2. Victoria can finish typing an essay in five hours while Jenny can type the same essay in eight hours. How long will it take them to type the essay if they work together?

Solution

one unknown only → time required to type the essay if the two girls work together

Let t represent this time (in hours)

Time (h)	Fraction of essay done by V	Fraction done by J	Together
1	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{5}+\frac{1}{8}=\frac{13}{40}$
t	$\frac{t}{5}=\frac{1}{5}t$	$\frac{t}{8}=\frac{1}{8}t$	$\frac{13}{40}t$

fraction of essay done when completed $= \frac{40}{40} = 1$

$$\frac{13}{40}t = 1$$

$$\frac{40}{13}\left(\frac{13}{40}t\right) = \frac{40}{13}(1)$$

Dividing B.S. by $\frac{13}{40}$ is equivalent to multiply B.S. by the reciprocal $(\frac{40}{13})$

$$t = \frac{40}{13} \text{ hours}$$

$$\therefore t = 3\frac{1}{13} \text{ hours}$$

$$= 3 \text{ hours} + \frac{1}{13} \text{ hour}$$

$$= 3\,h + \frac{1}{13}\,h\,(60 \text{ min}/h)$$

$$\doteq 3\,h,\ 5 \text{ minutes}$$

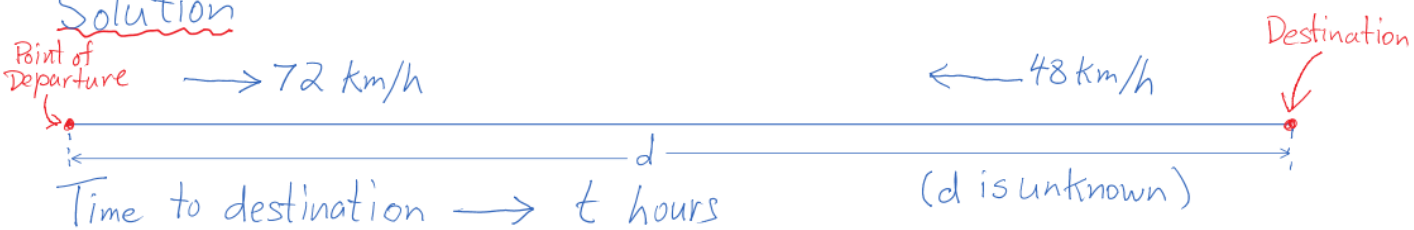
$\frac{40}{13} \doteq 3.08\ h$

$8 \text{ min} = \frac{8}{60}\,h$

$\doteq 3\,h + \frac{8}{100}\,h$

1. A train travelling nonstop to its destination makes the trip at an average speed of 72 km/h. On the return trip, the train makes several stops and is only able to average 48 km/h. If the return trip takes two hours longer than the initial trip to the destination, then what is the travel time each way?

Solution



Time to destination $\longrightarrow$ t hours

Return trip time $\longrightarrow$ $t+2$ hours

(d is unknown)

$$V = \frac{d}{t} \quad \frac{km}{h}$$

$$\therefore \; d = vt$$

$d = 72t$ (to destination)

$d = 48(t+2)$ (return trip)

Since the return trip distance equals the distance to the destination,

$$\therefore \; 72t = 48(t+2)$$

$$\therefore \; 72t = 48t + 96$$

$$\therefore \; 24t = 96$$

$$\therefore \; t = \frac{96}{24} = 4 \quad \text{(initial trip to destination)}$$

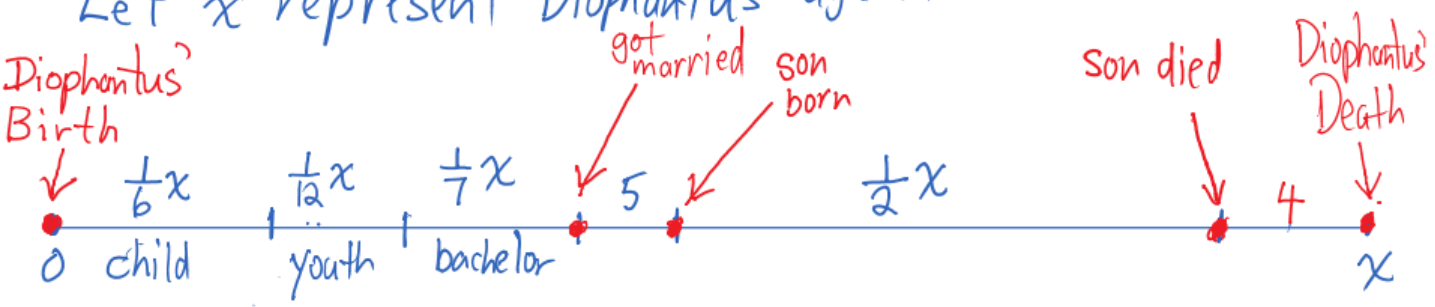
$$\therefore \; t+2 = 4+2 = 6 \quad \text{(return trip)}$$

The initial trip to the destination took 4 h and the return trip took 6 h.

2. Diophantus of Alexandria was a Greek mathematician who lived between 200 AD and 300 AD. He was a child for one-sixth of his life, a youth for one-twelfth of his life and a bachelor for one-seventh more. Five years after he married, his son was born. Diophantus' son died four years before his father at half his father's final age. How old was Diophantus when he died?

Solution

Let x represent Diophantus' age at the time he died



$$\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x$$

3. Basmati rice costs \$4.50/kg while wild rice costs \$5.40/kg. In what ratio should the basmati rice be mixed with wild rice to create a blend that costs \$5.00/kg?

Solution

1 kg altogether

	Fraction of Mixture	Cost of Fraction
Basmati	$1-v$	$4.5(1-v)$
Wild	v	$5.4v$
Total	1	$5(1)=5$

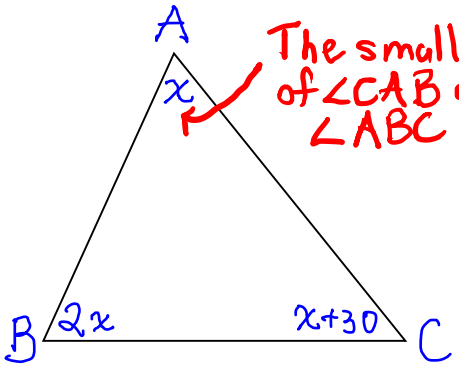
Wild v
Basmati $1-v$

Cost of Basmati + Cost of Wild = Total Cost

$$4.5(1-v) + 5.4v = 5$$

Solutions: TIPS Problems Involving Equations

1. In ΔABC, the measure of $\angle ABC$ is double the measure of $\angle CAB$. The measure of $\angle BCA$ is 30° greater than the measure of the smaller of the other two angles. Find the measure of each angle.



The smaller of $\angle CAB$ and $\angle ABC$

Let x represent the measure of $\angle CAB$.
Then, $2x$ represents the measure of $\angle ABC$
and $x+30$ represents the measure of $\angle BCA$.

(The sum of the interior angles of a triangle) is 180°

$$x + 2x + x + 30 = 180$$
$$\therefore 4x + 30 = 180$$
$$\therefore 4x + 30 - 30 = 180 - 30$$
$$\therefore 4x = 150$$
$$\therefore \frac{4x}{4} = \frac{150}{4}$$
$$\therefore x = 37.5$$

$\angle CAB = 37.5^\circ$
$\angle ABC = 2(37.5^\circ) = 75^\circ$
$\angle BCA = 37.5^\circ + 30 = 67.5^\circ$

2. Naquan is saving nickels and dimes in a jar. The jar contains 10 more nickels than dimes. Altogether, the value of the coins is $16.25. How many nickels and dimes are in the jar?

Coin	*Value of One Coin*	*Number of Coins*	*Value of Coins*
Dime	$0.10	d	$0.10d$
Nickel	$0.05	$d+10$	$0.05(d+10)$
Total	N/A	$d+d+10$	$16.25

Not relevant to solving this problem

(Value of dimes) + (Value of nickels) is 16.25

$$0.10d + 0.05(d+10) = 16.25$$
$$\therefore 0.10d + 0.05d + 0.5 = 16.25$$
$$\therefore 0.15d + 0.5 - 0.5 = 16.25 - 0.5$$
$$\therefore \frac{0.15d}{0.15} = \frac{15.75}{0.15}$$
$$\therefore d = 105$$
$$\therefore d + 10 = 115$$

Nickel = $5^\text{¢}$ =

Dime = $10^\text{¢}$ =

Conclusion
There are 105 dimes and 115 nickels in the jar.

Check
$105(0.10) + 115(0.05) = 10.5 + 5.75$
$= 16.25$ ✓

3. Solution A is 50% hydrochloric acid by volume, while solution B is 75% hydrochloric acid by volume. How many litres of each solution should be used to make 100 litres of a solution which is 60% hydrochloric acid by volume?

"a" litres of solution A
100 litres altogether
∴ 100 − a litres of solution B

Solution (of Problem)

Solution	*% of acid*	*Volume of the Solution (L)*	*Volume of Acid in the Solution (L)*
Solution A	50% = 0.5	a	$0.5a$
Solution B	75% = 0.75	$100-a$	$0.75(100-a)$
Mixture	60% = 0.6	100	$0.6(100)=60$

HCl = Hydrochloric Acid

(Volume of HCl in Solution A) + (Volume of HCl in Solution B) = (Total Volume of HCl in the mixture)

$$0.5a + 0.75(100-a) = 60$$

$$\therefore 0.5a + 75 - 0.75a = 60$$

$$\therefore -0.25a + 75 - 75 = 60 - 75$$

$$\therefore \frac{-0.25a}{-0.25} = \frac{-15}{-0.25}$$

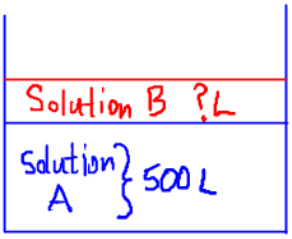
$$\therefore a = 60$$

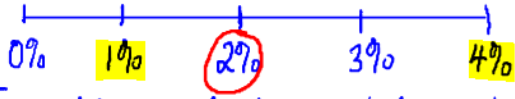
$$\therefore 100 - a = 100 - 60 = 40$$

The mixture should contain 60 L of solution A and 40 L of solution B.

4. To make lower-fat chocolate frozen yogurt, chocolate milk containing 2% butterfat is needed. To obtain the required percentage of butterfat, chocolate milk containing 4% butterfat is mixed with 500 litres of chocolate milk containing 1% butterfat.

(a) Without performing any calculations, predict whether the mixture will contain more of the 1% chocolate milk or more of the 4% chocolate milk. Explain.





I would expect the mixture to contain more of the 1% milk because 1% is closer to 2% than 4% is

(b) How many litres of the 4% chocolate milk are needed to create the required mixture? What is the total volume of the mixture?

Solution

Let x represent the number of litres of the 4% chocolate milk that must be mixed with 500 L of the 1% chocolate milk to produce a mixture containing 2% butterfat.

Type of Chocolate Milk	*Volume (L)*	*Amount of Butterfat in Given Volume (L)*
1% butterfat	500	$0.01(500) = 5$
4% butterfat	x	$0.04x$
2% butterfat mixture	$x+500$	$0.02(x+500)$

(Volume of butterfat in mixture from 1% milk) + (Volume of butterfat in mixture from 4% milk) = (Total volume of butterfat in the mixture)

$$5 + 0.04x = 0.02(x+500)$$

$$\therefore 5 + 0.04x = 0.02x + 10$$

$$\therefore 5 + 0.04x - 0.02x - 5 = 0.02x + 10 - 0.02x - 5$$

$$\therefore 0.02x = 5$$

$$\therefore \frac{0.02x}{0.02} = \frac{5}{0.02}$$

$$\therefore x = 250$$

To create the required mixture, 250 L of 4% chocolate milk must be added to 500 L of 1% chocolate milk, for a total volume of 750 L.

(c) Does your answer in part (b) agree with the prediction that you made in part (a)? What can you conclude from this?

My answer agrees with the prediction from part (a), which means that it is likely to be correct.

More TIPS Problems Involving Equations

Important Points to Keep in Mind

- There is a great deal of information that is embedded within the statement of a problem.
- Part of the statement will tell you how the unknowns are related to each other.
- Another part will tell you how to write an equation relating the unknowns.
- Yet another part will tell you what you are required to find.

1. In a two-digit number, the tens digit is four times the units digit. When the digits are reversed, the new number formed is fifty-four less than the original number. Find the original number.

tens, ones

a | b

$4x$ | x

value $= 10(4x) = 40x$

value $= 1x = x$

Let x represent the units (ones) digit of the original #.

	b	a
digit	x	$4x$
value	$10x$	$4x$

value new # = value original # − 54

$10x + 4x = 40x + x - 54$

2. Anil is nine years older than Amandeep. In ten years, Anil will be twice as old as Amandeep was ten years ago. Find their present ages.

Unknowns

Anil's age → a

Amandeep → $a - 9$

Equation

Anil's age in 10 years will be 2 times (Amandeep's age 10 years ago)

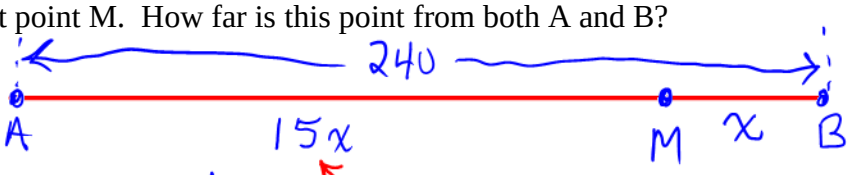
$a + 10 = 2(a - 9 - 10)$

3. The distance between two places A and B is 240 km. A motorcyclist starts from A at the same time that a pedestrian starts from B. The speed of the motorcyclist is fifteen times that of the pedestrian. Three hours after they start, they meet at point M. How far is this point from both A and B?

Hint: $v = \frac{d}{t}$

$\therefore d = vt$

and $t = \frac{d}{v}$



dist. = dist.

$15x + x = 240$

Motorcyclist moving 15x faster ∴ covers 15x distance pedestrian travels in same time.

4. Three integers are unknown but are related to each other. The second integer is three times the first integer and one-fifth of the third integer. If the sum of the three integers is 266, find the value of each integer.

x → smallest → $\frac{1}{15}\ell$ $\frac{1}{3}m$

$3x$ → middle → $\frac{1}{5}\ell$ m

$5(3x) = 15x$ → largest → ℓ $5m$

$x + 3x + 15x = 266$

Answers

1. 82, 28 **2.** ~~12, 21~~ 39, 48 **3.** 225 km, 15 km **4.** 14, 42, 210