MPM 1D9D **Semester 2, 2014 - 2015**

Grade 9 Pre-AP Math

Unit 0 – Introduction to Mathematical Thinking – Major Test (Period 3)

Mr. Nolfi

Victim: Mr. Solutions

Your work is inspiring Mr. S.!!

KU	APP	TIPS	COM
16 /16	22 /22	12 /12	10/10

Modified True or False (5 KU)

Indicate whether each statement is ***true*** or ***false***. If false, ***change*** the underlined part to make the statement true.

1. F ✓ Math is like a dating service because it's all about relaxedness. **Change:** relationships ✓
2. F ✓ If a cylinder has a volume of 90 cm^3, then the volume of a cone with the same radius and height is 270 cm^3. **Change:** 810 cm^3 ✓
3. F ✓ If a cube's length and width are not changed but its height is doubled, its surface area is quadrupled. **Change:** multiplied by $\frac{5}{3}$ ✓
4. F ✓ The basic elements of math are objectives, operations and relationships. **Change:** objects ✓ (✓ = ½ mark)
5. F ✓ $\frac{(A_{base})(\text{height})}{3}$ is the volume of any pyramid or prism. **Change:** cone ✓

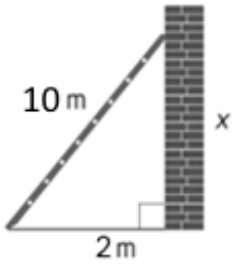
Multiple Choice (5 KU)

For questions 6 to 9, select the best answer. Write the letter of your choice in the provided blank space.

6. d ✓ Which of the following ***is not*** an expression for *volume*? (*r*=radius, *l*=length, *w*=width, *h*=height, *s*=distance)

(a) $\frac{4}{3}\pi r^3 + \pi r h^2$ **(b)** $2\pi r^3 + \pi r^2 h$ **(c)** $2lwh + 2w^2h + 10h^3$ **(d)** $\pi rs + 2\pi r^2$

7. b ✓ A window cleaner has placed a 10-m ladder against a wall. If the bottom of the ladder is 2 m away from the wall, how high above the ground is the top of the ladder?



(a) 12 m **(b)** 9.8 m **(c)** 10.2 m **(d)** 104 m

8. c ✓ Which statement is ***not*** true?

(a) The sum of the interior angles of an octagon is $6(180°)$. ✓

(b) The base angles of an isosceles triangle are equal. ✓

(c) The sum of the exterior angles of a convex hexagon is $720°$. ✗

(d) The sum of the exterior angles of a convex octagon is $360°$. ✓

9. b ✓ A triangle has a height of 3 m and a base of 15 cm. What is the area of the triangle?

$\frac{(300\text{cm})(15\text{cm})}{2} = \frac{4500}{2} = 2250\text{cm}^2$

(a) 22.5 m^2 **(b)** 2250 cm^2 **(c)** 0.225 cm^2 **(d)** 2250 m^2

10. d ✓ Which statement is ***true***?

(a) Corresponding angles are supplementary. **(b)** Alternate angles are supplementary.

(c) The area of a circle is equal to exactly π diameters. **(d)** $(3,4,5)$ is a Pythagorean triple.

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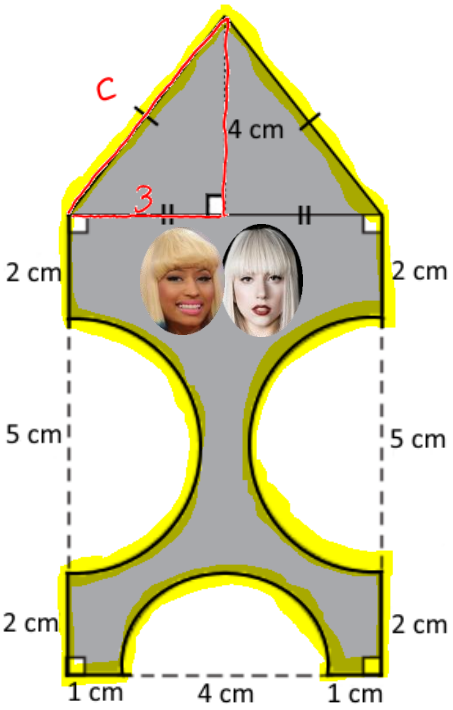
Full Solutions/Explanations

11. Niki Minaj and Lady Gaga have launched a new line of clothing called *MiGagaNaj BarelyDressed*. The "sew-on" logo for the *MiGagaNaj* line of clothing, shown at the right, is to be made of a blue denim material with a border of gold trim (around the boundary of the shape). How much gold trim is needed for ***one*** of these logos? **(6 KU)**

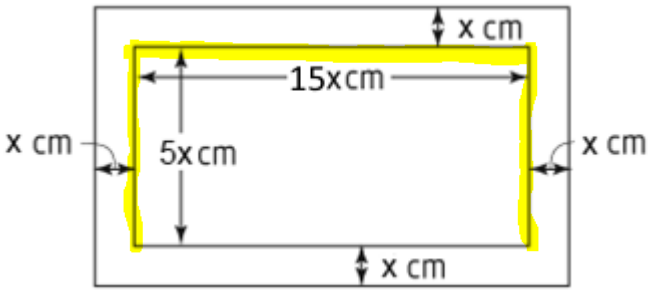


By the Pythagorean Theorem,
$c^2 = 3^2 + 4^2$ ✓
$\therefore c^2 = 25$
$\therefore c = \sqrt{25} = 5$ ✓

Gold trim needed
= perimeter of shape ✓
$= 2(5) + 4(2) + 2(1) + \pi(5) + \frac{\pi(4)}{2}$ ✓
$\doteq$ 42 cm ✓

About 42 cm of gold trim is needed for one logo

12. A picture is framed with a frame of unknown width, x cm. The length of the picture is 15 times the width of the frame and the width of the picture is five times the width of the frame. If the perimeter of the picture is 200 cm, find the ***width*** of the ***frame*** and the ***dimensions*** of the ***picture***. **(6 APP)**



The length of the picture is triple the width. By trial and error (see table at right), it is easy to determine that $w = 25$ and $\ell = 75$.

w	ℓ	P
20	60	160
25	75	200
30	90	240

Since $\ell = 15x$,
$15x = 75$ ✓
$\therefore x = 5$ ✓

Therefore, the width of the frame is 5 cm and the dimensions of the picture are 25 cm x 75 cm.

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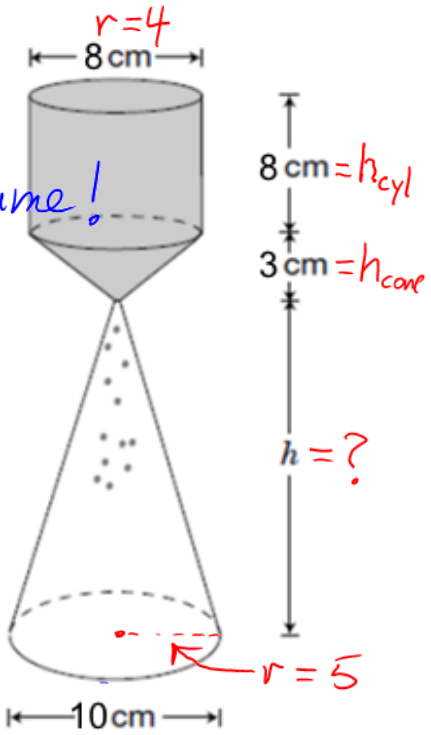
13. As shown at the right, sand is poured from one container into another. The sand flows from the shaded shape to the unshaded cone. The shaded shape starts full of sand and by the time it is empty, the unshaded cone is filled to the top. What is the height of the unshaded cone? **(6 APP)**



This means that the two shapes have the SAME volume!

$\therefore \pi(4)^2(8) + \frac{1}{3}\pi(4)^2(3) = \frac{1}{3}\pi(5)^2 h$ ✓

$\therefore \pi(16)(8) + \frac{1}{3}\pi(16)(3) = \frac{1}{3}\pi(25)h$

$\therefore 128\pi + 16\pi = \frac{25\pi}{3}h$

$\therefore 144\pi = \frac{25\pi}{3}h$

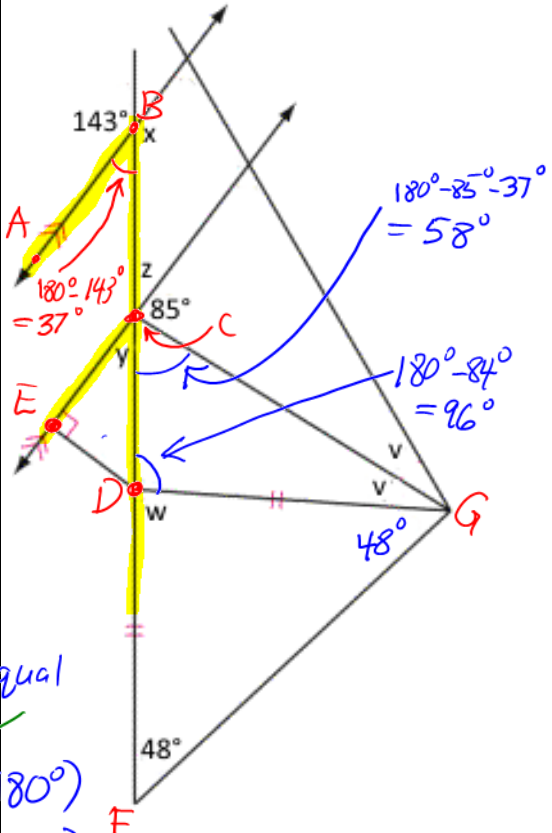
$\therefore 452.39 \div 26.18\, h$

$\therefore h \doteq \frac{452.39}{26.18} \doteq 17.28$

The height of the unshaded cone is about 17.3 cm.

14. Find the measures of each angle labelled with a letter. In each case, state your ***reasoning***. **(10 APP)**

Angle Measure	***Reasoning (State Why)***
$x =$ 143° ✓	Opposite angles (X pattern) are equal. ✓
$y =$ 37° ✓	$\angle ABC = 180° - 143° = 37°$ (supplementary angles) $\angle ECD = \angle ABC$ (corresponding angles) ⇉ pattern
$z =$ 37° ✓	$z = y$ (opposite angles) ✓
$w =$ 84° ✓	$\triangle DFG$ is isosceles ∴ base angles are equal ✓ $\therefore w = 180° - 48° - 48° = 84°$ (sum of interior angles of $\triangle$ is 180°)
$v =$ 26° ✓	$\angle CDG = 48° + 48° = 96°$ (exterior angle theorem) $\angle DCG = 180° - 85° - 37° = 58°$ (supplementary angles) ✓ $\therefore v = 180° - 96° - 58° = 26°$ (sum of interior angles of $\triangle$ is 180°)



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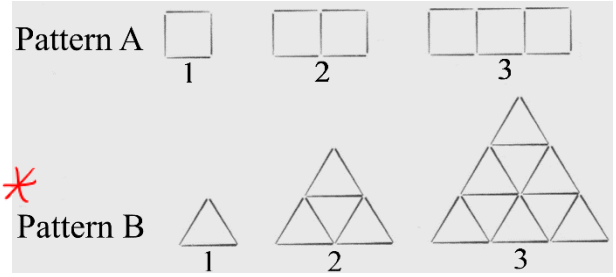
15. Shown at the right are two patterns that are formed by arranging toothpicks. For both patterns, consider the ***relation*** between the ***diagram number (d)*** and ***the number of regions (r) enclosed*** by the toothpicks. **(12 TIPS)**

*** NOT the # of toothpicks ***



(a) Complete the following table. **(4 TIPS)**

Diagram Number (d)	Number of Regions Enclosed (r)	
	Pattern A	Pattern B
1	1	1
2	2 (+1)	4 (+3)
3	3 (+1)	9 (+5)
4	4 (+1)	16 (+7)

(b) In words, describe the relationships between r and d for both patterns A and B. **(2 TIPS)**

Pattern A

When the diagram number increases by one, so does the # of regions. ✓ (Also, the diagram # equals the # of regions)

Pattern B

When the diagram number increases by one, the number of regions increases by two more than the previous time. ✓ The pattern suggests that the number of regions equals the diagram # multiplied by itself.

(c) Explain the connection(s) between the following patterns of numbers and the two patterns given above: **(4 TIPS)**

diagram # (1, 2, 3, 4)

① 1, 1+1, 1+1+1, 1+1+1+1, …

② 1, 1+3, 1+3+5, 1+3+5+7, …

① → Pattern A → the diagram # equals the number of "1's" in the sum, which also equals the # of regions ✓ ✓

② → Pattern B → each term of the sum is equal to the # of triangles (regions) in a particular row. The total # of regions is the sum of all odd integers from 1 up to one less than double the diagram # ✓ ✓

(d) For pattern B, how many regions are enclosed by the toothpicks in diagram 100? **(2 TIPS)**

regions = (diagram #) times itself
= 100 × 100 ✓ ✓
= 10 000

OR

regions = 1+3+5+7+…+2(100)−1
= 1+3+5+7+…+197+199
= (1+199)+(3+197)+(5+195)+…+(99+101)
= 200+200+200+…+200 (50)
= 50(200) = 10000

(e) ***Bonus Question.*** You are not required to attempt this question. If you do attempt it and the quality of your response warrants it, you will receive extra credit.

For pattern B, write an equation that relates r to d.

$r = d^2$ (or $r = d \times d$) +1 BONUS

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