

43. $\left(\frac{2x^{-1}}{y^{-3}}\right)^{-2}$

~~$\frac{4x^2}{y^6}$~~

~~$\frac{2x^2}{y^6}$~~

$\frac{x^2}{4y^6}$ ✓

(Answer to # 43)

$$= \frac{(2x^{-1})^{-2}}{(y^{-3})^{-2}}$$

$$= \frac{2^{-2}(x^{-1})^{-2}}{y^6}$$

$$= \frac{\frac{1}{2^2} x^2}{y^6}$$

$$= \frac{\frac{1}{4} x^2}{y^6}$$

$$= \frac{1}{4} \left(\frac{x^2}{1} \right) \div \frac{y^6}{1}$$

$$= \frac{x^2}{4} \times \frac{1}{y^6} = \frac{x^2}{4y^6}$$

* Please Note *

- No shortcuts are used in these solutions
- The use of shortcuts tends to interfere with the understanding of important mathematical principles
- Focus on the meaning of the expressions

Multiplying by $\frac{1}{4}$ is
 * equivalent to dividing by 4. *
 * This is the REASON that, *
 * the "4" ends up appearing *
 in the denominator.

$$\begin{aligned}
 32. \quad \frac{-22w^{-4}}{-11w^{-3}} &= \left(\frac{-22}{-11}\right) \left(\frac{w^{-4}}{w^{-3}}\right) \xrightarrow{-4+3} \\
 &= 2w^{-4-(-3)} \\
 &= 2w^{-1} \\
 &= \underline{2} \left(\frac{1}{w^1}\right) \\
 &= \frac{2}{w}
 \end{aligned}$$

~~$$\begin{aligned}
 2\left(\frac{1}{w}\right) \\
 2\left(\frac{1}{w}\right)
 \end{aligned}$$~~

$$\begin{aligned}
 18. \quad 2^{-1} + 4^{-1} &= \frac{1^{\times 2}}{2^{\times 2}} + \frac{1}{4} \\
 &= \frac{2}{4} + \frac{1}{4} \\
 &= \frac{3}{4}
 \end{aligned}$$

$$52. (ab^{-1})^2 - ab(-ab^{-3})$$

$$= a^2(b^{-1})^2 - a(-a)b^1b^{-3}$$

$$= a^2b^{-2} - (-a^2)b^{-2}$$

$$= \boxed{a^2b^{-2}} + \boxed{a^2b^{-2}} \quad \text{Like Terms!}$$

$$= 2a^2b^{-2}$$

$$= \frac{2a^2}{b^2}$$

$$\begin{aligned}
 53. \quad \frac{a^3 b^{-2}}{a^{-1}} + \left(\frac{b^6 a^{-2}}{b^5} \right)^{-2} &= \left(\frac{a^3}{a^{-1}} \right) \left(\frac{b^{-2}}{1} \right) + \frac{(b^6)^{-2} (a^{-2})^{-2}}{(b^5)^{-2}} \\
 &= a^4 b^{-2} + \frac{b^{-12} a^4}{b^{-10}} \\
 &= \frac{a^4}{b^2} + \left(\frac{b^{-12}}{b^{-10}} \right) \left(\frac{a^4}{1} \right) \\
 &= \frac{a^4}{b^2} + b^{-2} a^4 \\
 &= \frac{a^4}{b^2} + \frac{a^4}{b^2} = \frac{2a^4}{b^2}
 \end{aligned}$$

$$\left(\frac{a}{b} \right)^x = \frac{a^x}{b^x}$$

$$\begin{aligned}
 \left(\frac{a}{b} \right)^2 &= \left(\frac{a}{b} \right) \left(\frac{a}{b} \right) \\
 &= \frac{a^2}{b^2}
 \end{aligned}$$

$$52. (ab^{-1})^2 - ab(-ab^{-3})$$

$$= a^2(b^{-1})^2 - [-a(a)(b)(b^{-3})]$$

$$= a^2b^{-2} - (-a^2b^{-2})$$

$$= a^2b^{-2} + a^2b^{-2}$$

$$= 2a^2b^{-2} \quad \leftarrow \frac{2a^2}{1} \left(\frac{1}{b^2} \right)$$

$$= \frac{2a^2}{b^2}$$

Multiplying
by $\frac{1}{b^2}$ is the
same as
dividing by b^2

$$53. \frac{a^3b^{-2}}{a^{-1}} + \left(\frac{b^6a^{-2}}{b^5} \right)^{-2}$$

$$= \frac{a^3(\frac{1}{b^2})}{(\frac{1}{a})} + \frac{(b^6)^{-2}(a^{-2})^{-2}}{(b^5)^{-2}}$$

$$= \frac{a^3}{b^2} \div \frac{1}{a} + \frac{b^{-12}a^4}{b^{-10}}$$

$$= \frac{a^3}{b^2} \times \frac{a}{1} + \frac{\frac{1}{b^{12}}(\frac{a^4}{1})}{(\frac{1}{b^{10}})}$$

$$= \frac{a^4}{b^2} + \frac{a^4}{b^{12}} \div \frac{1}{b^{10}}$$

$$= \frac{a^4}{b^2} + \frac{a^4}{b^{12}} \times \frac{b^{10}}{1}$$

$$= \frac{a^4}{b^2} + \frac{a^4b^{10}}{b^{12}} \quad \leftarrow \frac{a^4}{1} \left(\frac{b^{10}}{b^{12}} \right)$$

$$= \frac{a^4}{b^2} + a^4b^{10-12}$$

$$= \frac{a^4}{b^2} + a^4b^{-2} \quad \leftarrow \frac{a^4}{1} \left(\frac{1}{b^2} \right)$$

$$= \frac{a^4}{b^2} + \frac{a^4}{b^2}$$

$$= \frac{2a^4}{b^2}$$

Common denominator!
Add the numerators,
don't change the
denominator!

54. $\left(\frac{x^{-3}y^{-1}}{2x}\right)^{-3} + \left(\frac{6x^9y^3}{-3x^{-3}}\right)$ ← Technically, the second term should be enclosed in brackets.

$$= \frac{(x^{-3}y^{-1})^{-3}}{(2x)^{-3}} + \left(\frac{6}{-3}\right)\left(\frac{x^9}{x^{-3}}\right)\left(\frac{y^3}{1}\right)$$

$$= \frac{x^{-3(-3)}y^{-1(-3)}}{2^{-3}x^{-3}} + (-2)x^{9-(-3)}y^3$$

$$= \frac{1x^9y^3}{\frac{1}{2^3}x^{-3}} - 2x^{12}y^3$$

$$= \left(\frac{1}{1} \div \frac{1}{8}\right)\left(\frac{x^9}{x^{-3}}\right)\left(\frac{y^3}{1}\right) - 2x^{12}y^3$$

$\frac{1}{1} \times \frac{8}{1} = \frac{8}{1} = 8$
Dividing by $\frac{1}{8}$ is the same as multiplying by 8.

$$= 8x^{12}y^3 - 2x^{12}y^3$$

LIKE TERMS!!

$$= 6x^{12}y^3$$