

Grade 9 Pre-AP Math
Unit 0 – Introduction to Mathematical Thinking – Major Test

+1 Bonus

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Victim: Mr. Solutions*Mr. S. has done it again!*

KU	APP	TIPS	COM
10/10	20/20	14/14	10/10

Modified True or False (5 KU)Indicate whether each statement is **true** or **false**. If false, **change** the underlined part to make the statement true.1. F ✓ Math is like a dating service because it's all about relativism.Change: relationships ✓2. F ✓ If a cone has a volume of 90 cm^3 , then the volume of a cylinder with the same radius and height is 30 cm^3 .Change: 270 ✓3. F ✓ A pentagonal pyramid has six **lateral** faces.Change: 5 ✓4. F ✓ The basic elements of math are objections, operations and relationships.Change: objects ✓5. F ✓ $\frac{(A_{\text{base}})(\text{height})}{3}$ is the volume of any cone or cylinder.Change: pyramid ✓**Multiple Choice (5 KU)**

For questions 6 to 10, select the best answer. Write the letter of your choice in the provided blank space.

6. C ✓ A cone has a **volume** of 314.16 cm^3 and a **height** of 5 cm. To one decimal place, what is its **radius**?

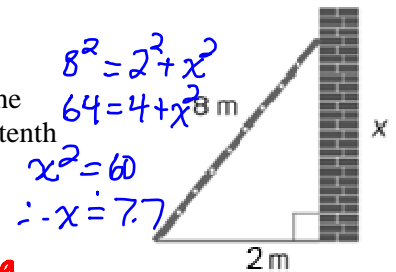
- (a) 60.0 cm (b) 20.0 cm (c) 7.7 cm (d) 12.0 cm

Too large!!

$$\frac{1}{3}\pi(7.7)^2(5) \doteq 310$$

7. C ✓ A window cleaner has placed an 8-m ladder against a wall. The bottom of the ladder is 2 m away from the wall. How high is the top of the ladder, to the nearest tenth of a metre, above the ground?

- (a) 60.0 m (b) 6 m (c) 7.7 m (d) 10 m

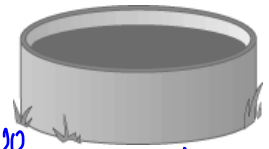
8. b ✓ A circular swimming pool has a diameter of 20 m. When filled completely the pool holds exactly 100,000 L of water. To one decimal place, what is the height of the pool?

- (a) 318.3 m (b) 3.2 m (c) 0.3 m (d) 79.6 m

$$100,000 \text{ L} = 100 \text{ m}^3$$

$$\pi(10)^2 h = 100$$

$$100\pi h = 100 \rightarrow h = \frac{1}{\pi} \doteq 0.3$$

9. C ✓ Which statement is **not** true?

- (a) The sum of the interior angles of a pentagon is $3(180^\circ)$.
 (b) The sum of the interior angles of a triangle is 180° .
 (c) The sum of the exterior angles of a triangle is 180° .
 (d) The sum of the exterior angles of a convex polygon is 360° .

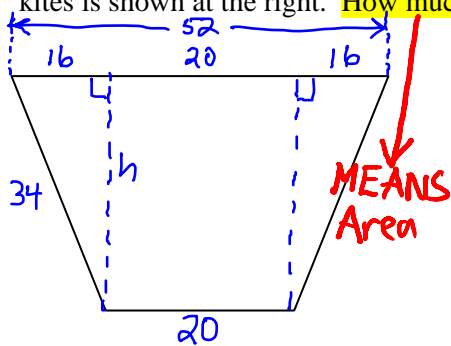
*original answer wrong**Must be 360° because a Δ is a convex polygon.*10. d ✓ When a transversal intersects a pair of parallel lines, which of the following is true?

- (a) Co-interior angles are supplementary.
 (b) Corresponding angles are equal.
 (c) Alternate angles are equal.
 (d) All of the above.

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Full Solutions/Explanations

11. Fatima and Nistha have a kite-making business that competes with Vyshna's and Uday's VUiTon Fashionable Kites. Their company is called FuNky kites because they specialize in oddly-shaped kites. The shape of one of their funkiest kites is shown at the right. **How much material** would be needed to make one of these kites? (6 APP)



By the Pythagorean Theorem,

$$h^2 + 16^2 = 34^2 \checkmark$$

$$\therefore h^2 = 34^2 - 16^2$$

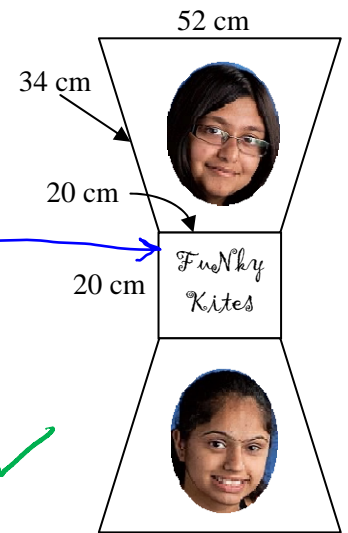
$$\therefore h^2 = 900$$

$$\therefore h = \sqrt{900} = 30 \checkmark$$

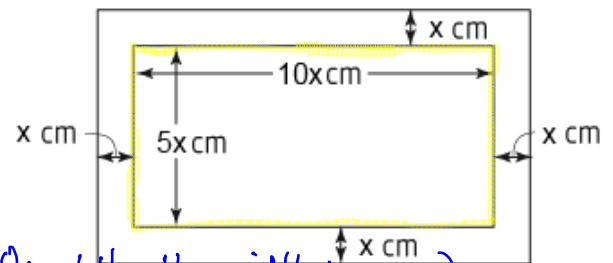
$$\begin{aligned} A_{\text{Trapezoid}} &= \frac{h(a+b)}{2} \\ &= \frac{30(52+20)}{2} \\ &= 1080 \text{ cm}^2 \checkmark \end{aligned}$$

$$\begin{aligned} A_{\text{square}} &= lw \\ &= 20(20) \\ &= 400 \text{ cm}^2 \checkmark \end{aligned}$$

$$\begin{aligned} \therefore A_{\text{kite}} &= 2A_{\text{Trapezoid}} + A_{\text{square}} \checkmark \\ &= 2(1080) + 400 \\ &= 2560 \text{ cm}^2 \checkmark \end{aligned}$$



12. A picture is framed with a frame of unknown width, x cm. The length of the picture is 10 times the width of the frame and the width of the picture is five times the width of the frame. If the perimeter of the picture is 120 cm, find the width of the frame and the dimensions of the picture. (6 APP)



The length of the picture must be twice the width (because the length is $10x$ while the width is $5x$)

By trial and error, we find that $l_{\text{picture}} = 40$, $w_{\text{picture}} = 20$. \checkmark

To verify this we note that

$$2(40) + 2(20) = 120 = P_{\text{picture}} \text{ and that } 40 \text{ is indeed double } 20. \checkmark$$

$$\text{Therefore, } 5x = 20$$

$$\therefore x = 4 \checkmark$$

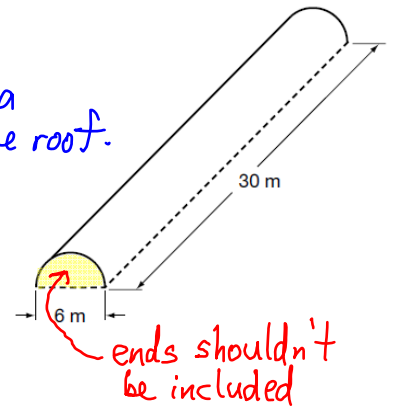
The dimensions of the picture are 40 cm \times 20 cm and the width of the frame is 4 cm. \checkmark

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13. The figure shows a greenhouse roof that is shaped like a half-cylinder. What is the surface area of the roof? (4 APP)

The "ends" should not be included in the surface area because vertical surfaces are not considered part of the roof.

$$\begin{aligned}\therefore A_{\text{Roof}} &= \frac{2\pi r h}{2} \quad \leftarrow + 2\pi r^2 \text{ not included because this term represents the area of the circular base and top} \\ &= \pi r h \quad \leftarrow \div 2 \text{ because this is a half-cylinder} \\ &= 3.14(3)(30) \\ &= 282.6 \text{ m}^2\end{aligned}$$



The surface area of the roof is about 282.6 m^2 .

14. Given that $\triangle ABC$ is equilateral, find the measure of $\angle ADC$.

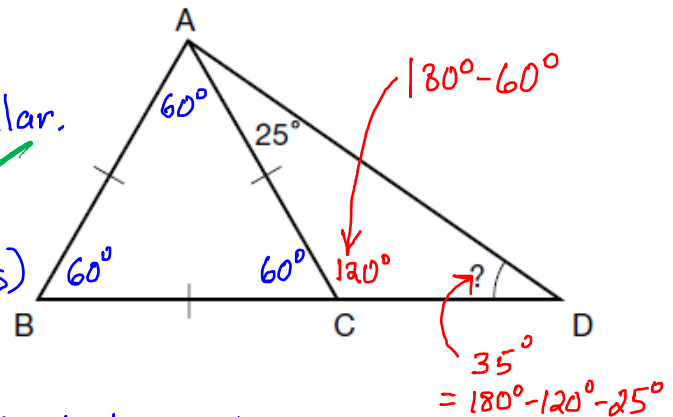
Explain your reasoning. (4 APP)

Since $\triangle ABC$ is equilateral, it is also equiangular.

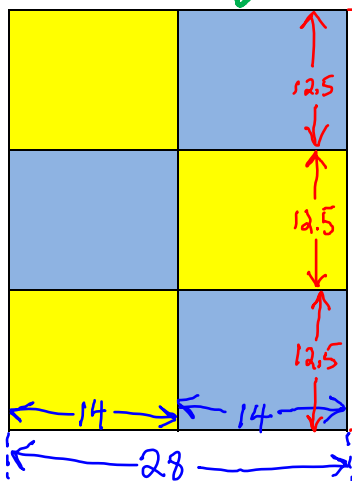
$$\therefore \angle ABC = \angle CAB = \angle BCA = \frac{180^\circ}{3} = 60^\circ$$

$$\begin{aligned}\therefore \angle ACD &= 180^\circ - \angle BCA \quad (\text{supplementary angles}) \\ &= 180^\circ - 60^\circ \\ &= 120^\circ\end{aligned}$$

$$\begin{aligned}\therefore \angle ADC &= 180^\circ - \angle CAD - \angle ACD \quad (\text{sum of interior angles of } \triangle \text{ is } 180^\circ) \\ &= 180^\circ - 120^\circ - 25^\circ \\ &= 35^\circ\end{aligned}$$



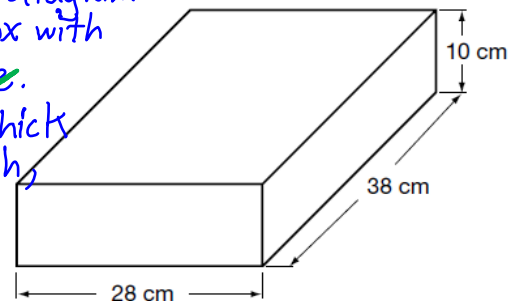
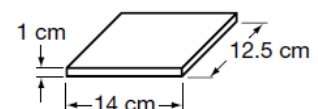
15. Elisa wants to pack CD cases into a storage box. What is the largest number of CD cases that she can pack inside the box? Explain your reasoning. (6 TIPS)



① Six CD's can be arranged as shown in the diagram and just fit inside the box with very little room to spare.

② Since the CD's are 1 cm thick and the box is 10 cm high, 10 such layers can be stacked up in the box.

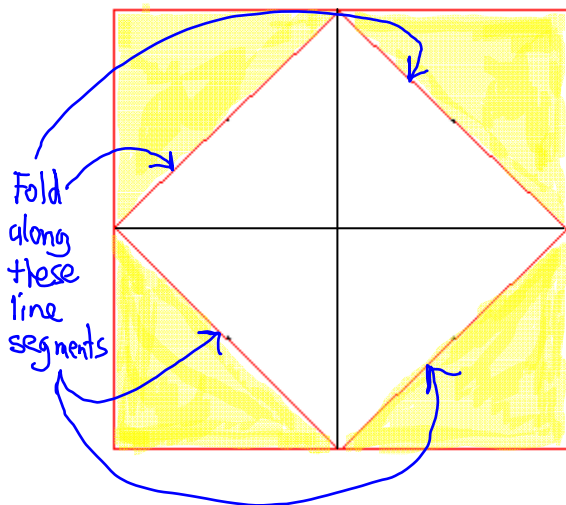
③ Therefore, the box should be able to hold about $10(6) = 60$ CD's.



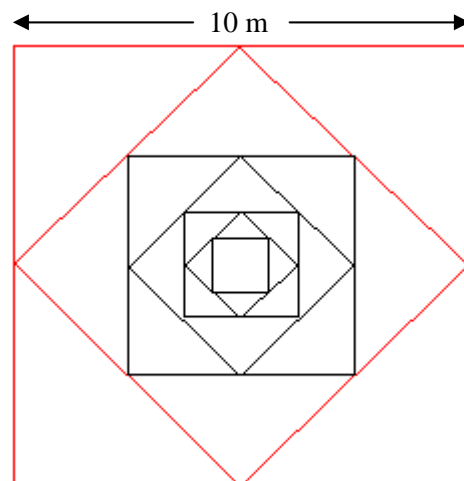
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16. Consider the diagram at the right. It consists of a series of squares within squares. Each square is formed by connecting the midpoints of the square that is immediately larger than it.

- (a) Explain why the area of each square is exactly **half** of the area of the square that is immediately larger than it. (Hint: There is a way of demonstrating this that involves much less work than using the Pythagorean Theorem!) (4 TIPS)



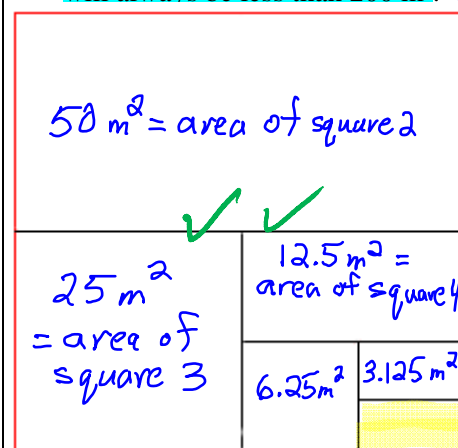
As shown in the diagram, "fold" along the sides of the smaller square. The highlighted triangles fit EXACTLY over the triangles within the smaller square. This means that the highlighted area is equal to the area of the smaller square. Therefore, the area of the smaller square is HALF the area of the larger square.



- (b) Complete the following table. (The squares are numbered according to their size. Square 1 is the largest, square 2 is the second largest, square 3 is the third largest and so on.) (2 TIPS)

Square Number	Area (m^2)
1	100
2	50
3	25
4	12.5
5	6.25
6	3.125

- (c) Imagine that the process of forming squares within squares continues indefinitely. This means that the squares get smaller and smaller and that there are infinitely many of them. Explain why no matter how many squares are formed, the **total area** of all the squares will always be less than 200 m^2 . (4 TIPS)



The highlighted rectangle shows the area left over after the areas of squares 2 to 6 are used to cover as much of the square as possible. When further areas of squares are added, the highlighted area will never be filled completely because the area that is added is always equal to half of what remains.

- (d) **Bonus Question.** You are not required to attempt this question. If you do attempt it and the quality of your response warrants it, you will receive extra credit.

Write an equation that relates the area of a square to its square number.

n = square number, A = area

$$A = 100 \left(\frac{1}{2}\right)^{n-1} + 1 \text{ BONUS}$$

e.g. If $n=3$, $A = 100 \left(\frac{1}{2}\right)^{3-1} = 100 \left(\frac{1}{2}\right)^2 = 100 \left(\frac{1}{4}\right) = \frac{100}{4} = 25$

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