

## MPM1D9 – LINEAR SYSTEMS: ALGEBRAIC APPROACH

### Example

The total cost of admission to a popular music concert was \$162 for 12 children and 3 adults. For 8 children and 3 adults, the total cost was \$122. What were the ticket prices for this concert?

### Solution

① Given:

\$162	12 children, 3 adults
\$122	8 children, 3 adults

② Unknown: Cost per child = ? → represent with  $c$   
Cost per adult = ? → " "  $a$

### ③ Write Equations

Cost for 12 children and 3 adults is 162  
→  $12c + 3a = 162$

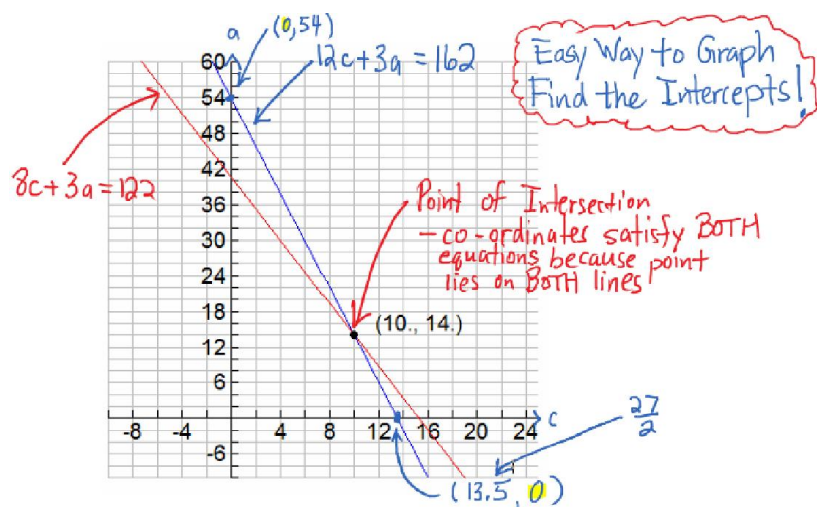
$$8c + 3a = 122$$

This is the  
MOST IMPORTANT  
part of the  
process!!

A solution of this system is any point  $(c, a)$  that satisfies BOTH equations!!

$$\begin{cases} 12c + 3a = 162 & \textcircled{1} \\ 8c + 3a = 122 & \textcircled{2} \end{cases}$$

This is called a system of two linear equations in two unknowns



### Graphical Solution

A system of two linear equations in two unknowns can be represented geometrically as a pair of lines.

$$\begin{cases} 12c + 3a = 162 & \textcircled{1} \\ 8c + 3a = 122 & \textcircled{2} \end{cases}$$

From  $\textcircled{1}$ ,

$$12c = 162 - 3a$$

$$\therefore c = \frac{162}{12} - \frac{3a}{12}$$

$$\therefore c = \frac{27}{2} - \frac{1}{4}a$$

Substitute for  $c$   
in equation  $\textcircled{2}$

Substitute in  $\textcircled{2}$ ,

$$8\left(\frac{27}{2} - \frac{1}{4}a\right) + 3a = 122$$

$$\therefore \frac{8}{1}\left(\frac{27}{2}\right) - \frac{8}{1}\left(\frac{1}{4}\right)a + 3a = 122$$

$$\therefore 108 - 2a + 3a = 122$$

$$\therefore 108 + a = 122$$

$$\therefore a = 122 - 108 = 14$$

Substitute in  $\textcircled{1}$  (doesn't matter which one)

$$12c + 3(14) = 162$$

$$\therefore 12c + 42 = 162$$

$$\therefore 12c = 120$$

$$\therefore c = 10$$

Therefore, the solution to the system is  $c=10, a=14$   
(sometimes just written as  $(10, 14)$ )

### Steps

- ① In either equation, solve for one of the unknowns in terms of the other
- ② Substitute into the other equation

Equation  $\textcircled{1}$  was used to express  $c$  in terms of  $a$   
( $c = \frac{27}{2} - \frac{1}{4}a$ ).

The expression  $\frac{27}{2} - \frac{1}{4}a$  is then substituted for  $c$  in equation  $\textcircled{2}$ .

Now  $a=14$  can be substituted into either equation to calculate the value of  $c$

The children's admission fee is \$10 and the adult admission fee is \$14.

$$\begin{cases} 12c + 3a = 162 & \textcircled{1} \\ 8c + 3a = 122 & \textcircled{2} \end{cases}$$

The method of elimination involves performing operations to eliminate one of the two variables.

$$\begin{array}{l} \textcircled{1} - \textcircled{2}, \quad \begin{array}{r} 12c + 3a = 162 \\ -(8c + 3a = 122) \\ \hline 4c = 40 \end{array} \quad \left. \vphantom{\begin{array}{r} 12c + 3a = 162 \\ -(8c + 3a = 122) \\ \hline 4c = 40 \end{array}} \right\} \text{This really means} \\ \quad \quad \quad 4c = 40 \\ \quad \quad \quad \therefore c = 10 \\ \quad \quad \quad \text{Sub in } \textcircled{2}, \\ \quad \quad \quad 8(10) + 3a = 122 \\ \quad \quad \quad \therefore 80 + 3a = 122 \\ \quad \quad \quad \therefore 3a = 122 - 80 \\ \quad \quad \quad \therefore 3a = 42 \\ \quad \quad \quad \therefore a = 14 \end{array}$$

### Problems

For each of the following problems:

- Write a system of two linear equations in two unknowns
- Solve the system graphically (e.g. use Desmos)
- Solve the system by substitution
- Solve the system by elimination

1. An investor buys a total of 360 shares of two stocks. The price of one stock is \$35 per share, while the price of the other stock is \$45 per share. The investor spends a total of \$15000. How many shares of each stock did the investor buy? [**Answer:** 120 shares of \$35-stock, 240 shares of \$45-stock]
2. The sum of two numbers is 90. The larger number is 14 more than 3 times the smaller number. Find the numbers. [**Answer:** 19, 71]

3. \$6000 is divided between two accounts, one paying 4% interest and the other paying 3% interest. At the end of one interest period, the interest earned by the 4% account exceeds the interest earned by the 3% account by \$65. How much was invested in each account? [**Answer:** \$3500, \$2500]
4. An airplane flying into a headwind travels the 3300-km flying distance between two cities in 3 hours and 36 minutes. On the return flight, the same distance is traversed in 3 hours and 18 minutes. Find the ground speed of the plane and the speed of the wind, assuming that both remain constant. (Ground speed is the speed of the plane if there were no wind.) [**Answer:** groundspeed  $\doteq$  958 km/h, wind speed  $\doteq$  42 km/h]
5. Naquan is saving nickels and dimes in a jar. The jar contains 10 more nickels than dimes. Altogether, the value of the coins is \$16.25. How many nickels and dimes are in the jar? [**Answer:** 105 dimes, 115 nickels]
6. Solution A is 50% hydrochloric acid by volume, while solution B is 75% hydrochloric acid by volume. How many litres of each solution should be used to make 100 litres of a solution which is 60% hydrochloric acid by volume? [**Answer:** 60 L of Solution A, 40 L of Solution B]
7. To raise money for a good cause, Smrithi puts her swimming talents to the test in a river race. On the downstream leg of her aquatic marathon, she swims 18 km in 3 hours. However, the return trip upstream takes her 6 hours. Find her average speed in still water and the speed of the river's current. [**Answer:** average speed in still water = 4.5 km/h, speed of current = 1.5 km/h]
8. To make lower-fat chocolate frozen yogurt, chocolate milk containing 2% butterfat is needed. To obtain the required percentage of butterfat, chocolate milk containing 4% butterfat is mixed with 500 litres of chocolate milk containing 1% butterfat. How many litres of the 4% chocolate milk are needed to create the required mixture? What is the total volume of the mixture? [**Answer:** 250 L of 4% milk, 750 L total volume]
9. Two airplanes fly in *opposite* directions from the same airport. The second plane leaves 30 minutes after the first. The second plane travels at a speed 60 km/h faster than the first. Find the groundspeed of each airplane if two hours after the first plane departs, the two planes are 2015 km apart. [**Answer:** 550 km/h, 610 km/h]