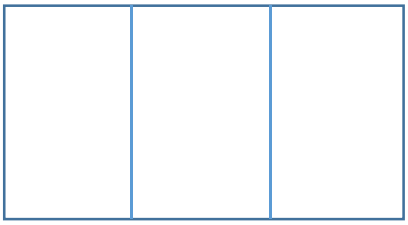
MPM1D9 – Even More Optimization Problems

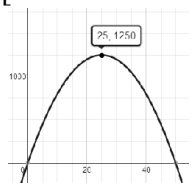
For each of the following optimization problems do the following:

- Solve each of the following problems using an algebraic/graphical model. A graphing calculator program such as Desmos is very helpful for the algebraic/graphical model.
- Use Wolfram Alpha to compute the derivative (slope function) of the algebraic expression that needs to be maximized or minimized. Use this to verify the answer that you obtained using the algebraic/graphical model.

1. Build a rectangular pen with two parallel partitions using 200 m of fencing. What dimensions will maximize the total area of the pen?



[**Answer:** width = 25 m, length = 50 m, A = 1250 m^2]



Derivative:

$$\frac{d}{dx}(x(100-2x)) = -4(x-25)$$

2. Find two nonnegative numbers whose sum is 9 such that the product of one number and the square of the other number is a maximum.

[**Answer:** 3, 6, maximum product is 108]

3. An open rectangular box with square base is to be made from 48 m^2 of material. What dimensions will result in a box with the largest possible volume?

[**Answer:** base is 4 m × 4 m, height is 2 m, maximum volume is 32 m^3]

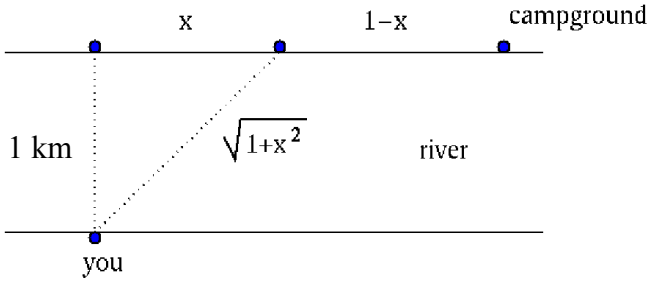
4. A container in the shape of a right circular cylinder with no top has surface area 3π m^2. What height h and base radius r will maximize the volume of the cylinder?

[**Answer:** r = 1 m, h = 1 m, maximum volume is π m^3]

5. A cylindrical can is to hold 20π m^3. The material for the top and bottom costs \$10/m^2 and the material for the side costs \$8/m^2. Find the radius r and height h of the can that can be built most cheaply.

[**Answer:** r = 2 m, h = 5 m, minimum cost is $\$240\pi \doteq \$754$]

6. You are standing at the edge of a slow-moving river, which is 1 km wide, and you wish to return to your campground on the opposite side of the river. First, you must swim across the river to any point on the opposite bank. From there, you walk to the campground, which is 1 km from the point directly across the river from where you start your swim. Assuming that you can swim at 2 km/h and walk at 3 km/h, what route will take the least amount of time?



[**Answer:** $x \doteq 0.89$ km, minimum time is about 0.71 h $\doteq$ 43 minutes]