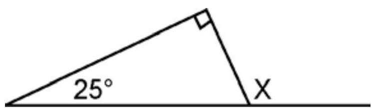
c) Yes, they are Pythagorean triples, with some restrictions on the values of m and n.

d) $m > n > 0$

Problems on Angle Relationships in Triangles

1. Find the measure of each indicated exterior angle.

(a)



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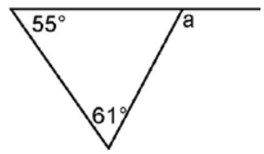

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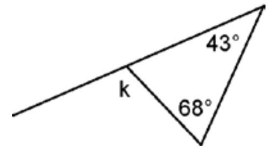

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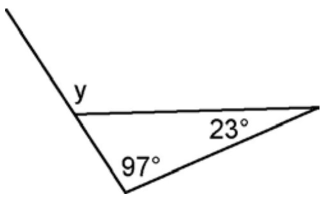

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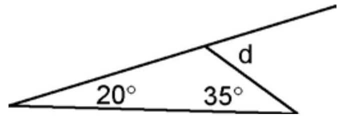

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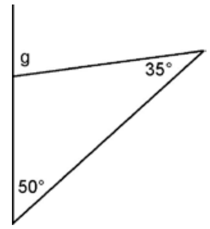
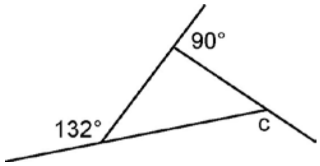


2. Find the measure of each indicated exterior angle.

(a)



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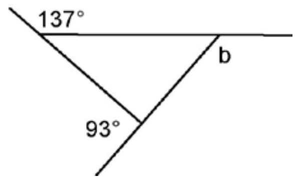

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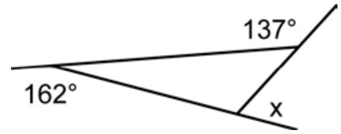

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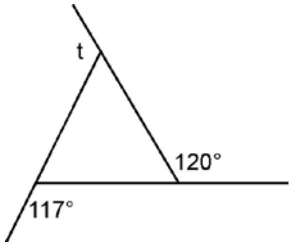

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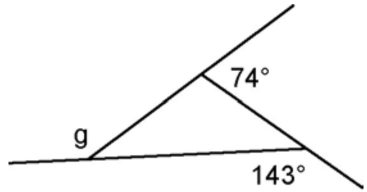

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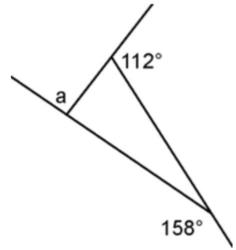
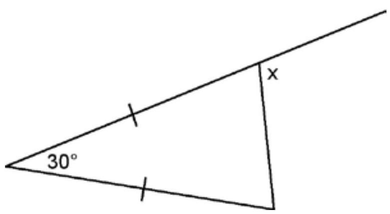


3. Find the measure of each indicated angle.

(a)



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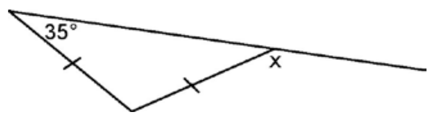
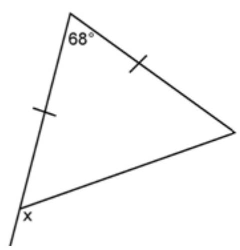
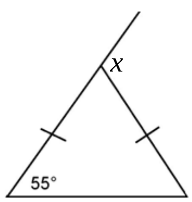


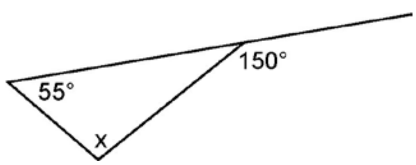
(c)



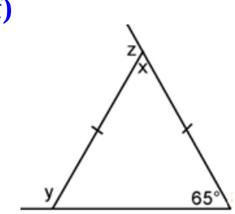
(d)



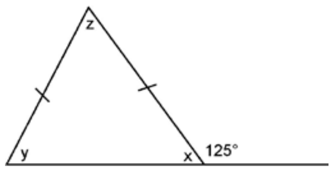
(e)



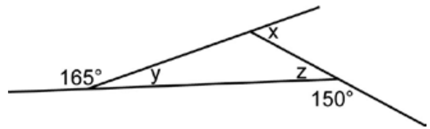
(f)



(g)



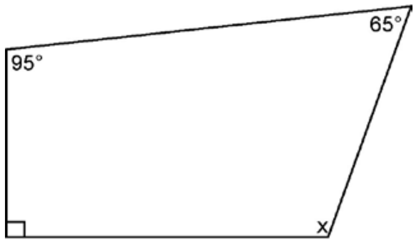
(h)



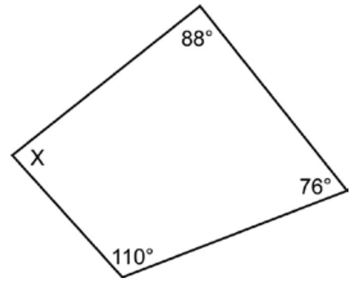
4. One interior angle in an isosceles triangle measures 42°. Find the possible measures for the exterior angles.

5. Find the measure of each indicated angle. **Hint:** Divide the quadrilaterals into triangles.

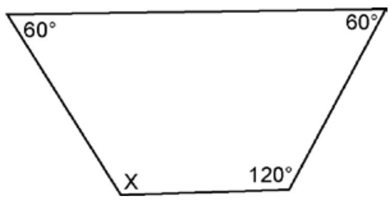
(a)



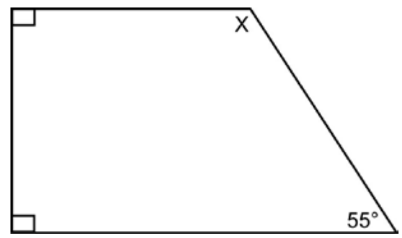
(b)



(c)

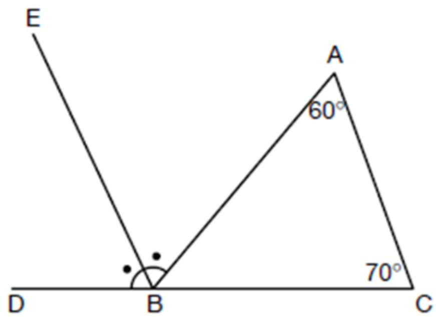


(d)

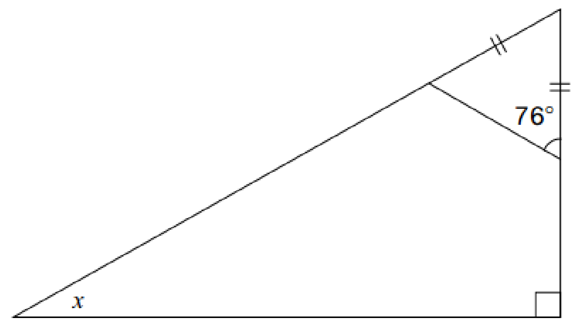


6. Find the measure of each indicated angle.

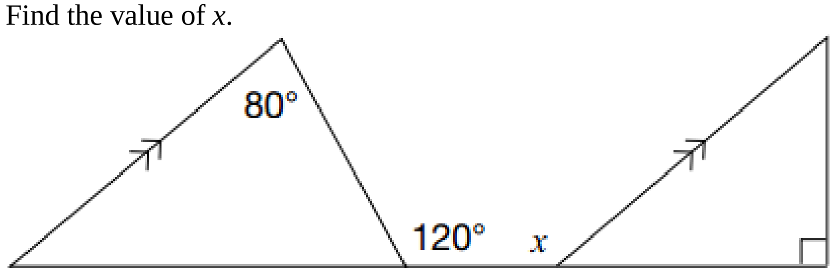
(a) In the following diagram, line segment *EB* ***bisects*** (divides into two equal angles) $\angle ABD$. What is the measure of $\angle ABE$.



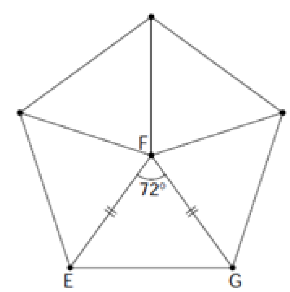
(b) Find the value of *x*.



(c) Find the value of *x*.



(d) What is the measure of $\angle FEG$?



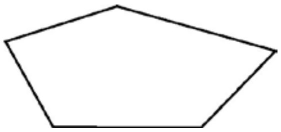
Answers

1. **a)** 115° **b)** 116° **c)** 111° **d)** 120° **e)** 55° **f)** 85°
2. **a)** 138° **b)** 130° **c)** 61° **d)** 123° **e)** 143° **f)** 90°
3. **a)** 105° **b)** 145° **c)** 124° **d)** 110° **e)** $x = 95°$ **f)** $x = 50°$; $y = 115°$; $z = 130°$ **g)** $x = y = 55°$; $z = 70°$ **h)** $x = 45°$; $y = 15°$; $z = 30°$
4. 138°, 138°, 84° or 138°, 111°, 111°
5. **a)** 110° **b)** 86° **c)** 120° **d)** 125°
6. **a)** 65° **b)** 62° **c)** 140° **d)** 54°

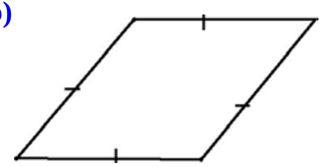
Problems on Angle Relationships in Polygons

1. Find the sum of the interior angles of each polygon.

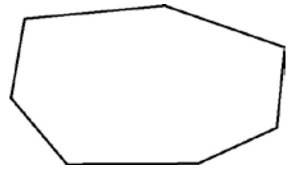
a)



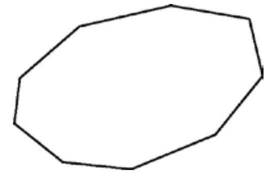
b)



c)

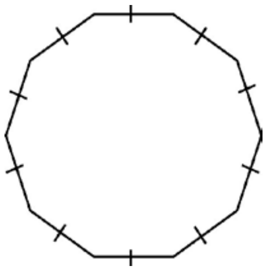


d)

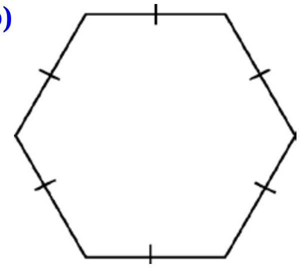


2. Find the sum of the interior angles of each polygon.

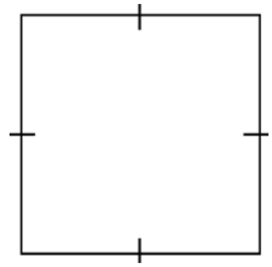
a)



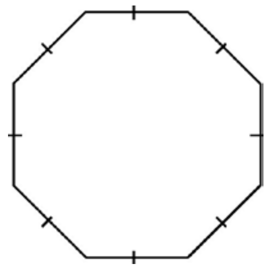
b)



c)



d)



3. Find the sum of the interior angles of a polygon with each number of sides.

a) 11 sides **b)** 14 sides
c) 18 sides **d)** 24 sides

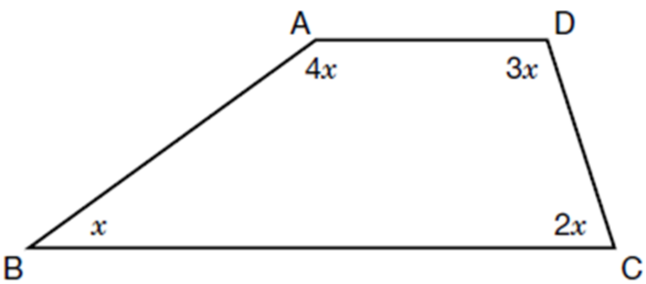
4. Find the measure of each interior angle of a ***regular polygon*** with each number of sides.

a) 3 sides **b)** 20 sides
c) 9 sides **d)** 16 sides

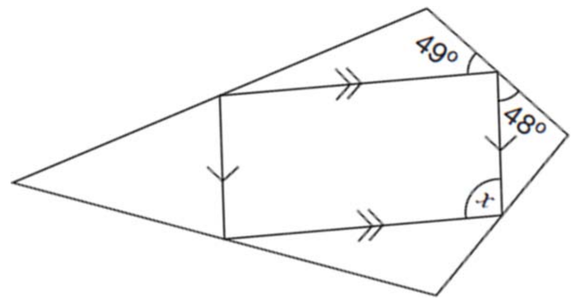
5. Find the number of sides in each polygon given the sum of its interior angles.

a) 720° **b)** 1980°
c) 2340° **d)** 4140°

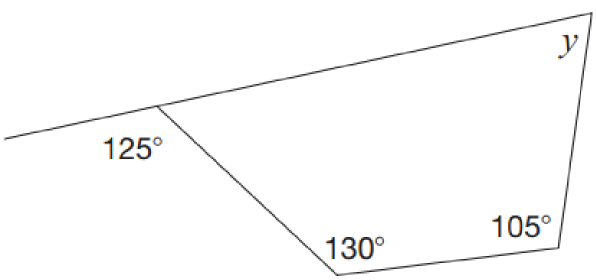
6. Determine the value of x.



7. Determine the value of x.

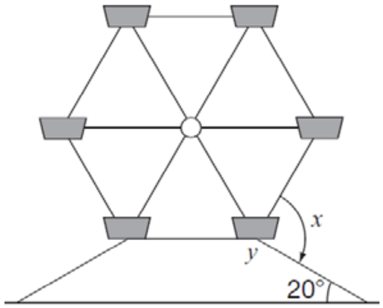


8. Determine the value of y.



9. A Ferris wheel has six sides of equal length. The exit ramp of the Ferris wheel is in the shape of a trapezoid and has an angle of incline of 20°.

Determine the values of x and y.



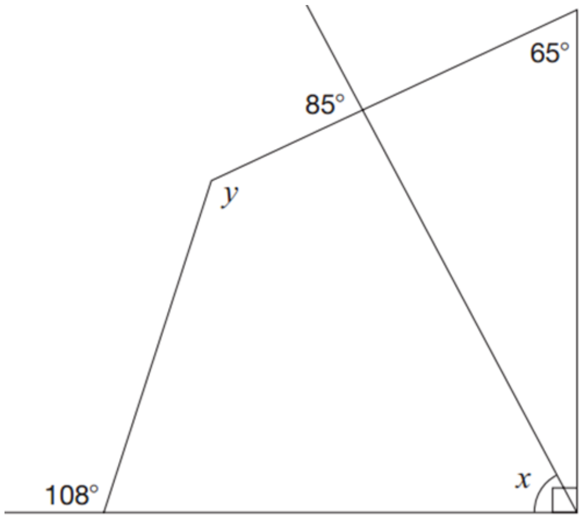
Answers

1. **a)** 540° **b)** 360° **c)** 900° **d)** 1260°
2. **a)** 1440° **b)** 720° **c)** 360° **d)** 1080°
3. **a)** 1620° **b)** 2160° **c)** 2880° **d)** 3960°
4. **a)** 60° **b)** 162° **c)** 140° **d)** 157.5°
5. **a)** 6 sides **b)** 13 sides
c) 15 sides **d)** 25 sides

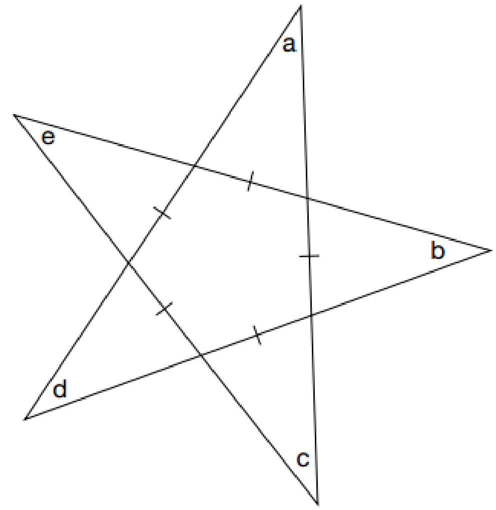
6. $x = 36°$
7. $x = 97°$
8. $y = 70°$
9. $x = 80°$, $y = 160°$

More Challenging Problems on Angle Relationships in Polygons

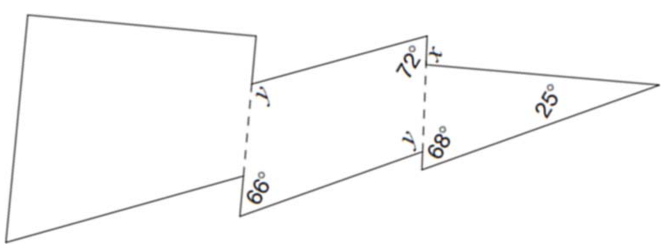
1. Determine the values of *x* and *y*.



2. Determine the value of $a+b+c+d+e$.



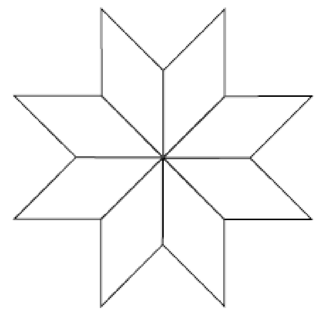
3. Pravin designs a lightning bolt using two quadrilaterals and one triangle as shown below. Determine the values of *x* and *y*.



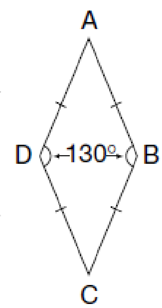
4. An eight-pointed star quilt is to be made using quilt pieces exactly like the one shown at the far right.

Is it possible to make the quilt using pieces like the given piece? Explain.

Eight-Pointed Star



Quilt Piece



Answers

1. $x=60°$, $y=133°$

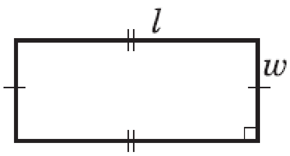
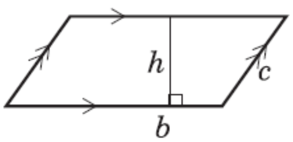
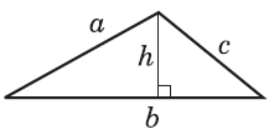
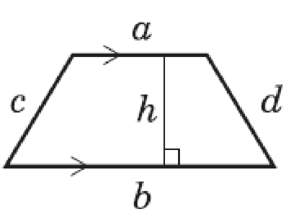
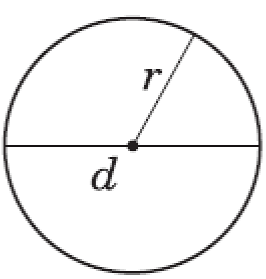
2. $a+b+c+d+e=180°$

3. $x=93°$, $y=111°$

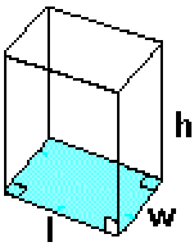
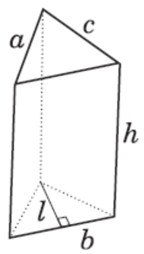
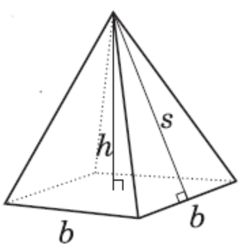
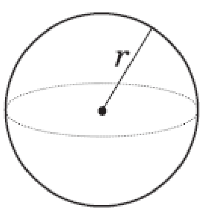
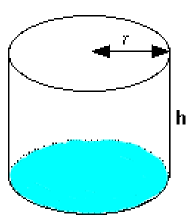
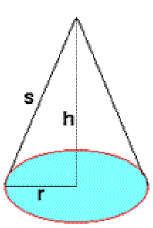
4. Since eight pieces are needed to make the quilt, $\angle DAB$ and $\angle DCB$ should both have a measure of $\frac{360°}{8}=45°$. Using the fact that the sum of the interior angles of a quadrilateral is 360°, it follows that $\angle DAB$ and $\angle DCB$ actually both have a measure of $\frac{360°-2(130°)}{2}=50°$. Therefore, it is ***not possible*** to make the quilt using the given pieces.

What Happens If…

1. Complete the following table. The first row has been done for you.

Shape	***Name of the Shape***	***What Happens to the Perimeter if …***	***What Happens to the Area if …***
	Rectangle	…the length is doubled? ***Solution*** $P = 2l + 2w$ If the length is doubled, the new length is $2l$. Then, the perimeter becomes $P = 2(2l) + 2w = 4l + 2w = (2l + 2w) + 2l$ The perimeter increases by $2l$.	…the width is tripled? ***Solution*** $A = lw$ If the width is tripled, the new width is $3w$. Then, the area becomes $A = l(3w) = 3lw = 3(lw)$ The area is also tripled.
		…the base is doubled?	…the height is quadrupled?
		…the base is tripled? (If this can be done without changing the values of *a* and *c*.)	…the height is tripled?
		…the base is tripled? (If this can be done without changing the values of *c* and *d*.)	…the height is doubled?
		…the radius is doubled?	…the radius is doubled?

2. Complete the following table. The first row has been done for you.

Shape	***Name of the Shape***	***What Happens to the Surface Area if …***	***What Happens to the Volume if …***
	Rectangular Prism	…the length is doubled? ***Solution*** $A = 2lw + 2lh + 2wh$ If the length is doubled, the new length is $2l$. Then, the surface area becomes $A = 2(2l)w + 2(2l)h + 2wh$ $= 4lw + 4lh + 2wh$ $= (2lw + 2lh + 2wh) + 2lw + 2lh$ The surface area increases by $2lw + 2lh$.	…the width is tripled? ***Solution*** $V = lwh$ If the width is tripled, the new width is $3w$. Then, the volume becomes $V = l(3w)h = 3lwh = 3(lwh)$ The volume is also tripled.
		…*b* is doubled? (If this can be done without changing the values of *a* and *c*.)	…the height is quadrupled?
		…the slant height is tripled?	…the height is tripled?
		…the radius is doubled?	…the radius is doubled?
		…the radius is doubled?	…the radius is doubled?
		…the radius is doubled?	…the radius is doubled?

OPTIMIZATION PROBLEMS

Definition: Optimize

- Make ***optimal*** (i.e. the best, most favourable or desirable, ***especially under some restriction***); get the most out of; use best
- In a mathematical context, to ***optimize*** means either to ***maximize*** (make as great as possible) or to ***minimize*** (make as small as possible), subject to a restriction called a ***constraint***.

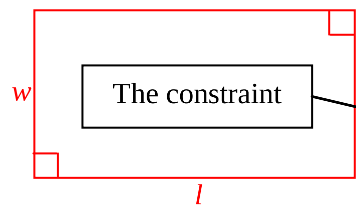
Optimization Problem 1

You have 400 m of fencing and you would like to enclose a rectangular region of ***greatest possible area***. What dimensions should the rectangle have?

(a) What is the ***constraint*** in this problem?

The constraint is the length of fencing available. Since only 400 m of fencing are available, the region enclosed by the fence will have a limited size.

(b) Draw a diagram describing this situation. Then write an equation that relates the unknowns to the constraint.



Length of fencing = 400 m

$\therefore$ perimeter = 400 m

$\therefore \boxed{2l + 2w = 400}$

(c) What quantity needs to be optimized? State whether the quantity needs to be maximized or minimized. Then write an equation describing the quantity that needs to be optimized.

In this problem, the area needs to be ***maximized***. Therefore, the equation must describe the area of the rectangular region.

$$A = lw$$

(d) The equation in (c) cannot be used directly to maximize the area because there are too many variables. Use the constraint equation to solve for l in terms of w. Then rewrite the equation in (c) in such a way that A is expressed entirely in terms of w.

$\because 2l + 2w = 400$

$\therefore \frac{2l}{2} + \frac{2w}{2} = \frac{400}{2}$

$\therefore l + w = 200$

$\therefore l + w - w = 200 - w$

$\therefore l = 200 - w$

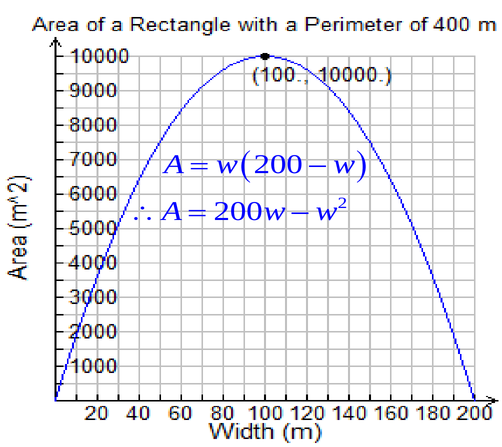
$\therefore A = lw$

$\therefore A = (200 - w)w$

$\therefore A = w(200 - w)$

Now the area has been expressed ***in terms of one variable only*** (i.e. the width).

(e) Sketch a graph of area of the rectangle versus width. Label the axes and include a title.



(f) Is the relationship between A and w linear or non-linear? Give ***three*** reasons to support your answer.

The relation is ***non-linear***. We know this because of the following reasons.

- The graph is curved.
- The equation has a squared term (w^2).
- The first differences are ***not constant***.

w	A	ΔA
20	3600	-
30	5100	1500
40	6400	1300
50	7500	1100
60	8400	900

(g) State the dimensions of the rectangular region having a perimeter of 400 m and a ***maximal*** area.

From the graph it can be seen that the maximum area is 10000 m^2, which is attained when the width is 100 m. Therefore, for maximum area,

$w = 100$ and $l = 200 - w = 200 - 100 = 100$.

For maximal area, both the length and the width should be 100 m. In other words, the region should be a square with side length of 100 m.

Optimization Problem 2

Design a cylindrical pop can that has the ***greatest possible capacity*** but can be manufactured using at most 375 cm^2 of aluminum.

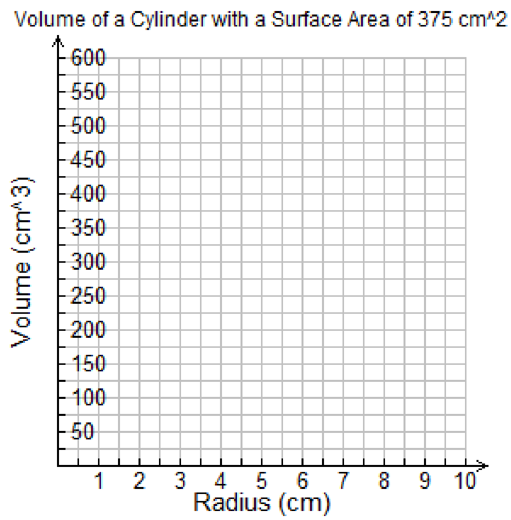
(a) What is the ***constraint*** in this problem?

(b) Draw a diagram describing this situation. Then write an equation that relates the unknowns to the constraint. (Let *r* represent the radius of the cylinder and let *h* represent its height.)

(c) What quantity needs to be optimized? State whether the quantity needs to be maximized or minimized. Then write an equation describing the quantity that needs to be optimized.

(d) The equation in (c) cannot be used directly to maximize the volume because there are too many variables. Use the constraint equation to solve for *h* in terms of *r*. Then rewrite the equation in (c) in such a way that *V* is expressed entirely in terms of *r*.

(e) Sketch a graph of volume of the cylindrical can versus radius. Label the axes and include a title.



(f) Is the relationship between *V* and *r* linear or non-linear? Give ***three*** reasons to support your answer.

r	*V*	ΔV

(g) State the dimensions of the cylindrical can having a surface area of 375 cm^2 and a ***maximal*** volume.

Optimization Problem 3

A container for chocolates must have the shape of a ***square prism*** and it must also have a volume of 8000 cm^3. Design the box in such a way that it can be manufactured using the ***least amount of material***.

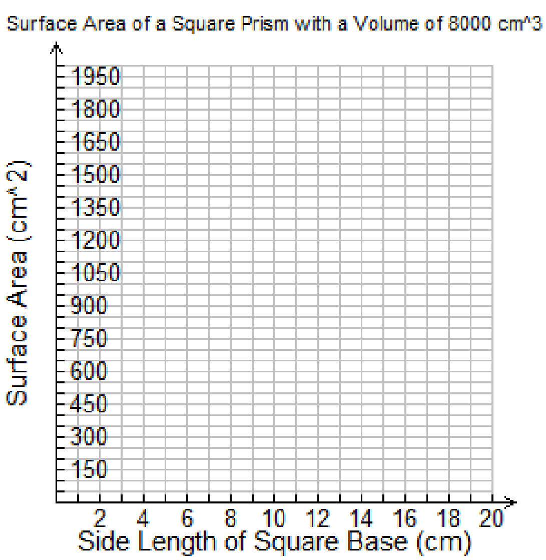
(a) What is the ***constraint*** in this problem?

(b) Draw a diagram describing this situation. Then write an equation that relates the unknowns to the constraint. (Let x represent the side length of the square base and let h represent the height.)

(c) What quantity needs to be optimized? State whether the quantity needs to be maximized or minimized. Then write an equation describing the quantity that needs to be optimized.

(d) The equation in (c) cannot be used directly to ***minimize*** the surface area because there are too many variables. Use the constraint equation to solve for h in terms of x. Then rewrite the equation in (c) in such a way that A is expressed entirely in terms of x.

(e) Sketch a graph of volume of the square prism versus width. Label the axes and include a title.



(f) Is the relationship between A and x linear or non-linear? Give ***three*** reasons to support your answer.

x	A	ΔA

(g) State the dimensions of the square prism with a volume of 8000 cm^3 and a ***minimal*** surface area.