MPM1D9 – Final Culminating Project

Question 1 – Optimization (50%)

Design a container that is made up of ***two different solids*** that are joined in such a way that they have ***one*** dimension in common. Then determine the dimensions of the container that could be made using a ***fixed*** amount of material and having the greatest possible capacity. Your solution should include the following:

(a) a detailed diagram of your container with the dimensions labelled clearly

(b) the fixed surface area of the material that is used to make the container (e.g. 10000 cm^2)

(c) an equation (the constraint equation) that relates the dimensions of the container to the ***fixed*** surface area

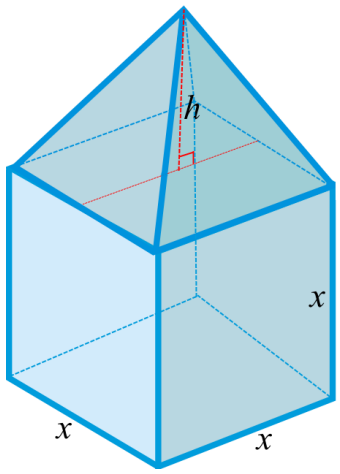
(d) an equation that relates the volume of the container to ***one*** of the dimensions of the container

(e) a graph of the equation from (d)

(f) a conclusion stating the maximum volume and the dimensions that produce that volume

Example

In the diagram at the right, a square pyramid is combined with a cube to form a composite three-dimensional shape. Notice that there are only two unknowns, x and h; this is an intentional aspect of the design. If there were three or more unknowns, the problem could not be solved without additional equations relating the unknowns.

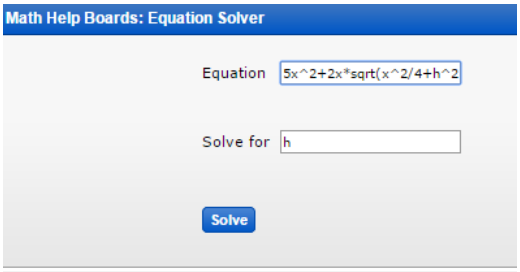


Suppose that this container needed to be manufactured using at most 10000 cm^2 of material. With a little thought and careful application of the Pythagorean Theorem, it can be shown that

$$5x^2 + 2x\sqrt{\frac{x^2}{4} + h^2} = 10000 \text{ (surface area must equal 10000)}$$

This equation relates the unknowns x and h to each other. To proceed, one of the unknowns must be expressed in terms of the other. This is no easy task, however, and is somewhat beyond the skillset of most grade nine students. Fortunately, it is not beyond the skillset of "Wolfram Alpha's Math Help Boards Equation Solver!"

(See http://www.wolframalpha.com/widgets/view.jsp?id=bc455327d0772719486c1a3ecf2e96d3 or simply use Google to search for "Wolfram Alpha Math Help Boards Equation Solver")



Input interpretation:

solve $5x^2 + 2x\sqrt{\frac{x^2}{4} + h^2} = 10000$ for h

Results:

$$h = -\frac{\sqrt{2}\sqrt{3x^4 - 12500x^2 + 12500000}}{x}$$

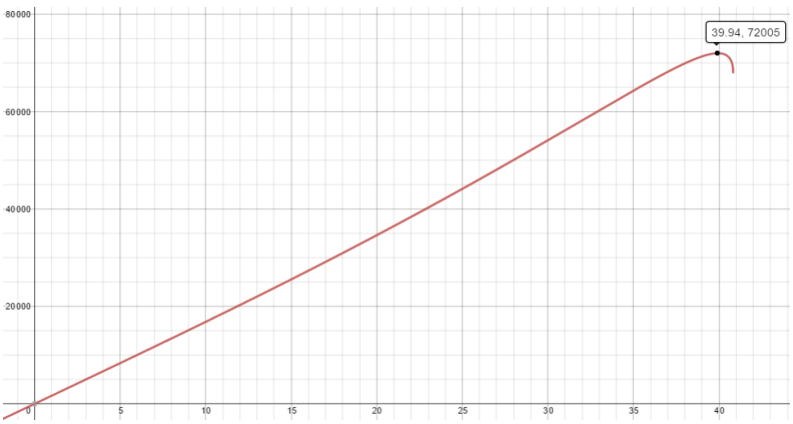
$$h = \frac{\sqrt{2}\sqrt{3x^4 - 12500x^2 + 12500000}}{x}$$

$$V = x^3 + \frac{1}{3}x^2h$$

$$\therefore V = x^3 + \frac{1}{3}x^2\left(\frac{\sqrt{2}\sqrt{3x^4 - 12500x^2 + 12500000}}{x}\right)$$

$$= x^3 + \frac{x\sqrt{2}\sqrt{3x^4 - 12500x^2 + 12500000}}{3}$$

Now use a graphing tool such as Desmos.



Question 2 – Systems of Two Linear Equations in Two Unknowns (25%)

Create a problem that can be solved by using a system of two linear equations in two unknowns. Then solve the problem graphically, by substitution and by elimination.

The problem that you create should be at least of moderate difficulty. Humour is greatly appreciated!

Question 3 – Proof of the Pythagorean Theorem (25%)

Find any proof of the Pythagorean Theorem that we have ***not*** discussed in class. Explain it clearly using words and diagrams.

- Most proofs of the Pythagorean rely on the concept of area. If you select such a proof, your explanation should emphasize the connection between side length and area.
- You ***do not*** need to create your own proof of the Pythagorean Theorem! However, I would be very impressed if you were able to do so!