

## Grade 9 Pre-AP Math

## Unit 0 – Introduction to Mathematical Thinking – Major Test (Period 1)

Mr. Nolfi

Victim:

*Mr. Solutions**inspiring work Mr. S.!!*

| KU    | APP   | TIPS  | COM   |
|-------|-------|-------|-------|
| 16/16 | 22/22 | 12/12 | 10/10 |

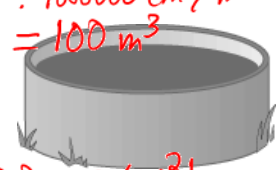
## Modified True or False (5 KU)

Indicate whether each statement is **true** or **false**. If false, **change** the underlined part to make the statement true.

- F Math is like a dating service because it's all about relativity. Change: relationships ✓
- F If a cone has a volume of  $150 \text{ cm}^3$ , then the volume of a cylinder with the same radius and height is 50  $\text{cm}^3$ . Change:  $450 \text{ cm}^3$  ✓  
*( $\checkmark = \frac{1}{2}$  mark)*
- F A hexagonal pyramid has five **lateral** faces. Change: six ✓
- F If a sphere's radius is doubled, its volume doubles. Change: multiplied by 8 ✓
- F  $\frac{(A_{\text{base}})(\text{height})}{3}$  is the volume of any cone or cylinder. Change: pyramid ✓

## Multiple Choice (5 KU)

For questions 6 to 10, select the best answer. Write the letter of your choice in the provided blank space.

- d A **cone** has a **volume** of  $314.16 \text{ cm}^3$  and a **height** of 5 cm. To one decimal place, what is its **radius**?  
~~(a) 60.0 cm~~ ~~(b) 20.0 cm~~ ~~(c) 12.0 cm~~ (d) 7.7 cm  
*a, b, c are much too large → only d makes sense*
- c Which of the following is **NOT** an expression for **volume**?  
 (a)  $2lwh + \frac{2}{3}\pi r^3$  (b)  $\frac{1}{2}\pi r^2 h$  (c)  $2\pi rs + 3\pi r^2$  (d)  $x^2 h + x^3$
- a A circular swimming pool has a **diameter** of 20 m. When filled completely the pool holds exactly **100,000 L** of water. To one decimal place, **what is the height** of the pool?  
~~(a) 0.3 m~~ (a) 0.3 m (b) 3.2 m (c) 318.3 m (d) 79.6 m  
*100,000 L × 1000  $\text{cm}^3/\text{L}$  = 100,000,000  $\text{cm}^3$  ÷ 1,000,000  $\text{cm}^3/\text{m}^3$  = 100  $\text{m}^3$*   
  
 $100 = \pi(10)^2 h$   
 $\therefore 100 = 100\pi h$   
 $\therefore h = \frac{100}{100\pi} = \frac{1}{\pi} \approx 0.3$
- b Which statement is **NOT true**?  
 (a) The sum of the interior angles of a pentagon is  $3(180^\circ)$ . ✓  
(b) Co-interior angles are equal. ✗  
 (c) The measure of any exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles. ✓  
 (d) The sum of the exterior angles of a convex polygon is  $360^\circ$ . ✓
- c Which statement is **true**?  
~~(a) Corresponding angles are supplementary.~~ ~~(b) Alternate angles are supplementary.~~  
(c) The circumference of a circle is equal to exactly  $\pi$  diameters. ~~(d) (3, 4, 7) is a Pythagorean triple.~~

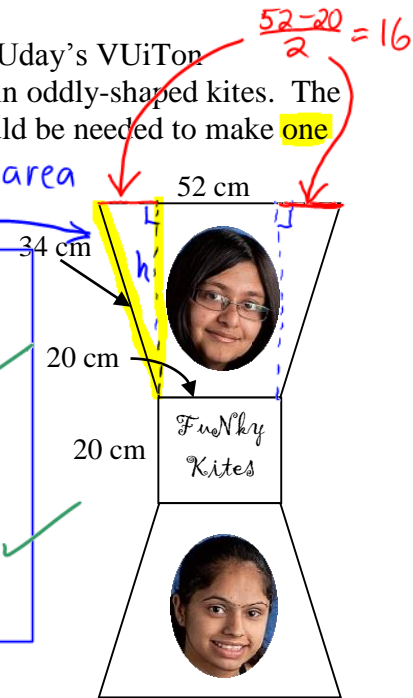
## Full Solutions/Explanations

11. Fatima and Nistha have a kite-making business that competes with Vyshna's and Uday's VUiTon Fashionable Kites. Their company is called FuNky kites because they specialize in oddly-shaped kites. The shape of one of their funkiest kites is shown at the right. **How much material** would be needed to make **one** of these kites? (6 KU)

$A \rightarrow$  total area,  $A_{\text{Trap}} \rightarrow$  area of trapezoidal part  
 $A_{\text{square}} \rightarrow$  area of square part

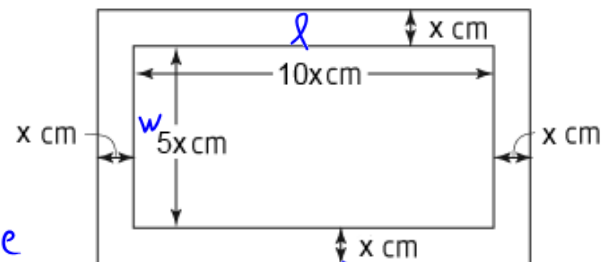
$$\begin{aligned} A &= 2A_{\text{Trap}} + A_{\text{square}} \\ &= 2 \left[ \frac{30(20+52)}{2} \right] + 20(20) \\ &= 2560 \end{aligned}$$

By the Pythagorean theorem,  
 $h^2 + 16^2 = 34^2$   
 $\therefore h^2 = 34^2 - 16^2$   
 $\therefore h^2 = 900$   
 $\therefore h = \sqrt{900} = 30$



Exactly 2560 cm<sup>2</sup> of material is needed to make each kite.

12. A picture is framed with a frame of unknown width,  $x$  cm. The length of the picture is 10 times the width of the frame and the width of the picture is five times the width of the frame. If the **perimeter of the picture is 120 cm**, find the **width of the frame** and the **dimensions of the picture**. (6 APP)



The length of the picture is twice the width. By trial and error (see table at right), it is easy to determine that  $w=20$  and  $l=40$ .

Since  $l = 10x$ ,

$$10x = 40$$

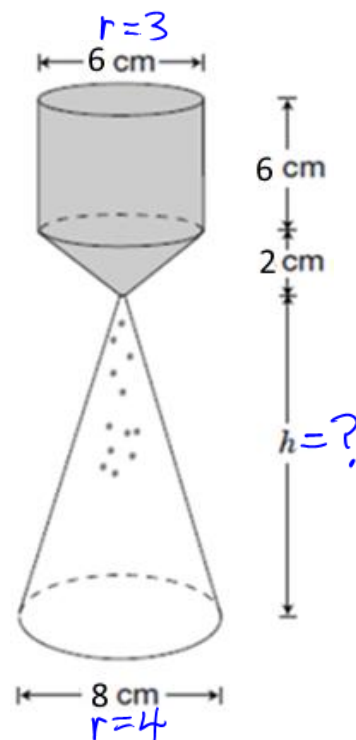
$$\therefore x = 4$$

Therefore, the width of the frame is 4 cm and the dimensions of the picture are 40 cm x 20 cm.

| w  | l  | P   |
|----|----|-----|
| 10 | 20 | 60  |
| 20 | 40 | 120 |
| 30 | 60 | 180 |

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13. As shown at the right, sand is poured from one container into another. The sand flows from the shaded shape to the unshaded cone. The shaded shape starts full of sand and by the time it is empty, the unshaded cone is filled to the top. What is the height of the unshaded cone? (6 APP)



$$\therefore V_{\text{shaded shape}} = V_{\text{unshaded cone}}$$

$$\text{Now, } V_{\text{shaded shape}} = \pi(3^2)(6) + \frac{1}{3}\pi(3^2)(2) \\ \doteq 188.5 \text{ cm}^3$$

$$\therefore V_{\text{unshaded cone}} \doteq 188.5 \text{ cm}^3$$

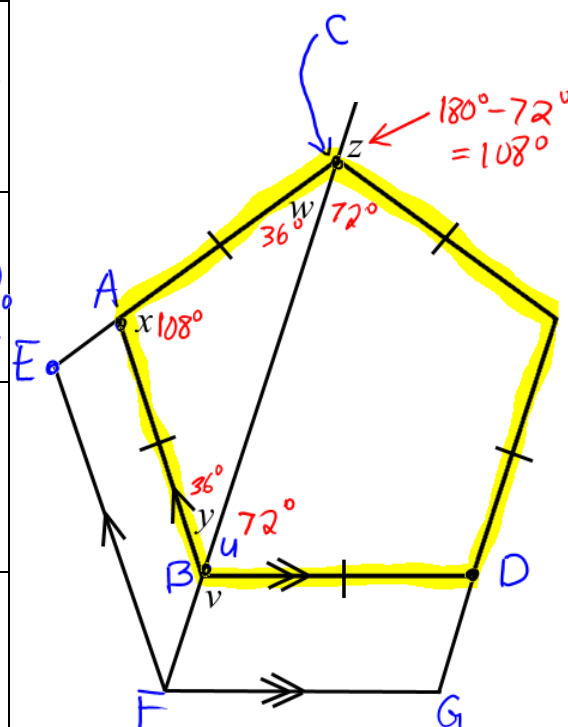
$$\therefore \frac{1}{3}\pi(4^2)h \doteq 188.5$$

$$\therefore h \doteq \frac{188.5}{\left(\frac{16\pi}{3}\right)} \doteq 11.3$$

The height of the unshaded cone is about 11.3 cm.

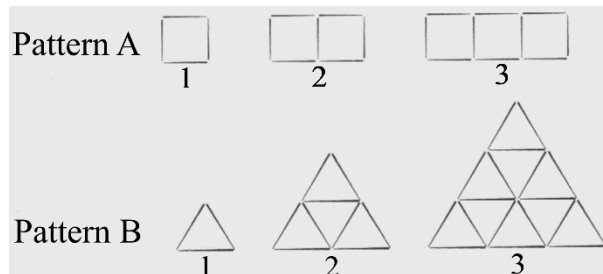
14. Find the measures of each angle labelled with a letter. In each case, state your reasoning. (10 APP)

| Angle Measure     | Reasoning (State Why)  |
|-------------------|--|
| $x = 108^\circ$ ✓ | Sum of interior angles of pentagon = $3(180^\circ) = 540^\circ$<br>Since the pentagon is regular, ✓<br>$x = \frac{540^\circ}{5} = 108^\circ$   |
| $y = 36^\circ$ ✓  | $\triangle ABC$ is isosceles $\rightarrow y = w$<br>In $\triangle ABC$ , $x + y + w = 180^\circ$ (ASTT) ✓<br>$\therefore 108^\circ + y + y = 180^\circ$ , so $y = \frac{180^\circ - 108^\circ}{2}$   |
| $z = 108^\circ$ ✓ | Same reasoning as for $v$ ✓  |
| $w = 36^\circ$ ✓  | See reasoning for $y$ ✓  |
| $v = 108^\circ$ ✓ | $u + y = 108^\circ$ (interior angle of pentagon)<br>$\therefore u + 36 = 108^\circ \rightarrow u = 72^\circ$ ✓<br>But $u + v = 180^\circ$ (supplementary angles)<br>$\therefore 72^\circ + v = 180^\circ$<br>$\therefore v = 180^\circ - 72^\circ = 108^\circ$ |



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15. Shown at the right are two patterns that are formed by arranging toothpicks. For both patterns, consider the **relation** between the **diagram number ( $d$ )** and **the number of toothpicks**. (12 TIPS)



- (a) Complete the following table. (4 TIPS)

| Diagram Number ( $d$ ) | Number of Toothpicks ( $n$ ) |           |
|------------------------|------------------------------|-----------|
|                        | Pattern A                    | Pattern B |
| 1                      | 4                            | 3         |
| 2                      | 7                            | 9         |
| 3                      | 10                           | 18        |
| 4                      | 13                           | 30        |

- (b) In words, describe the relationships between  $n$  and  $d$  for both patterns A and B. (2 TIPS)

Pattern A

When the diagram # increases by 1, the # of toothpicks increases by 3.  
OR The # of toothpicks is one more than triple the diagram #.

Pattern B

When the diagram # increases by 1, the # of toothpicks increases by 3 more than the previous time.  
 Specifically, the number of toothpicks increases first by 6, then by 9, then by 12 and so on.

- (c) Explain the connection(s) between the following patterns of numbers and the two patterns given above: (4 TIPS)

Diagram # 1 2 3 4  
 ① 4, 4+3, 4+3+3, 4+3+3+3, ... Pattern A  
 ② 3, 3+6, 3+6+9, 3+6+9+12, ... Pattern B

① This pattern of numbers corresponds to pattern A. The "4" in each sum is the number of toothpicks in diagram #1. The number of "3's" in each sum is the # of times 3 more toothpicks must be added to complete the picture.

② This pattern of numbers corresponds to pattern B. Each term in the sum is the number of toothpicks in a particular row of the picture.

- (e) **Bonus Question.** You are not required to attempt this question. If you do attempt it and the quality of your response warrants it, you will receive extra credit.

For pattern B, write an equation that relates  $n$  to  $d$ .

+2 Bonus

$$n = \frac{3}{2}d(d+1) \quad \text{OR} \quad n = 1.5d(d+1)$$

$$\text{OR} \quad n = \frac{3}{2}d^2 + \frac{3}{2}d \quad \text{OR} \quad n = 1.5d^2 + 1.5d$$

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