

Grade 9 Pre-AP Math

Unit 0 – Introduction to Mathematical Thinking – Major Test

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Victim: Mr. Solutions*Once again, your work is impressive Mr. S.!!*

KU	APP	TIPS	COM
10/10	26/26	16/16	10/10

Modified True or False (6 KU)

Indicate whether each statement is **true** or **false**. If false, **change** the underlined part to make the statement true.

- F ✓ Math is like a dry cleaning service because it's all about relationships.
- F ✓ If a cone has a volume of 90 cm³, then the volume of a cylinder with the same radius and height is 30 cm³.
- F ✓ If a rectangle's width is not changed but its length is doubled its area is quadrupled.
- F ✓ The basic elements of math are objects, operatics and relationships. *✓ = 1/2 mark*
- F ✓ $\frac{(A_{\text{base}})(\text{height})}{3}$ is the volume of any cone or prism.
- F ✓ $(A_{\text{base}})(\text{height})$ is the volume of any solid with a uniform.

Change: dating ✓Change: 10 cm³ ✓Change: doubled ✓Change: operations ✓Change: pyramid ✓Change: uniform cross-section ✓

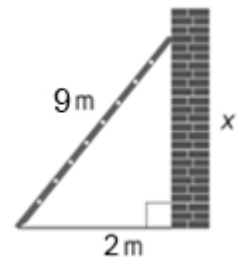
Multiple Choice (5 KU)

For questions 7 to 10, select the best answer. Write the letter of your choice in the provided blank space.

7. b ✓ Which of the following **is not** an expression for area? (r =radius, l =length, w =width, h =height, s =distance)
- (a) $4\pi r^2 + 2\pi rh$ (b) $2\pi r^2 + \pi r^2 h$ *Volume* (c) $2lh + 2wh + 10s^2$ (d) $\pi rs + 2\pi r^2$

8. b ✓ A window cleaner has placed an 9-m ladder against a wall. The bottom of the ladder is 2 m away from the wall. How high is the top of the ladder, to the nearest tenth of a metre, above the ground?

~~(a) 77.0 m~~ (b) 8.8 m (c) 7.7 m ~~(d) 7 m~~



9. a ✓ Which statement is **not** true?

(a) The sum of the interior angles of an octagon is ~~8~~(180°). *6(180°)*

(b) The measure of an exterior angle of a triangle is the sum of the measures of the two opposite interior angles. ✓

(c) The sum of the exterior angles of a convex hexagon is 360°. ✓

(d) The sum of the exterior angles of a convex octagon is 360°. ✓

} True for (EAT) ALL convex polygons

10. b ✓ When a transversal intersects a pair of parallel lines, which of the following is true?

~~(a) Co-interior angles are equal.~~ *supp.*

(b) Corresponding angles are equal. ✓

(c) Alternate angles are supplementary. ✓

~~(d) All of the above.~~

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Full Solutions/Explanations

11. Jinkle and Yashleen have a rug-making business that they call JinkLeen Rugs. Their company specializes in oddly-shaped rugs like the one shown at the right. (The diagram is NOT drawn to scale.) How much material would be needed to make this rug? (6 APP)

- ① Calculate the height of the trapezoid.

By the Pythagorean Theorem,

$$h^2 + 35^2 = 120^2$$

$$\therefore h^2 = 120^2 - 35^2$$

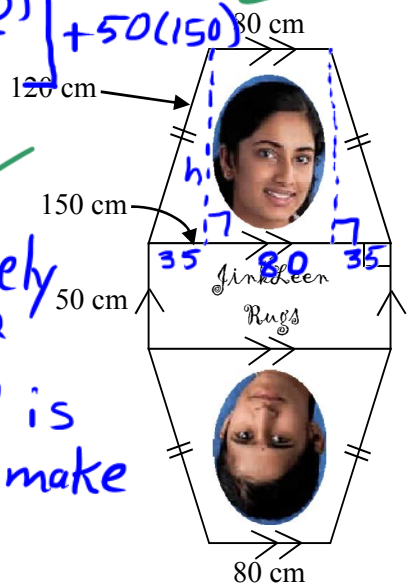
$$\therefore h^2 = 13175$$

$$\therefore h = \sqrt{13175} \approx 114.8$$

$$= 2 \left[\frac{114.8(80+150)}{2} \right] + 50(150)$$

$$= 33904$$

Approximately 33904 cm² of material is needed to make the rug.



- ② Calculate Total Area

$$A_{\text{Total}} = 2A_{\text{Trapezoid}} + A_{\text{rectangle}}$$

Sphere surface area: $4\pi r^2$
 $\therefore$ Hemisphere surface area: $2\pi r^2$

12. A hemispherical wooden bowl has an outer radius of 10 cm and an inner radius of 9 cm. Calculate the surface area of the bowl.

Hint: Include both the outer and the inner surfaces. (6 APP)

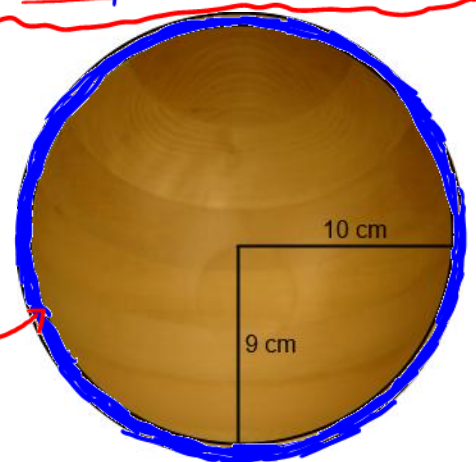
Total surface area

= Surface area of outer part of hemisphere + surface area of inner part of hemisphere + area of rim

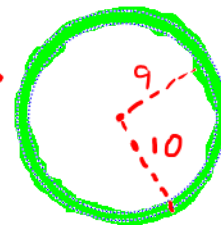
$$= 2\pi(10)^2 + 2\pi(9)^2 + \pi(10)^2 - \pi(9)^2$$

$$\approx 1196.9 \text{ cm}^2$$

The surface area of the bowl is about 1196.9 cm².



Top View of Bowl



Note This expression can also be simplified before being evaluated (will learn how to do this in algebra unit)

$$200\pi + 162\pi + 100\pi - 81\pi = 381\pi \approx 1196.9$$

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13. As shown at the right, sand is poured from one container to another. The sand flows from the shaded part to the unshaded cone. The shaded part starts full of sand and by the time it is empty, the unshaded cone is filled to the top. What is the height of the unshaded cone? (6 APP)

Since the cone is filled completely by the time the shaded part is empty, the two shapes must have the same volume. ✓

$$\therefore V_{\text{cone}} = V_{\text{cylinder}} + V_{\text{shaded cone}}$$

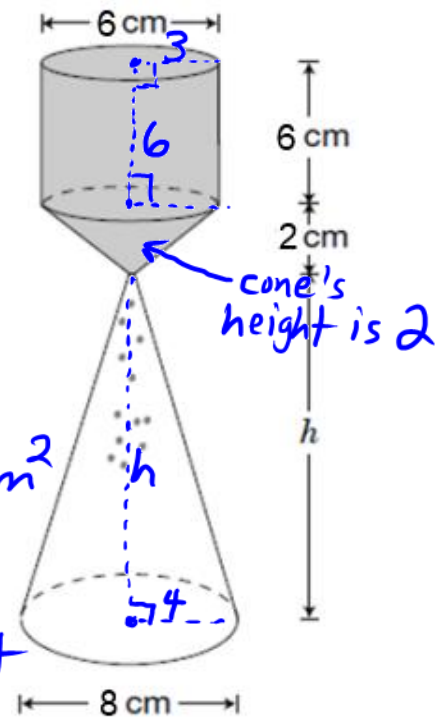
$$= \pi(3)^2(6) + \frac{1}{3}\pi(3)^2(2) \doteq 188.5 \text{ cm}^2$$

$$\therefore 188.5 \doteq \frac{\pi(4)^2 h}{3}$$

$$\therefore 188.5 \doteq \frac{16\pi h}{3}$$

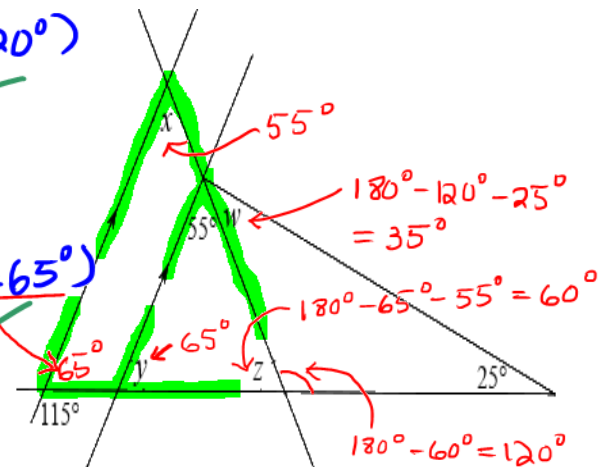
$$\therefore h \doteq \frac{3(188.5)}{16\pi} \doteq 11.25$$

The height of the cone is about 11.25 cm.



14. Find the measures of each angle labelled with a letter. In each case, state your reasoning. (8 APP)

Angle Measure	Reasoning (State Why)
$w = 35^\circ$ ✓	Supplementary angles ($180^\circ - 60^\circ = 120^\circ$) $w + 120^\circ + 25^\circ = 180^\circ$ (ASTT) ✓
$x = 55^\circ$ ✓	Corresponding angles are equal ($\overleftrightarrow{F\leftrightarrow}$) ✓
$y = 65^\circ$ ✓	Supplementary angles ($180^\circ - 115^\circ = 65^\circ$) Corresponding angles are equal ✓
$z = 60^\circ$ ✓	$z + y + 55^\circ = 180^\circ$ (ASTT) $\therefore z + 65^\circ + 55^\circ = 180^\circ$ ✓



15. Mr. Confusedalot needs your help to understand an expression shown to him by his six-year-old-genius daughter. She told him that the expression gives the perimeter, area or volume of a shape but she didn't tell him anything about the nature of the shape. Sketch a diagram of the shape for Mr. Confusedalot. Then explain to him why the expression gives the perimeter, area or volume of the shape.

Expression: $\frac{2}{3}\pi r^3 + \pi r^2 h + r^3$ (6 TIPS)

$\frac{2}{3}\pi r^3 \rightarrow$ volume of a hemisphere ✓

$\pi r^2 h \rightarrow$ volume of a cylinder ✓

$r^3 \rightarrow$ volume of a cube ✓

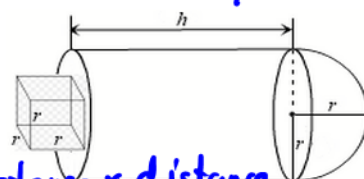


Volume is $\pi r^2 h$

Volume is $\frac{2}{3}\pi r^3$

Shape can be any combination of these three shapes.

e.g.

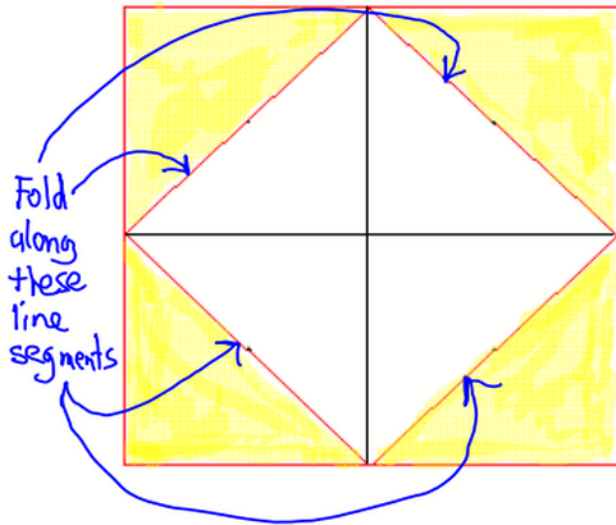


Each of these terms evaluates to a volume because each involves distance x distance x distance



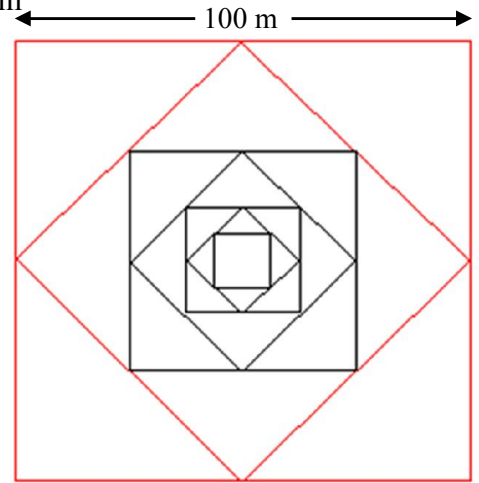
16. Consider the diagram at the right. It consists of a series of squares within squares. Each square is formed by connecting the midpoints of the square that is immediately larger than it.

- (a) Explain why the area of each square is exactly **half** of the area of the square that is immediately larger than it. (Hint: There is a way of demonstrating this that involves much less work than using the Pythagorean Theorem!) (4 TIPS)



As shown in the diagram, "fold" along the sides of the smaller square.

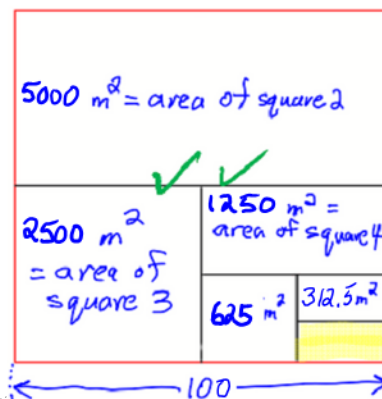
The highlighted triangles fit EXACTLY over the triangles within the smaller square. This means that the highlighted area is equal to the area of the smaller square. Therefore, the area of the smaller square is HALF the area of the larger square.



- (b) Complete the following table. (The squares are numbered according to their size. Square 1 is the largest, square 2 is the second largest, square 3 is the third largest and so on.) (2 TIPS)

Square Number	Area (m ²)
1	10000
2	5000
3	2500
4	1250
5	625
6	312.5

- (c) Imagine that the process of forming squares within squares continues indefinitely. This means that the squares get smaller and smaller and that there are infinitely many of them. Explain why no matter how many squares are formed, the **total area** of all the squares will always be less than 20000 m². (4 TIPS)



The highlighted rectangle shows the area left over after the areas of squares 2 to 6 are used to cover as much of the square as possible. When further areas of squares are added, the highlighted area will never be filled completely because the area that is added is always equal to half of what remains.

- (d) **Bonus Question.** You are not required to attempt this question. If you do attempt it and the quality of your response warrants it, you will receive extra credit.

Write an equation that relates the area of a square to its square number.

$A = 10000 \left(\frac{1}{2}\right)^{n-1}$, where n represents square number and A represents area

+1 Bonus

e.g. $n = 3$, $A = 10000 \left(\frac{1}{2}\right)^{3-1} = 10000 \left(\frac{1}{4}\right) = 2500$

(Equation can also be written $A = \frac{10000}{2^{n-1}}$)

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