Conceptual Retrospective: Number Sense and Algebra

1. Complete the following statements:
Two algebraic expressions are said to be ***equivalent*** if ______________________________________

The following is an example of two equivalent expressions: ______________________________

2. The expressions $2x$ and x^2 are ***not equivalent***. Show this in the following ways.

(a) $2x$ ***means*** ______________________

while x^2 ***means*** ______________________.

For example, if $x = -7$,

$2x =$ __________ $=$ __________

and $x^2 =$ __________ $=$ __________.

(b) $2x$ ***means*** ________ groups of ________ while

x^2 ***means*** ________ groups of ________.

For example, if $x = 4$, $2x = 2(4)$, which means

______ groups of ______ and $x^2 = 4^2 = (4)(4)$,

which means _____ groups of _____.

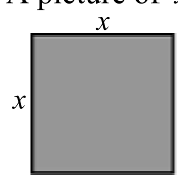
(c) Complete the table. Then draw conclusions by completing the statement to the right of the table.

x	$2x$	x^2
−5		
−4		
−3		
−2		
−1		
0		
1		
2		
3		
4		
5		

From the table, we can see that $2x$ and x^2 agree ***only when*** $x =$ _____ and when $x =$ _____. For all other values of x, $2x$ and x^2 ________________.

Therefore, $2x$ and x^2 ***cannot be*** __________.

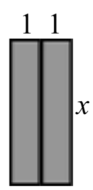
(d) A picture of x^2 could look like the following:



This picture can represent x^2 because ____________________

____

On the other hand, a picture of $2x$ could look like the following:



This picture can represent $2x$ because ____________________

____

From these pictures we must conclude that $2x$ and x^2 are ***not equivalent*** because ________________

______________________________________.

3. Using both a ***logical argument*** and ***pictures***, explain why $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$. Then complete the statement.

(a) ***Logical Argument***

(b) ***Pictures***

Whenever I ***add*** or ***subtract fractions***, I must always remember to ______________

____

because ____________________

____

4. First complete the statements found below. Then ***evaluate*** the expression shown at the right. Show all steps!

$$\frac{-2\left[4^2-3(-7)^2\right]-\left(3^2-2^4\right)}{-6^2+(-6)^2+3(-7)(-8)-4(3-7)}$$

Whenever I ***evaluate expressions***, I must always remember:

1. ***Adding*** and ***subtracting*** involve ________ and ________.
2. ***Multiplying*** and ***dividing*** involve ___________________.
3. I ***should not*** use the ___________ ___________ because
   ____.
4. I ***should*** use ______________ so that I'll know how to
   apply the operations in the correct _________________.
5. I ***should*** separate the expression into ________________.

5. First complete the statements found below. Then ***substitute*** the given values into the expression shown at the right and ***evaluate***. Show all steps!

$$\frac{-a^2+3ab^3-6ab^2}{(a-b)(a+b)},\ a=4,\ b=-\frac{1}{2}$$

Whenever I ***substitute values into expressions***, I must:

1. Replace the ***variables*** with empty __________________ ,
   taking care to ensure that __________________ are not
   changed and exponents remain the _______________ .
2. Then the given values should be inserted into the empty
   ________________ , taking care to ensure that the correct
   values are used.
3. Finally, the resulting expression should be ____________
   using ______________ and keeping in mind all the points
   made in question _________ .

6. First complete the statements found below. Then ***simplify*** the expression shown at the right. Show all steps!

$$\frac{\left(-ab^2\right)^3\left(-2a^2b\right)^4}{\left(2a\right)^{-3}}-2a^5b^3\left(3a^9b^7-6a^{-3}b^{-3}\right)-\left(a-b\right)\left(2a+7b\right)$$

Whenever I ***simplify expressions***, I must remember:

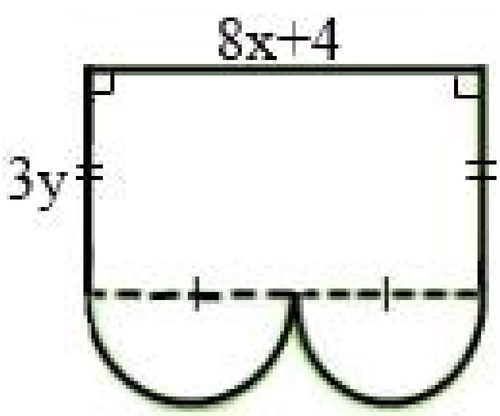
1. When ***adding*** and ***subtracting expressions***, I must __________ ___________ __________.

I should add the opposite ***only when*** _________ ____.

2. When ***multiplying*** and ***dividing*** expressions, I must put ___________ _________ together and use the laws of __________________. In addition, if I multiply a monomial by a polynomial with two or more (unlike) terms, I should use the _____________ _____________.

3. I ***must never confuse*** addition and subtraction with multiplication and division. For example, $x^2+x^2=$ ________ but $x^2\left(x^2\right)=$ ________

7. ***Write expressions*** for the area and perimeter of the following shape.



8. Complete the following table. By doing so ***correctly***, you will demonstrate your understanding of how basic mathematical principles can be used to ***justify*** or ***refute assertions***.

Assertion	***True or False?***	***If the assertion is true, justify it. If it is false, refute it. Then provide a correction.***
$(-3xy^3)^2$ $=(-3)^2(x^2)(y^3)^2$ $=9x^2y^6$	True	$(-3xy^3)^2$ $=(-3xy^3)(-3xy^3)$ $=(-3)(-3)xxy^3y^3$ $=9xxyyyyyy$ $=9x^2y^6$ ***Expanded form*** can always be used to verify a simplification performed by using laws of exponents. Although expanded form makes the simplification more time-consuming, it has the advantage of relying only on very basic principles such as the ***meaning*** of mathematical operations.
$2a+3a=5a^2$	False	When like terms are added/subtracted, the coefficients must be added/subtracted but the variable part ***must not change***. Thus, $2a+3a$ simplifies to $5a$, which is analogous to stating that 2 apples plus 3 apples is 5 apples. To strengthen the argument, we can use a ***counterexample*** as shown below. Suppose that $a=2$. Then $2a+3a=2(2)+3(2)=4+6=10$ but $5a^2=5(2)^2=5(4)=20$. Since we can confidently assert that $10\neq 20$, this example ***proves*** that $2a+3a$ ***is not*** equivalent to $5a^2$.
$(3x+y^3)^2$ $=(3x)^2+(y^3)^2$ $=9x^2+y^6$		
$2(x+3)(x+5)$ $=(2x+6)(2x+10)$ $=4x^2+32x+60$		
$2(x+3)(x+5)$ $=(2x+6)(x+5)$ $=2x^2+16x+30$		
$-2(-3c^7d^{-4})+2(3c^7d^{-4})$ $=6c^7d^{-4}+6c^7d^{-4}$ $=(6)(6)c^7c^7d^{-4}d^{-4}$ $=\frac{36c^{14}}{d^8}$		
$3[5-4(7)]^2$ $=3(5)^2-3[4(7)]^2$ $=75-2352$ $=-2277$		