

## Grade 9 Pre-AP Math

## Unit 0 – Introduction to Mathematical Thinking – Major Test (Period 3)

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Victim: Mr. Solutions

Super work Mr. S.!!

KU	APP	TIPS	COM
15/15	22/22	12/12	10/10

## Modified True or False (5 KU)

✓ = 1/2 mark

Indicate whether each statement is true or false. If false, change the underlined part to make the statement true.

- F ✓ Math is like a dating service because it's all about relaxation. Change: relationships ✓
- F ✓ If a cylinder has a volume of 90 cm<sup>3</sup>, then the volume of a cone with the same radius and height is 270 cm<sup>3</sup>. Change: 810 ✓
- F ✓ If a rectangular prism's length and width are not changed but its height is doubled, its surface area is quadrupled. Change: Almost doubled but not quite ✓  
Oops! I meant volume!
- F ✓ The basic elements of math are objectives, operations and relationships. Change: objects ✓
- F ✓  $\frac{(A_{\text{base}})(\text{height})}{3}$  is the volume of any prism or cone. Change: pyramid ✓

## Multiple Choice (4 KU)

For questions 6 to 9, select the best answer. Write the letter of your choice in the provided blank space.

6. b ✓ Which of the following is not an expression for volume? ( $r$ =radius,  $l$ =length,  $w$ =width,  $h$ =height,  $s$ =distance)

~~(a)~~  $\frac{4}{3}\pi r^3 + \pi r h^2$

(b)  $\pi r s + 2\pi r^2$

~~(c)~~  $2lwh + 2w^2h + 10h^3$

~~(d)~~  $2\pi r^3 + \pi r^2 h$

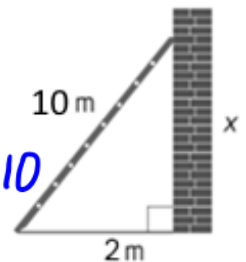
7. b ✓ A window cleaner has placed a 10-m ladder against a wall. If the bottom of the ladder is 2 m away from the wall, how high above the ground is the top of the ladder?

~~(a)~~ 12 m

(b) 9.8 m

~~(c)~~ 10.2 m

~~(d)~~ 104 m

 $x$  must be smaller than 10

8. b ✓ Which statement is not true?

(a) The sum of the interior angles of an octagon is  $6(180^\circ)$ . ✓~~(b)~~ The sum of the exterior angles of a convex hexagon is  $720^\circ$ . ✗(c) The base angles of an isosceles triangle are equal. ✓(d) The sum of the exterior angles of a convex octagon is  $360^\circ$ . ✓

9. a ✓ A triangle has a height of 3 m and a base of 15 cm. What is the area of the triangle?

(a) 2250 cm<sup>2</sup>

~~(b)~~ 22.5 m<sup>2</sup>

~~(c)~~ 2250 m<sup>2</sup>

~~(d)~~ 0.225 cm<sup>2</sup>

$$A = \frac{(300\text{ cm})(15\text{ cm})}{2}$$

$$= \frac{4500}{2} \text{ cm}^2$$

$$= 2250 \text{ cm}^2$$

$$A = \frac{bh}{2}$$

$$= \frac{(0.15\text{ m})(3\text{ m})}{2}$$

$$= \frac{0.45}{2}$$

$$= 0.225 \text{ m}^2$$

$$= 0.225 \text{ m}^2 = (0.225 \text{ m}^2)(10000 \text{ cm}^2/\text{m}^2) = 2250 \text{ cm}^2$$

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## Full Solutions/Explanations

10. Niki Minaj and Lady Gaga have launched a new line of clothing called *MiGagaNaj BarelyDressed*. The “sew-on” logo for the *MiGagaNaj* line of clothing, shown at the right, is to be made of a blue denim material with a border of gold trim (around the boundary of the shape). How much gold trim is needed for *one* of these logos? (6 KU)

By the Pythagorean Theorem,

$$c^2 = 3^2 + 4^2$$

$$\therefore c^2 = 25$$

$$\therefore c = 5$$

Length of gold trim needed

= perimeter of shape

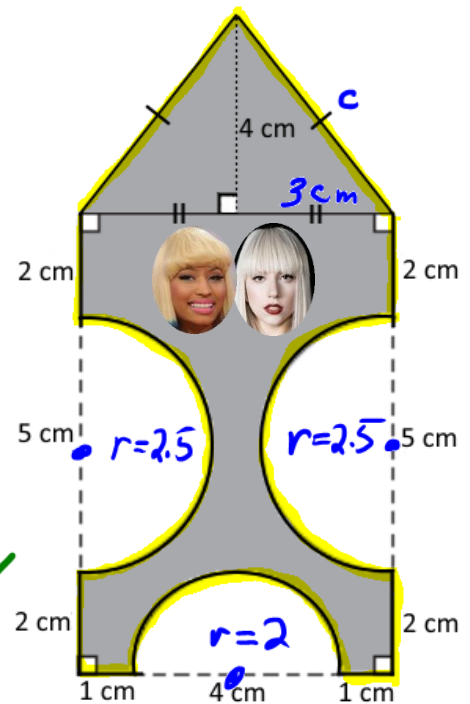
$$= 2(5) + 4(2) + 2(1) + \pi d_{5\text{cm}} + \frac{\pi d_{4\text{cm}}}{2}$$

$$= 10 + 8 + 2 + \pi(5) + \frac{\pi(4)}{2}$$

$$= 20 + 5\pi + 2\pi$$

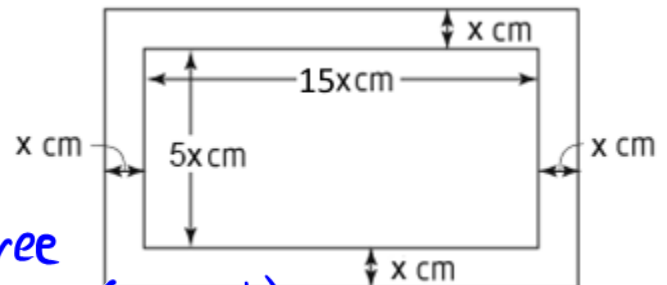
$$= 20 + 7\pi$$

$$\approx 42 \text{ cm}$$



11. A picture is framed with a frame of unknown width,  $x$  cm. The length of the picture is 15 times the width of the frame and the width of the picture is five times the width of the frame. If the perimeter of the picture is 200 cm, find the width of the frame and the dimensions of the picture. (6 APP)

The length of the picture is three times its width. By trial and error (see table), it's easy to determine that the picture has a perimeter of 200 cm when  $w = 25$  and  $l = 75$ .



w	l = 3w	P
10	30	80
20	60	160
25	75	200

Since  $w = 5x$

$$5x = 25$$

$$\therefore x = 5$$

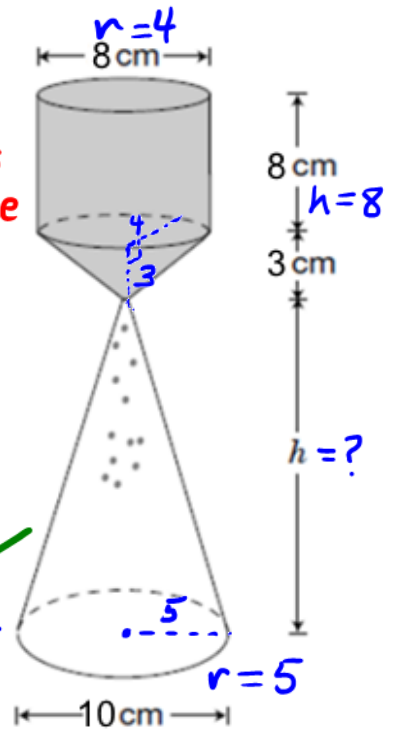
The dimensions of the picture are 75 cm by 25 cm and the width of the frame is 5 cm.

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12. As shown at the right, sand is poured from one container into another. The sand flows from the shaded shape to the unshaded cone. The shaded shape starts full of sand and by the time it is empty, the unshaded cone is filled to the top. What is the height of the unshaded cone? (6 APP)

$$\begin{aligned}
 V_{\text{shaded}} &= V_{\text{cyl}} + V_{\text{cone}} \\
 &= \pi(4^2)(8) + \frac{1}{3}\pi(4^2)(3) \\
 &= 128\pi + 16\pi \\
 &= 144\pi \approx 452.4 \text{ cm}^3
 \end{aligned}$$

$\therefore$  shapes have same volume



Since the unshaded cone is filled to the top by the time the shaded shape is empty,

$$\therefore V_{\text{unshaded cone}} = V_{\text{shaded}} \approx 452.4 \text{ cm}^3 = \frac{\pi(5^2)h}{3} = \frac{25\pi h}{3}$$

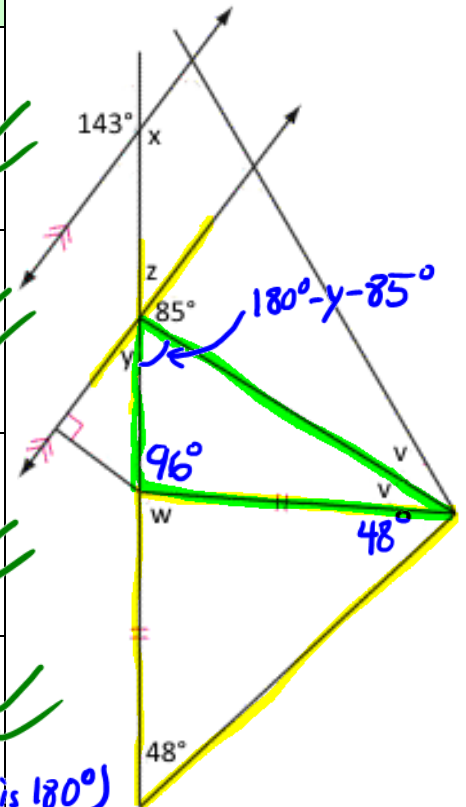
$$\therefore h = \frac{3(452.4)}{25\pi} \approx 17.3$$

The height of the unshaded cone is about 17.3 cm.

13. Find the measures of each angle labelled with a letter. In each case, state your reasoning. (10 APP)

Angle Measure	Reasoning (State Why)
$x = 143^\circ$	Opposite angles are equal
$y = 37^\circ$	$y = z$ because opposite angles are equal
$z = 37^\circ$	$  \begin{aligned}  x + z &= 180^\circ \\  143^\circ + z &= 180^\circ \\  z &= 37^\circ  \end{aligned}  $ PLT pattern
$w = 84^\circ$	Base angles of isosceles angles are equal $w = 180^\circ - 48^\circ - 48^\circ = 84^\circ$ (sum of interior angles of $\Delta$ is $180^\circ$ )
$v = 26^\circ$	$  \begin{aligned}  180^\circ - y - 85^\circ &= 58^\circ \text{ (supp. angles)} \\  v &= 180^\circ - 58^\circ - 96^\circ \\  &= 26^\circ \text{ (sum of int. angles of } \Delta \text{ is } 180^\circ)  \end{aligned}  $

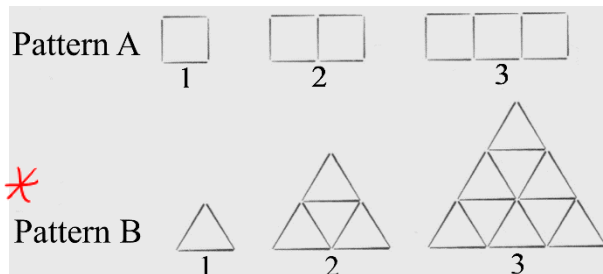
$$180^\circ - 84^\circ = 96^\circ \text{ (supp. angles)}$$



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14. Shown at the right are two patterns that are formed by arranging toothpicks. For both patterns, consider the relation between the diagram number ( $d$ ) and the number of regions ( $r$ ) enclosed by the toothpicks. (12 TIPS)

\*\*\* NOT the # of toothpicks \*\*\*



- (a) Complete the following table. (4 TIPS)

Diagram Number ( $d$ )	Number of Regions Enclosed ( $r$ )	
	Pattern A	Pattern B
1	1	1
2	2	4
3	3	9
4	4	16

- (b) In words, describe the relationships between  $r$  and  $d$  for both patterns A and B. (2 TIPS)

Pattern A

When the diagram number increases by one, so does the # of regions. (Also, the diagram # equals the # of regions) ✓

Pattern B

When the diagram number increases by one, the number of regions increases by two more than the previous time. The pattern suggests that the number of regions equals the diagram # multiplied by itself. ✓

- (c) Explain the connection(s) between the following patterns of numbers and the two patterns given above: (4 TIPS)

1 2 3 4 ← diagram #  
 ① 1, 1+1, 1+1+1, 1+1+1+1, ...  
 ② 1, 1+3, 1+3+5, 1+3+5+7, ...

① → Pattern A → the diagram # equals the number of "1's" in the sum, which <sup>also</sup> equals the # of regions ✓

② → Pattern B → each term of the sum is equal to the # of triangles (regions) in a particular row. The total # of regions is the sum of all odd integers from 1 up to one less than double the diagram # ✓

- (e) Bonus Question. You are not required to attempt this question. If you do attempt it and the quality of your response warrants it, you will receive extra credit.

- (d) For pattern B, how many regions are enclosed by the toothpicks in diagram 100? (2 TIPS)

# regions = (diagram #) times itself  
 $= 100 \times 100$   
 $= 10\,000$  ✓

OR

# regions =  $1 + 3 + 5 + 7 + \dots + 2(100) - 1$   
 $= 1 + 3 + 5 + 7 + \dots + 197 + 199$   
 $= (1 + 199) + (3 + 197) + (5 + 195) + \dots + (99 + 101)$   
 $= 200 + 200 + 200 + \dots + 200$   
 $= 50(200) = 10\,000$

For pattern B, write an equation that relates  $r$  to  $d$ .

$$r = d^2 \quad (\text{or } r = d \times d)$$

+1 BONUS

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