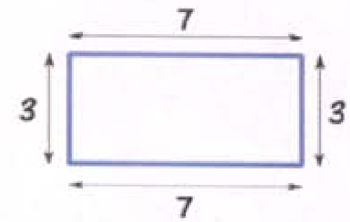
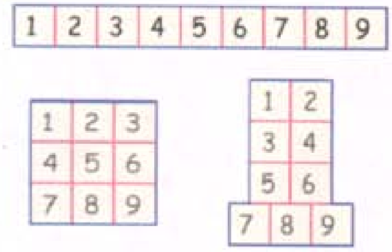
Understanding the Concepts of Perimeter, Area and Volume

Perimeter



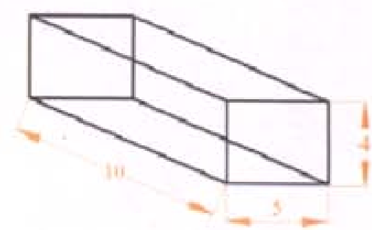
- The ***distance*** around a two-dimensional shape.
- **Example:** the perimeter of this rectangle is $3+7+3+7=20$
- The perimeter of a circle is called the ***circumference***.
- Perimeter is measured in ***linear units*** such as mm, cm, m, km.

Area



- The "size" or "amount of space" inside the boundary of a two-dimensional surface, including curved surfaces. In the case of a curved surface, the area is usually called ***surface area***.
- **Example:** If each small square at the left has an area of 1 cm^2, the larger shapes all have an area of 9 cm^2.
- Area is measured in ***square units*** such as mm^2, cm^2, m^2, km^2.

Volume



- The "amount of space" contained within the interior of a three-dimensional object. (The ***capacity*** of a three-dimensional object.)
- **Example:** The volume of the "box" at the right is $4\times5\times10=200$ m^3. This means, for instance, that 200 m^3 of water could be poured into the box.
- Volume is measured in ***cubic units*** such as mm^3, cm^3, m^3, km^3, mL, L.
  Note: 1 mL = 1 cm^3

Questions

1. You have been hired to renovate an old house. For each of the following jobs, state whether you would measure perimeter, area or volume and explain why.

Job	*Perimeter, Area or Volume?*	*Why?*
Replace the baseboards in a room. 	Perimeter	A length needs to be measured. An appropriate unit is metres (m).
Paint the walls.	Area	A wall is a two-dimensional surface. An appropriate unit is square metres (m^2).
Pour a concrete foundation. 	Volume	A foundation is a three-dimensional space. An appropriate unit is cubic metres (m^3).

2. Convert 200 m^3 to litres. (**Hint:** Draw a picture of 1 m^3.)

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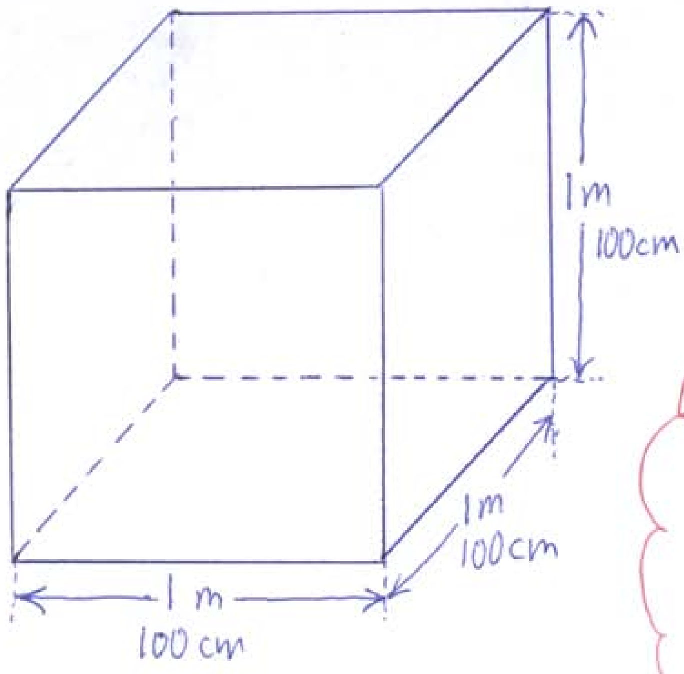
Solution to # 2 (page MG-2)



$1\,m^3$

$= (100\,cm)(100\,cm)(100\,cm)$

$= 1\,000\,000\,cm^3$

∴ 1 cubic metre is equivalent to 1 million cubic centimetres!

Now, $1\,L = 1000\,mL = 1000\,cm^3$ $(1\,mL = 1\,cm^3)$.

$$\therefore 200\,m^3 = (200\,m^3)(1\,000\,000\,cm^3/m^3)$$

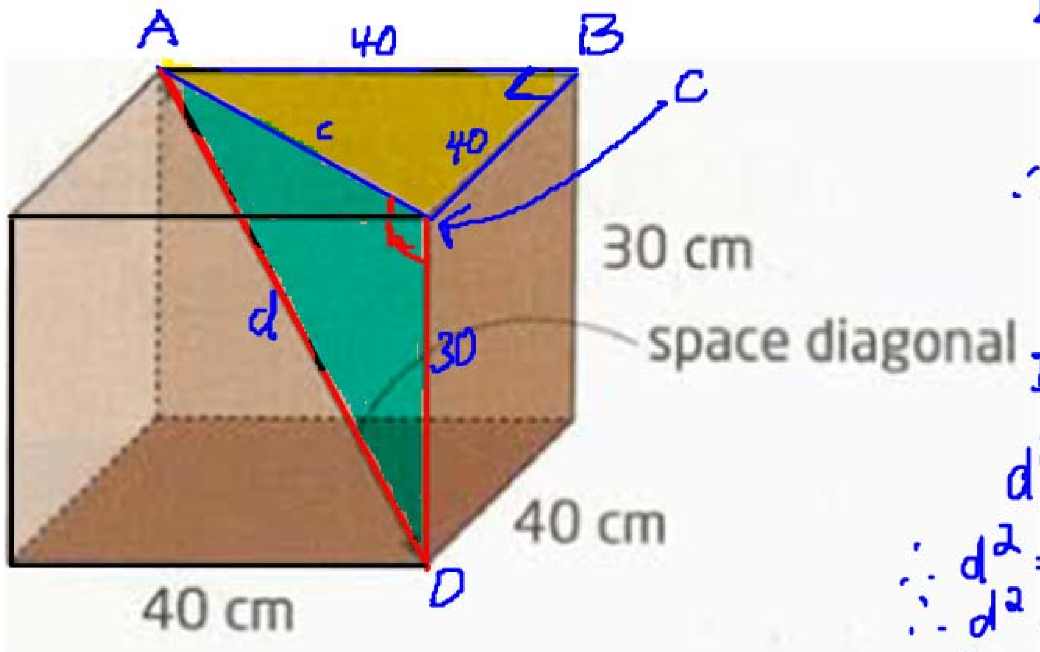
$$= 200\,000\,000\,cm^3$$

$$= \frac{200\,000\,000\,cm^3}{1000\,cm^3/L}$$

$$= 200\,000\,L$$

$200\,m^3 = 200\,000\,L$

Cardboard Box Space Diagonal Solution



In right $\triangle ABC$,

$c^2 = 40^2 + 40^2$

$\therefore c^2 = 1600 + 1600$

$\therefore c^2 = 3200$

In right $\triangle ACD$,

$d^2 = c^2 + 30^2$

$\therefore d^2 = 3200 + 900$

$\therefore d^2 = 4100$

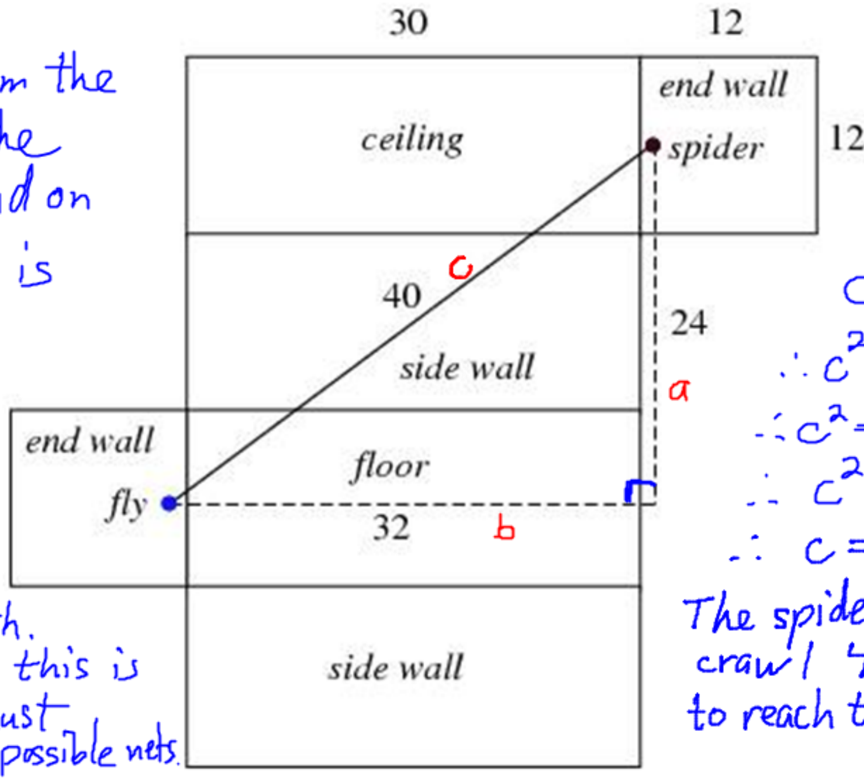
$\therefore d = \sqrt{4100} \doteq 64$

Spider and the Fly Solution

The path from the spider to the fly will depend on how the net is constructed.

The net at the right gives the shortest possible path.

To prove that this is the case, one must construct all possible nets.



$c^2 = a^2 + b^2$

$\therefore c^2 = 24^2 + 32^2$

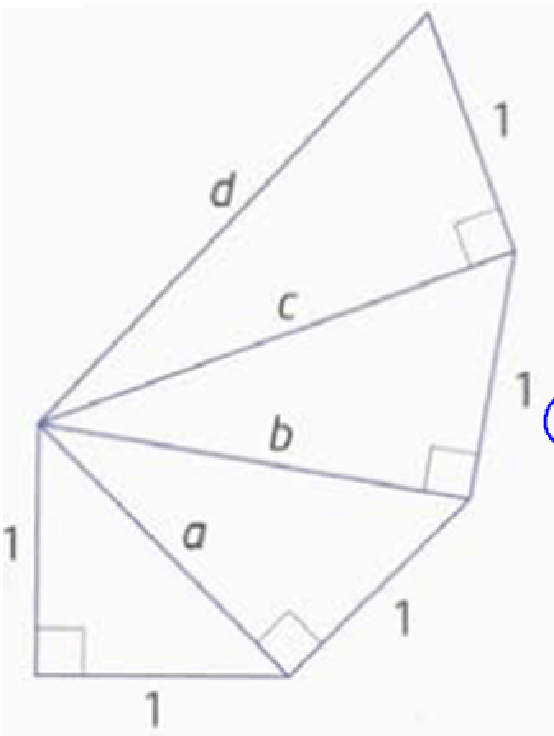
$\therefore c^2 = 576 + 1024$

$\therefore c^2 = 1600$

$\therefore c = 40$

The spider must crawl 40 feet to reach the fly.

Spiral Solution



By the Pythagorean Theorem,

① $a^2 = 1^2 + 1^2$
$\therefore a^2 = 2$
$\therefore a = \sqrt{2}$

② $b^2 = a^2 + 1^2$
$\therefore b^2 = 2 + 1$
$\therefore b^2 = 3$
$\therefore b = \sqrt{3}$

③ $c^2 = b^2 + 1^2$
$\therefore c^2 = 3 + 1$
$\therefore c^2 = 4$
$\therefore c = 2$

④ $d^2 = c^2 + 1^2$
$\therefore d^2 = 4 + 1$
$\therefore d^2 = 5$
$\therefore d = \sqrt{5}$

(a) The hypotenuse lengths are in the ratio
$\sqrt{2} : \sqrt{3} : \sqrt{4} : \sqrt{5}$ (or $\sqrt{2} : \sqrt{3} : 4 : \sqrt{5}$)

(b) area $= \frac{1(1)}{2} + \frac{1(\sqrt{2})}{2} + \frac{1(\sqrt{3})}{2} + \frac{1(\sqrt{4})}{2}$

$= \frac{\sqrt{1}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{4}}{2}$

(c) If a fifth triangle were added, the area would increase by $\frac{\sqrt{5}}{2}$

If a sixth triangle were added, the area would increase by $\frac{\sqrt{6}}{2}$

$\therefore$ the area increases by $\frac{\sqrt{n}}{2}$, where n represents the total # of triangles

13. **Math Contest**

a) The set of whole numbers (5, 12, 13) is called a *Pythagorean triple*. Explain why this name is appropriate.

b) The smallest Pythagorean triple is (3, 4, 5). Investigate whether multiples of a Pythagorean triple make Pythagorean triples.

c) Substitute values for m and n to investigate whether triples of the form $(m^2 - n^2, 2mn, m^2 + n^2)$ are Pythagorean triples.

d) What are the restrictions on the values of m and n in part c)?

(a) (5, 12, 13) is an ordered triple (i.e. an ordered set consisting of 3 numbers)
The values 5, 12 and 13 satisfy the Pythagorean theorem:

$$5^2+12^2 = 25+144 = 169 = 13^2$$

(b) Any multiple of (3, 4, 5) is also a Pythagorean triple

Proof: Let n represent any positive integer.
Then $(3n, 4n, 5n)$ represents any multiple of $(3, 4, 5)$

Now, $(3n)^2+(4n)^2$ and $(5n)^2$

$= 3^2n^2+4^2n^2$ $\quad$ $=5^2n^2$

$= 9n^2+16n^2$ $\quad$ $=25n^2$

$= 25n^2$

$\therefore (3n)^2+(4n)^2 = (5n)^2$

$\therefore (3n, 4n, 5n)$ is a Pythagorean Triple

(c) The ordered triple $(m^2-n^2, 2mn, m^2+n^2)$ is ALWAYS a PYTHAGOREAN TRIPLE.

Proof: Requires algebra taught in grade 10 math:

$$\boxed{(x+y)^2 = x^2+2xy+y^2}$$

squaring a binomial

Recall BEDMAS! On the L.S., addition is performed before squaring! $\therefore (x+y)^2 \neq x^2+y^2$

$(m^2-n^2)^2+(2mn)^2$

$=(m^2)^2-2m^2n^2+(n^2)^2+2^2m^2n^2$

$= m^4 - 2m^2n^2+n^4+4m^2n^2$

$= m^4+2m^2n^2+n^4$

$$(m^2+n^2)^2 = (m^2)^2 + 2m^2n^2 + (n^2)^2$$
$$= m^4 + 2m^2n^2 + n^4$$

$$\therefore (m^2-n^2)^2 + (2mn)^2 = (m^2+n^2)^2$$

$\therefore$ $(m^2-n^2,\ 2mn,\ m^2+n^2)$ is a Pythagorean triple.

As shown below, a spreadsheet can be used to generate Pythagorean triples.

	A	B	C	D	E	F	G	H
1								
2								
3			*a*	*b*	*c*			
4	*m*	*n*	*m*^2-*n*^2	2*mn*	*m*^2+*n*^2		*a*^2+*b*^2	*c*^2
5	2	1	3	4	5		25	25
6	3	1	8	6	10		100	100
7	4	1	15	8	17		289	289
8	5	1	24	10	26		676	676
9	3	2	5	12	13		169	169
10	4	2	12	16	20		400	400
11	5	2	21	20	29		841	841
12	4	3	7	24	25		625	625
13	5	3	16	30	34		1156	1156
14	6	3	27	36	45		2025	2025
15	7	3	40	42	58		3364	3364
16								

(d) **Restrictions on m and n**

a, b and c must all be positive

$$\underbrace{m^2-n^2}\qquad 2mn \qquad m^2+n^2$$

will be negative or 0 if $m \leq n$

$\therefore$ $m > n$ (m must be greater than n)